**JURONG PIONEER JUNIOR COLLEGE**
**9749 H2 PHYSICS**

# CURRENT OF ELECTRICITY

## Content



## Learning Outcomes

Students should be able to:

(a) show an understanding that electric current is the rate of flow of charge.

(b) derive and use the equation $I = nAvq$ for a current-carrying conductor, where $n$ is the number density of charge carriers and $v$ is the drift velocity.

(c) recall and solve problems using the equation $Q = It$.

(d) recall and solve problems using the equation $V = \frac{W}{Q}$.

(e) recall and solve problems using the equations $P = VI$, $P = I^2R$ and $P = \frac{V^2}{R}$.

(f) define the resistance of a circuit component as the ratio of the potential difference across the component to the current passing through it and solve problems using the equation $V = IR$.

(g) sketch and explain the $I$-$V$ characteristics of various electrical components such as an ohmic resistor, a semiconductor diode, a filament lamp and a negative temperature coefficient (NTC) thermistor.

(h) sketch the resistance-temperature characteristic of an NTC thermistor.

(i) recall and solve problems using the equation $R = \frac{\rho l}{A}$.

(j) distinguish between electromotive force (e.m.f.) and potential difference (p.d.) using energy considerations.

(k) show an understanding of the effects of the internal resistance of a source of e.m.f. on the terminal potential difference and output power.

## Introduction

- In everyday life, we are familiar with electric currents flowing in wires and other conductors. Electric current is said to be set up along a conductor when there is a flow of charges in a particular direction.
- We first examine electric current from a macroscopic point of view: i.e. current measured using laboratory apparatus. Then, we examine it from a microscopic point as flows of electrons and their drift velocity.
- Fig. 1.1 shows a circuit diagram where a direct current is flowing, driven by a cell. It also shows electrons in the wire moving away from the negative terminal (since electrons are negatively-charged and we know that like charges repel) and move towards the positive terminal (since unlike charges attract) of the cell.

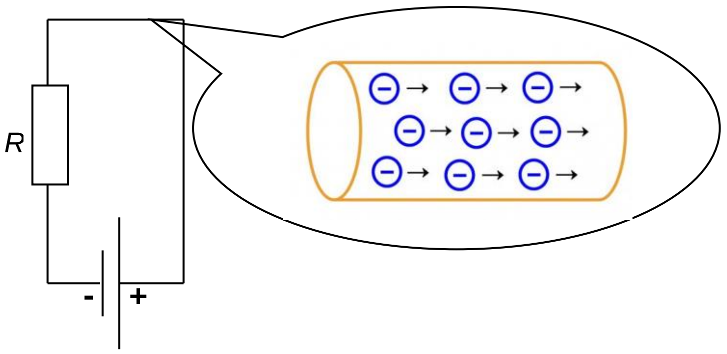



**Fig. 1.1** Electron movement in an electric circuit

<u>Note:</u>

❖ Scientists first thought that positive charges flow from the positive terminal of a cell to the negative terminal. This is the **conventional current** direction. The direction of conventional current is thus the direction of the flow of positive charges in a circuit.

❖ However, it was later found that a current in a metal wire is in fact a flow of negatively-charged electrons in the opposite direction. That is, electrons flow from the negative (−) to the positive (+) terminal <u>outside</u> the battery.

❖ Nevertheless, the conventional current is still being used.

❖ When solving problems involving circuit diagrams, it is the <u>conventional current</u> (and not the electron flow) that is indicated on the circuit diagrams. Its direction is from the positive (+) to the n(−) terminal <u>outside</u> the battery as shown in Fig. 1.2.

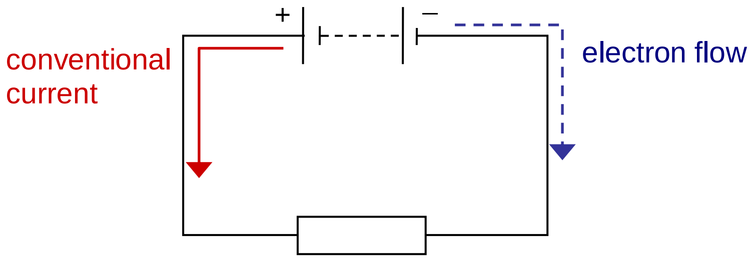



**Fig. 1.2** Conventional current in an electric circuit

## 1 Electric Current

*(a) To show an understanding that electric current is the rate of flow of charge.*
*(c) To recall and solve problems using the equation Q = It.*

### 1.1 Charge and the coulomb

- Charge $Q$ refers to a quantity of electricity.
- **Charge flowing past** a given **cross-section** of a conductor is the **product** of the **steady current and time during which** the current flows.
- The SI unit of charge is the coulomb (C).
- The **coulomb** is the **quantity of charge which passes** a given cross-section **in one second** when a **current of one ampere** is flowing, i.e. **1 C = 1 A s**.
- $e$ is the elementary charge, and it has a magnitude of $1.60 \times 10^{-19}$ C.
- Protons and positive ions are positively-charged particles. Charge of a proton is $+1.60 \times 10^{-19}$ C or $+e$.
- Electrons and negative ions are negatively-charged particles. Charge of an electron is $-1.60 \times 10^{-19}$ C or $-e$.

