Anglo - Chinese School
(Independent)

FINAL EXAMINATION 2015
YEAR THREE EXPRESS
ADDITIONAL MATHEMATICS
PAPER 1

Wednesday **30 September 2015** **1 hour 30 minutes**

Additional Materials: Answer Paper (7 Sheets)

READ THESE INSTRUCTIONS FIRST

Write your index number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

Answer all the questions.

1 A cylinder with volume v cm^3 has base radius $(3+2\sqrt{3})$ cm and height $(5\sqrt{3}+\frac{1}{\sqrt{3}})$ cm.

Express $\frac{v}{\pi}$ in the form $a+b\sqrt{3}$ where a and b are integers. [4]

2 **(i)** Find the coordinates of the point of intersection of $y=3x^{\frac{1}{2}}$ and $y=\frac{5}{x^2}$ for $x \geq 0$. [2]

(ii) Sketch the graph of $y=\frac{5}{x^2}$ and $y=3x^{\frac{1}{2}}$ on the same diagram, indicating the point of intersection and any intercepts. [2]

3 The curves $y=\frac{1}{3x}$ and $2x-15xy+6y=0$ cut at two distinct points *A* and *B*. Find the equation of the perpendicular bisector of *AB*. [6]

4 The equation $2x^2-3x=6$ has roots α and β. Find

(i) the value of $\alpha+\beta$ and $\alpha\beta$, [2]

(ii) the quadratic equation whose roots are $\alpha^3\beta$ and $\alpha\beta^3$. [4]

5 Given the function $f(x)=4\sin 2x+2$,

(i) state the period and the amplitude of the function, [2]

(ii) solve the equation $4\sin 2x+2=0$ for $0° \leq x \leq 180°$, [3]

(iii) sketch the graph of $f(x)=|4\sin 2x+2|$ for $0° \leq x \leq 180°$, showing the intercepts clearly. [2]

6 **(a)** Find the range of values of k for which $x(x+k)>k$ for all real values of x. [3]

(b) Find the range of values of *p* for which the line $y=p+2x$ intersect the curve $x^2+y^2=5$ at two distinct points. [4]

7 The equation of a circle, C_1, is $x^2+y^2-10x-6y-15=0$.

(i) Find the coordinates of the centre and the radius of C_1. [3]

(ii) Show that the point (-1, 2) lies inside the circle C_1. [2]

(iii) Find the equation of the circle C_2 which is a reflection of C_1 in the line $x=-1$. [2]

8 Solve the following equations.

(a) $\log_3(18-7x) - \log_3 2 = \log_{\sqrt{3}}(x-2)$. [4]

(b) $e^{x-1} + 4 = 21\left(\frac{1}{e}\right)^{x-1}$ [4]

9 **(a)** **(i)** Solve the equation $3x^3 - 2x^2 - 7x - 2 = 0$. [4]

(ii) Hence, find the values of θ between 0^o and 360^o inclusive such that $\sin^2\theta(3\sin\theta - 2) = 7\sin\theta + 2$. [3]

(b) The expressions $x^3 + 14x + 6$ and $x^3 - 2x^2 - 8x - 30$ leave the same remainder when divided by $x - a$. Find the possible values of a. [4]

_____END OF PAPER 1 _____

Answers

1 $112\sqrt{3} + 192$

2(i) $x = 1.23$ $y = 3.32$

3 $(2, \frac{1}{6})$, $(\frac{1}{2}, \frac{2}{3})$

Equation: $y = 3x - \frac{10}{3}$

4(i) $\alpha + \beta = \frac{3}{2}$, $\alpha\beta = -3$ (ii) $4x^2 + 99x + 324 = 0$

5(i) amplitude = 4 Period = 180°

(ii) $x = 105, 165$

6(i) $-4 < k < 0$ (ii) $-5 < p < 5$

7(i) Centre = (5, 3)
Rad = 7

(iii) $x^2 + y^2 + 14x - 6y + 9 = 0$

8(i) $x = \frac{5}{2}$

(ii) $x = 2.10$

9(a)(i) $x = -1, -\frac{1}{3}, 2$ (ii) $\theta = 199.5°, 270°, 340.5°$

(b) $a = -2$ *or* -9