### 1.2 Flow of charged particles

- Positive particles move from a position of high potential (+) to a position of low potential (−). Negative particles move from a position of low potential (−) to a position of high potential (+). The flow of charged particles constitutes an electric current, denoted by the symbol $I$.
- The SI unit of electric current is the ampere (A).
- **The electric current $I$ is the rate of flow of charge.**

$$\boxed{I = \frac{dQ}{dt}}$$

Amount of charge $Q = \int I \, dt$ = area under $I$-$t$ graph, shown in Fig. 2.

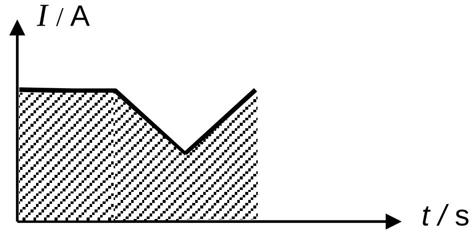



**Fig. 2** Variation of current $I$ (non-constant) with time

- A constant current exists when there is a constant rate of flow of charge.
- For constant current, $\boxed{I = \dfrac{Q}{t}}$

Amount of charge $Q = It$ = area under $I$-$t$ graph, as shown in Fig. 3.

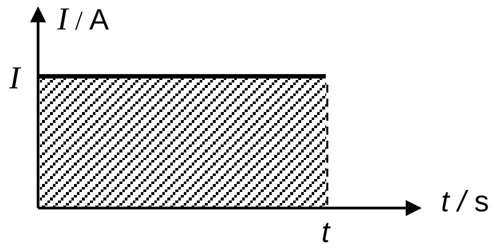



**Fig. 3** Variation of current $I$ (constant) with time

**Example 1**

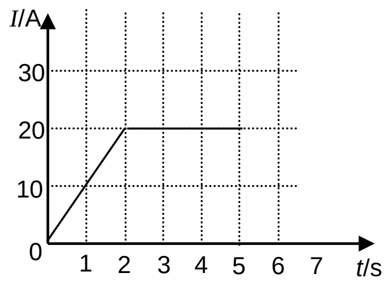
Calculate the (steady) current in a circuit when a charge of 40 C passes in 5.0 s.

**Solution:**

$$I = \frac{Q}{t} = \frac{40\ \text{C}}{5.0\ \text{s}} = 8.0\ \text{A}$$

**Example 2**

A car battery is used to supply a varying current. Calculate the total charge delivered from the battery if the variation in current supplied with time is given by the graph as shown on the right.




**Solution:**

Total charge, $Q$ = area under $I$-$t$ graph

$$Q = \frac{1}{2}(3 + 5) \times 20 = 80\ \text{C}$$

- For a constant current, $I = \dfrac{Q}{t}$.
- If $N$ is the number of charged particles passing a cross-section of a conductor in time $t$, and $q$ is the charge of each charged particle, the total charge flow is $Q = Nq$.
- Thus, $I = \dfrac{Q}{t} = \dfrac{Nq}{t} = \left(\dfrac{N}{t}\right)q$.
- $\dfrac{N}{t}$ is the number of charged particles per unit time passing through a cross-section of the conductor.

**Example 3**

Determine the number of electrons are passing through a wire per second if the current is 1.00 mA?

**Solution:**

$I = \frac{Ne}{t}$ → $\frac{N}{t} = \frac{I}{e}$

$= \frac{1.00 \times 10^{-3}}{1.60 \times 10^{-19}}$

$= 6.25 \times 10^{15}\ s^{-1}$

- Fig. 4 shows oppositely-charged particles in a gas or liquid moving in opposite directions under the influence of an electric field.

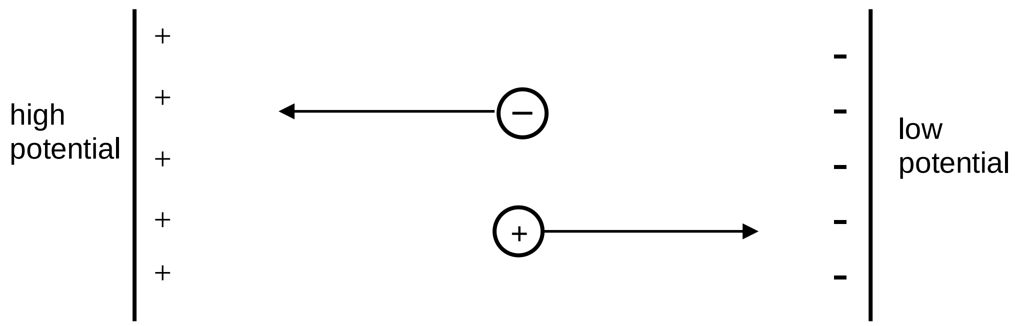



**Fig. 4** Charged particle movement in a gas or liquid, under influence of electric field

**Example 4**

A high potential is applied between the electrodes of a hydrogen discharge tube so that the gas is ionised. Electrons then move towards the positive electrode and protons towards the negative electrode. In each second, $5.0 \times 10^{18}$ electrons and $2.0 \times 10^{18}$ protons pass a cross-section of the tube. Determine the current flowing in the discharge tube.

**Solution:**

Total number of charged particles passing in 1 s, $\frac{N}{t} = (5.0 + 2.0) \times 10^{18} = 7.0 \times 10^{18}\ s^{-1}$

Current passing the tube, $I = \frac{Q}{t}$

$= \frac{Ne}{t}$

$= (7.0 \times 10^{18})(1.60 \times 10^{-19})$

$= 1.12$ A

### 1.2.1 Flow of charges in a metallic conductor

- Fig. 5 shows free (mobile) electrons within the lattice structure of a metallic conductor, e.g. a copper wire.

- The electrons wander randomly and haphazardly among the metal atoms/ions of the lattice structure.

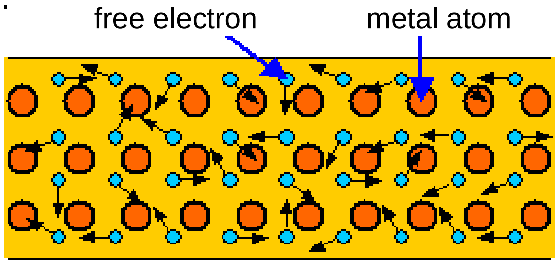



**Fig. 5** Free electrons within the lattice structure of a metallic conductor

- The <u>net</u> flow of charge is <u>zero</u> and thus, there is no current.

- Focusing on a single electron, its net displacement after multiple collisions with the metal atoms is also zero, as shown in Fig. 6.

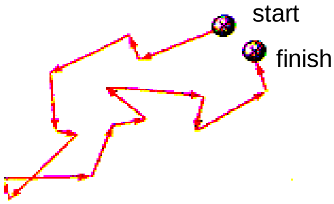



**Fig. 6** Zero net displacement of an electron after multiple collisions

- If the copper wire is connected to the positive and negative terminals of a cell, the cell provides an electric field through the wire.

- Under the influence of this electric field, the free electrons in the wire accelerate, and are attracted or 'drift' towards the region of high potential (i.e. the positive terminal).

- As they move, the electrons encounter lattice ions, and are deflected repeatedly. The resultant motion is slow, intermittent and jerky, as shown in Fig. 7.

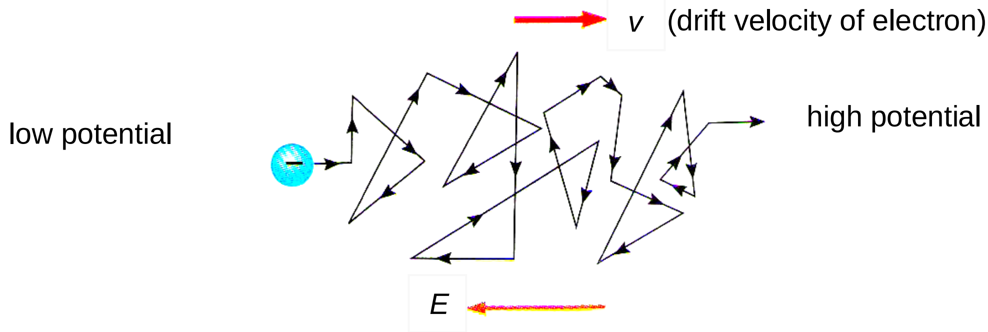



**Fig. 7** Effect of electric field and lattice ions on electron movement

- However, there is a net movement of the electrons to the right, resulting in a current being present. The velocity at which the electrons drift toward the higher potential is known as the <u>drift velocity</u> of the electrons.

### 1.2.2 Derivation of $I = nAvq$

*(b) To derive and use the equation $I = nAvq$ for a current-carrying conductor, where $n$ is the number density of charge carriers and $v$ is the drift velocity.*

- Fig. 8 shows a section of a current-carrying conductor of cross-sectional area $A$.

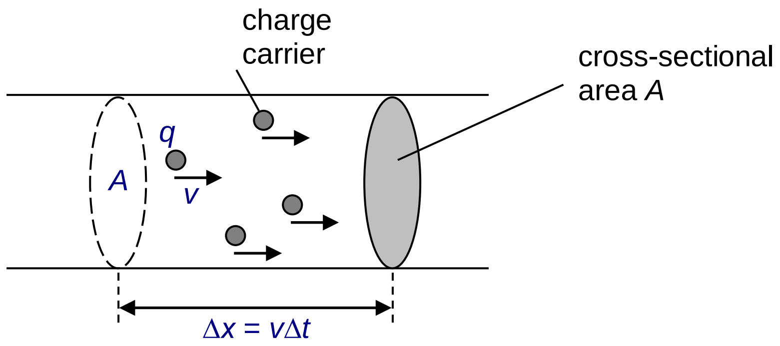



**Fig. 8** A section of a current-carrying conductor

- The charge carriers are moving to the right with drift velocity $v$.
- In a time interval $\Delta t$, they have a displacement $\Delta x = v\Delta t$ to the right.
- All the charge carriers within the volume of conductor of length $\Delta x$ will cross the shaded cross-sectional area $A$ during the time $\Delta t$, as shown in Fig. 8.
- The volume of a section of the conductor of length $\Delta x$ is $V = A\Delta x$.
- If $n$ is the number density of charge carriers (i.e. the number of charge carriers per unit volume, $n = \frac{N}{V}$), the number of charge carriers in this section $N = nV = nA\Delta x$.
- Therefore, the total charge $\Delta Q$ in this section is

$$\begin{aligned}\Delta Q &= \text{number of charge carriers in section} \times \text{charge per carrier} \\ &= Nq \\ &= (nA\Delta x)q \\ &= (nAv\Delta t)q\end{aligned}$$

- If we divide both sides of this equation by $\Delta t$, the current in the conductor is

$$\boxed{I = \frac{\Delta Q}{\Delta t} = nAvq}$$

**Example 5**

The electric current through a cylinder of semiconductor material is supplied through copper wires, as shown.


The charge carriers in the semiconductor material are electrons. The semiconductor material and the copper have the same cross-sectional area.

The table gives details of the number of free electrons per cubic metre in each material.

| | copper wire | semiconductor |
|---|---|---|
| number of free electrons per cubic metre | $8.6 \times 10^{28}$ | $4.3 \times 10^{21}$ |

The mean speed of electrons through the copper wire is 0.58 mm $s^{-1}$.

Calculate the mean speed of electrons through the semiconductor material.

**Solution:**

The same current flows through the copper and the semiconductor.

$I = nAvq$

$nAvq$ for copper = $nAvq$ for semiconductor

$(8.6 \times 10^{28})A(0.58 \times 10^{-3})q = (4.3 \times 10^{21})Avq$

$v = 1.16 \times 10^{4}$ m $s^{-1}$

## 2 Potential Difference

*(d) To recall and solve problems using the equation* $V = \frac{W}{Q}$.

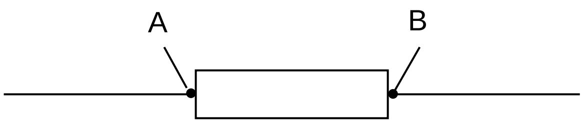
- The **potential difference (p.d.) between two points** in a circuit is the **energy converted from electrical to other forms per unit charge** passing from one point to the other.
- The SI unit of potential difference is the volt (V).
- **One volt (V)** is the potential difference between two points in a circuit in which **one joule** of energy is **converted** when **one coulomb** passes from one point to the other.
- Potential difference is denoted by $V$. It can be expressed as:

  $$\boxed{V = \frac{W}{Q}}$$

  where

  $W$ is the energy converted from electrical to other forms, and

  $Q$ is the amount of charge moved from one point to the other.
- Since potential difference refers to two points in a circuit, the potential difference can only be used if the two points are clearly stated (e.g. the potential difference between points A and B, i.e. $V_{AB}$).
- For a single component (e.g. a resistor) in a circuit, the two points A and B are normally <u>immediately before</u> the component and <u>immediately after</u> the component.




*Self-attempt question:*

A potential difference of 3.0 V is applied across the above resistor, and 30.0 mA of current is allowed to flow through the resistor for 2.0 s.

What is the amount of electrical energy converted to thermal energy in the resistor in this 2.0 s?

Answer:

$$\begin{aligned} W &= VQ = V \times It \\ &= (3.0)\left(30.0 \times 10^{-3}\right)(2.0) \\ &= 0.180 \text{ J} \end{aligned}$$

**Example 6**

A current of 0.50 A flows through a conductor in 100 ms and produces 0.10 J of heat.

Calculate the potential difference across the conductor?

**Solution:**

Using $Q = It$

Amount of charge flowing through the conductor $= (0.50)(100 \times 10^{-3})$
$= 0.050\ \text{C}$

Using $V = \dfrac{W}{Q}$

Potential difference across the conductor $= \dfrac{0.10}{0.050} = 2.0\ \text{V}$

## 3 Resistance and Resistivity

(e) *To recall and solve problems using the equations $P = VI$, $P = I^2R$* and $P = \dfrac{V^2}{R}$.

(f) define the resistance of a circuit component as the ratio of the potential difference across the component to the current passing through it and solve problems using the equation $V = IR$.

### 3.1 Resistance

- The **resistance of a resistor** is the **ratio** of the **potential difference across** the resistor **to** the **current flowing through** the resistor.

$$\boxed{R = \frac{V}{I}}$$

- The SI unit of resistance is the ohm ($\Omega$).

- The **ohm** is the resistance of a resistor through which a current of **one ampere is flowing** when the potential difference **across it is one volt**.

  **(i.e. $1\ \Omega = 1\ \text{V A}^{-1}$)**

- Fig. 9 shows a circuit used to determine the resistance of a conductor.

- A low resistance ammeter is connected in series with the conductor to measure the current $I$ through it.

- A high resistance voltmeter is connected across the conductor to measure the potential difference $V$ across it.

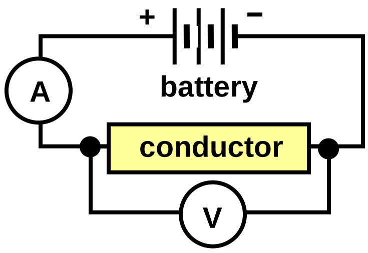
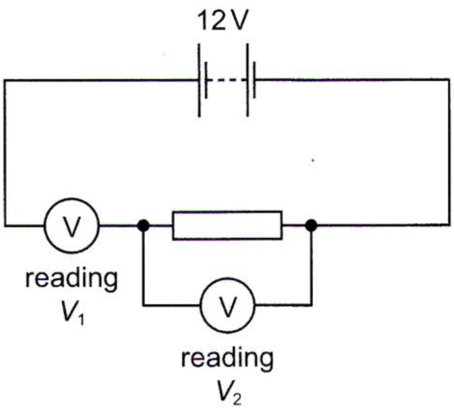



**Fig. 9** Circuit used to determine the resistance of a conductor

**Example 7** [H1 N2016/I/22]

A 12 V battery, with negligible internal resistance, is connected to a resistor and two voltmeters.




If both voltmeters are ideal, which readings do they show?

| | $V_1$ / V | $V_2$ / V |
|---|---|---|
| **A** | 0 | 12 |
| **B** | 6 | 6 |
| **C** | 12 | 0 |
| **D** | 12 | 12 |

**Solution:**

Ideal voltmeters have infinite resistance. Therefore no current flows through the resistor.

The voltmeter on the right will read $V_2 = IR = (0)R = 0$

Therefore the voltmeter on the left will read 12 V.

Answer: C

## 3.2 Electrical power

- Recall: $V = \frac{W}{Q}$ and $Q = It$

  Hence, $V = \frac{W}{It} = \frac{P}{I}$ (since $P = \frac{W}{t}$)

- The potential difference $V$ between two points in a circuit can be determined from the rate of dissipation of electrical energy per unit current flowing (or the electrical power dissipated per unit current flowing).

$$\boxed{V = \frac{P}{I}} \quad \text{or} \quad \boxed{P = VI}$$

- The above equation can be expressed as: $\boxed{P = I^2R}$ (since $V = IR$)

- Alternatively, it can be expressed as $\boxed{P = \frac{V^2}{R}}$ (since $I = \frac{V}{R}$)

**Example 8**

Calculate the current $I$, for

(a) 45 W dissipated in a resistor of p.d. 15 V across it,

(b) 250 V across a 12 MΩ resistor,

(c) 1200 W dissipated in a 600 Ω resistor.

**Solution:**

(a) $P = VI \quad \Rightarrow \quad I = \frac{P}{V} = \frac{45}{15} = 3.0\ \text{A}$

(b) $V = IR \quad \Rightarrow \quad I = \frac{V}{R} = \frac{250}{12 \times 10^6} = 2.1 \times 10^{-5}\ \text{A}$

(c) $P = I^2R \quad \Rightarrow \quad I = \left(\frac{P}{R}\right)^{\frac{1}{2}} = \left(\frac{1200}{600}\right)^{\frac{1}{2}} = 1.41\ \text{A}$

**Example 9**

Calculate the resistance $R$, in ohms, for
(a) 134 mA produced by 220 V,
(b) 800 W dissipated with 120 V.

**Solution:**

(a) $V = IR \quad \Rightarrow \quad R = \dfrac{V}{I} = \dfrac{220}{134 \times 10^{-3}} = 1540\ \Omega$

(b) $P = \dfrac{V^2}{R} \quad \Rightarrow \quad R = \dfrac{V^2}{P} = \dfrac{120^2}{800} = 18\ \Omega$

**Example 10**

A resistor of resistance $R$ has a power rating $P$ when the current in the resistor is $I$.

Calculate the resistance of a resistor that has the power rating $P$ for a current of $2I$.

**Solution:**

For first resistor: $P = I^2R$

For second resistor: $P = (2I)^2R_1$

Therefore $R_1 = \dfrac{1}{4}R$

## 3.3 Classification of materials

### 3.3.1 Conductors (e.g. metals, $CuSO_4$ solution)

- Conductors are materials that have many mobile charge carriers that will drift in a given direction to constitute an electric current under an applied electric field.
- The charge carriers in metals are electrons, those in $CuSO_4$ solution are ions.

### 3.3.2 Insulators (e.g. sulphur, rubber)

- Insulators are materials with little or no mobile charge carriers.
- The electrons are tightly bounded and not free to move about.

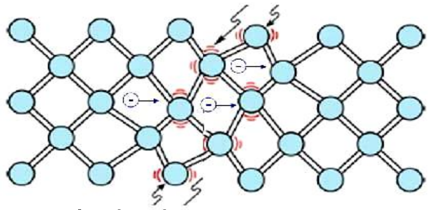
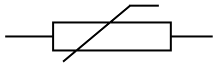
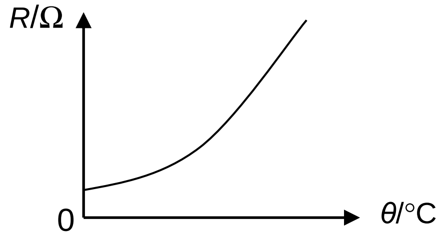
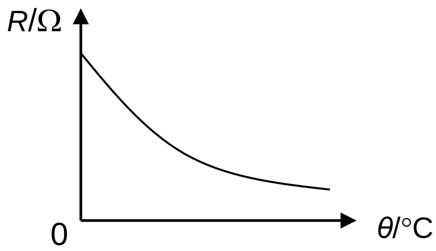
### 3.3.3 Semiconductors (e.g. silicon, germanium)

- Semiconductors are materials that have properties of conductors and insulators under different temperature conditions.
- The charge carriers in semiconductors are electrons and holes.

## 3.4 Change of resistance with temperature

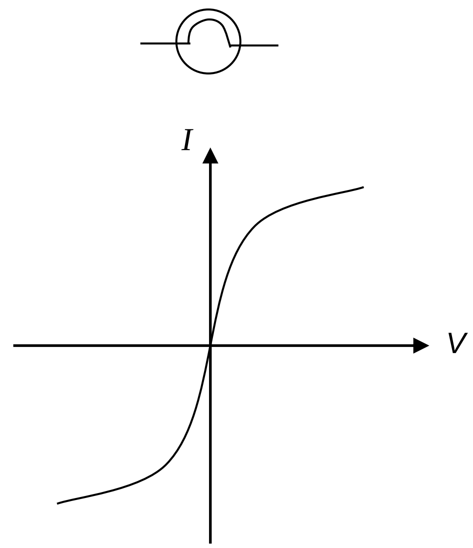
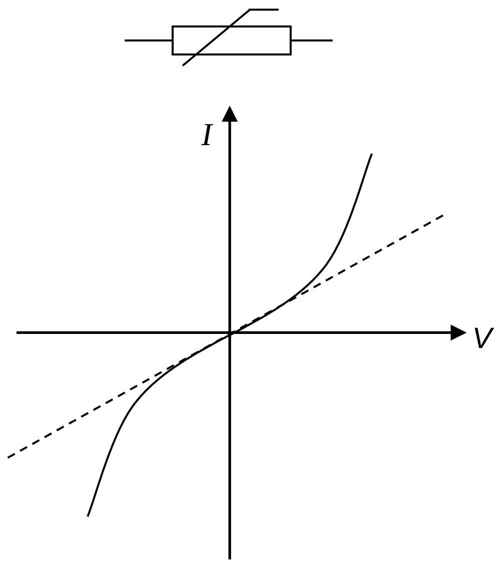
*(h) To sketch the resistance-temperature characteristic of an NTC thermistor.*

For a material with a constant applied potential difference:

| metals | semiconductors |
|---|---|
| lattice ions<br><br>lattice ions | <br>**negative temperature coefficient (NTC) thermistor**<br>(made from semiconductor) |
| ❖ Charge-carriers are mobile electrons.<br><br>❖ Increase in temperature leads to an increase in thermal energy of the lattice ions and this enables the magnitude of vibration of the lattice ions to increase.<br><br>❖ This causes the mobile electrons to collide more frequently with the lattice ions, reducing their drift velocity, which in turn reduces the current (even though number of mobile electrons has also increased, the effect is not significant).<br><br>❖ Therefore, the resistance of a metal **increases** as temperature increases. | ❖ Increase in temperature leads to an increase in thermal energy of the electrons and this enable more of them to become mobile electrons.<br><br>❖ The vacancies left behind by the newly-freed electrons behave like positive charges known as 'holes'. Both the mobile electrons and the holes are charge-carriers.<br><br>❖ As temperature increases, more electron-hole pairs are formed, leading to an increase in the number of charge-carriers.<br><br>❖ This increases the current (even though magnitude of vibration of the lattice ion has also increased, the effect is not significant).<br><br>❖ Therefore, the resistance of a semiconductor **decreases** as temperature increases. |
| <br><br>**Fig. 10** Variation of resistance with temperature for metal | <br><br>**Fig. 11** Variation of resistance with temperature for NTC thermistor |

**3.5 *I-V* graphs**

*(g) To sketch and explain the $I$-V characteristics of various electrical components such as an ohmic resistor, a semiconductor diode, a filament lamp and a negative temperature coefficient (NTC) thermistor.*

| | |
|---|---|
| **(a) Filament lamp (made of metal)**<br><br>**Fig. 12** Variation of current $I$ in a filament lamp with potential difference $V$ | **(b) Negative temperature coefficient (NTC) thermistor**<br><br>**Fig. 13** Variation of current $I$ in a thermistor with potential difference $V$ |
| ❖ As p.d. across filament increases, current $I$ increases.<br><br>❖ Rate of thermal energy dissipated increases. Hence, temperature increases.<br><br>❖ *R* increases with increase in temperature.<br><br>❖ Ratio $\frac{V}{I}$ increases and thus, ratio of $\frac{I}{V}$ decreases. | ❖ As p.d. across thermistor increases, current $I$ increases.<br><br>❖ Rate of thermal energy dissipated increases. Hence, temperature increases.<br><br>❖ *R* decreases with increase in temperature.<br><br>❖ Ratio $\frac{V}{I}$ decreases and thus, ratio of $\frac{I}{V}$ increases. |

<table>
<tr>
<td>

**(c) <u>Ohmic resistor (a resistor with constant resistance at constant temperature)</u>**

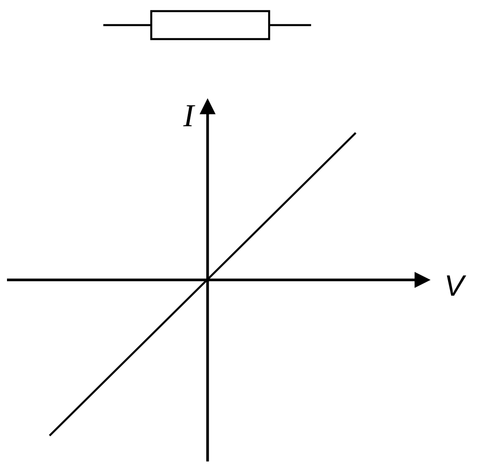



**Fig. 14** Variation of current $I$ in an ohmic resistor with potential difference $V$

</td>
<td>

**(d) <u>Semiconductor diode</u>**

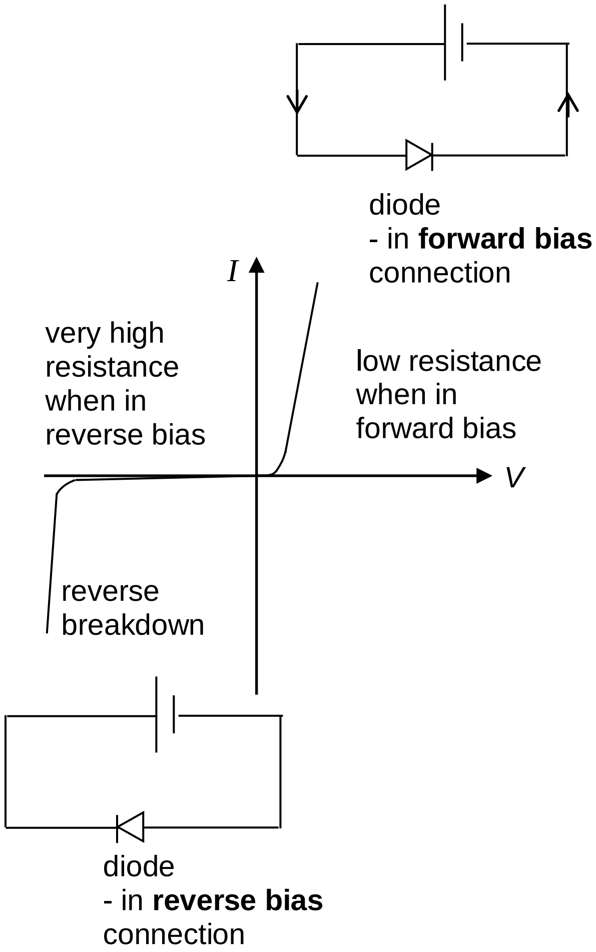



**Fig. 15** Variation of current $I$ in a semiconductor diode with potential difference $V$

</td>
</tr>
<tr>
<td>

- Ohmic resistors obey Ohm's law.
- Ohm's Law states that the current flowing in a resistor is proportional to the potential difference applied across it, provided that the physical conditions (such as temperature, stress, etc) are constant.
- Therefore $V \propto I$.
- The $I$-$V$ characteristic is a straight line through the origin.
- Ratio $\frac{V}{I}$ = resistance is constant.
- If the temperature of the resistor is constant, the magnitude of vibration of the lattice ions remains constant and hence the resistance is constant.

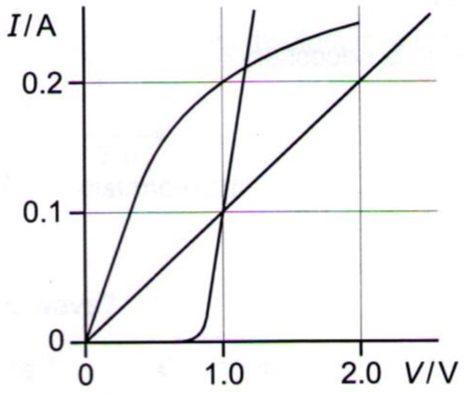
</td>
<td>

- A diode has low resistance when connected such that it is forward biased, and a very high resistance when connected such that it is reverse biased.
- When in forward bias, the $I$-$V$ characteristic is similar to that of a thermistor.
- If reverse bias potential difference exceeds its 'breakdown voltage' (e.g. 5.1 V), the diode breaks down and allows a relatively large current to flow.

</td>
</tr>
</table>

**<u>Example 11</u>** [H1 N2013/I/23]

The graph shows the $I$-$V$ characteristics of three electrical components, a diode, a filament lamp and a resistor, plotted on the same axes.




Which statement is correct?

**A** The resistance of the diode equals that of the filament lamp at about 1.2 V.

**B** The resistance of the diode is constant above 0.8 V.

**C** The resistance of the filament lamp is twice that of the resistor at 1.0 V.

**D** The resistance of the resistor equals that of the filament lamp when $V$ = 0.8 V.

**Solution:**

The $I$-$V$ characteristics of the diode and the filament lamp intersect at 1.2 V.

Since they have the same potential difference and the same current, both the diode and the filament lamp have the same resistance at 1.2 V.

Answer: A

## 3.6 Resistivity

*(i) To recall and solve problems using the equation* $R = \dfrac{\rho l}{A}$.

- The resistance of a wire $R$ is directly proportional to its length $l$, and inversely proportional to its cross-sectional area $A$. We can write the resistance of a wire in terms of the constant of proportionality called the resistivity $\rho$ of the material.

$$\boxed{R = \frac{\rho l}{A}}$$

where $R$ = resistance, $l$ = length, $A$ =area of cross-section,

$\rho$ = resistivity of the material (SI unit is Ω m).

- The table below shows the resistivity for a selection of different materials.

| | **substance** | **resistivity at 25 °C / Ω m** | **uses** |
|---|---|---|---|
| **conductors** | | | |
| metals | copper<br>gold<br>aluminium<br>tungsten | $1.72 \times 10^{-8}$<br>$2.42 \times 10^{-8}$<br>$2.82 \times 10^{-8}$<br>$5.51 \times 10^{-8}$ | connecting wires<br>microphone contacts<br>power cables<br>light-bulb filaments |
| alloys | steel<br>constantan<br>nichrome | $20 \times 10^{-8}$<br>$49 \times 10^{-8}$<br>$100 \times 10^{-8}$ | transformers<br>standard resistors<br>heating elements |
| | | | |
| **semiconductors** | carbon<br>germanium<br>silicon | $3.5 \times 10^{-5}$<br>0.60<br>2300 | resistors<br>transistors<br>transistors, chips |
| | | | |
| **insulators** | glass<br>polythene | $\sim 10^{13}$<br>$\sim 10^{14}$ | power grid insulators<br>wire insulation |

**Example 12**

The overhead wire used to supply power to a factory is made of copper of resistivity $1.72 \times 10^{-8}$ Ω m and has a cross-sectional area of $5.00 \times 10^{-5}$ m$^2$.

Calculate the resistance of one kilometre length of the wire.

**Solution:**

$$R = \frac{\rho l}{A}$$
$$= \frac{(1.72 \times 10^{-8})(1000)}{5.00 \times 10^{-5}}$$
$$= 0.344\ \Omega$$

## 4 Sources of Electromotive Force

*(j) To distinguish between electromotive force (e.m.f.) and potential difference (p.d.) using energy considerations.*

- *Recall:*

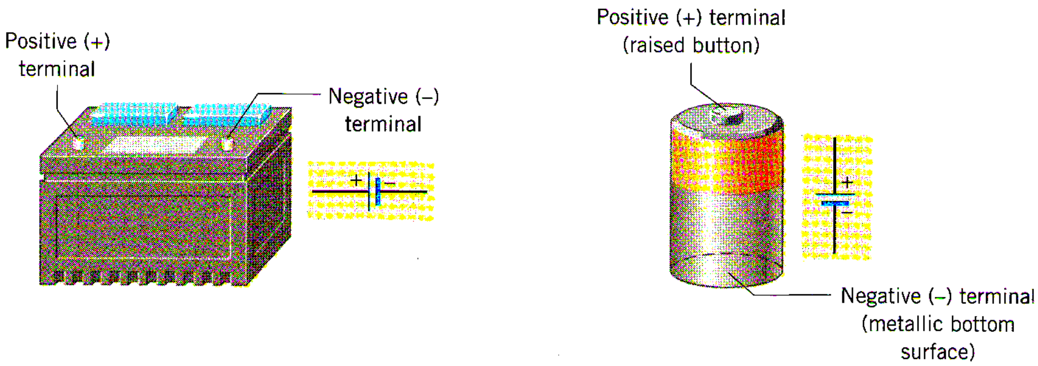
  The potential difference (p.d.) between two points in a circuit is the energy converted from electrical to other forms per unit charge passing from one point to the other.

### 4.1 Electromotive force (e.m.f.)

- The term electromotive force (e.m.f.) is used when electrical energy is being produced from other forms of energy (e.g. chemical energy).

- For example, when a battery is being used, chemical energy is converted to electrical energy and the store of chemical energy within the battery is reduced.

- Fig. 16 shows some typical batteries.




Physics. Cutnell & Johnson.

**Fig. 16** Typical batteries

- **Electromotive force (e.m.f.) of a source** is the **energy converted from other forms to electrical per unit charge** driven through the source.

- The SI unit of e.m.f. is the volt (V).

- The e.m.f. $E$ may be expressed as:

$$\boxed{E = \frac{W}{Q}}$$

where

$W$ is the energy converted from other forms to electrical, and

$Q$ is the amount of charge driven through the source (i.e. driven round the circuit).

From $\quad E = \dfrac{W}{Q}$ and $Q = It$,

hence $\quad E = \dfrac{W}{It} = \dfrac{P}{I} \qquad$ (since $P = \dfrac{W}{t}$)

**The electromotive force of a source** is the **electrical power generated per unit current** flowing through it.

Note:

- ❖ The electrical power generated by a source may be expressed as $P = EI$.
- ❖ For a practical battery, not all of the battery's chemical energy is converted to electrical energy. This is due to the presence of internal resistance in the battery.

## 4.2 Internal resistance

*(k) To show an understanding of the effects of the internal resistance of a source of e.m.f. on the terminal potential difference and output power.*

- Fig. 17 shows a cell with e.m.f. $E$ and internal resistance $r$.

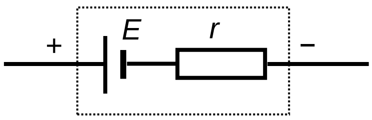
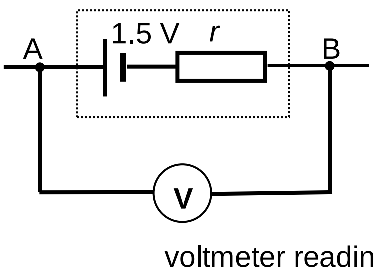



**Fig. 17** Cell with e.m.f. $E$ and internal resistance $r$

### 4.2.1 Effects of internal resistance on the terminal p.d. across a cell

- If a voltmeter is connected across the positive and negative terminals of the cell, the voltmeter will read the terminal p.d. across the cell.
- Fig. 18 shows an ideal voltmeter measuring the terminal p.d. across a cell of e.m.f. 1.5 V. Since the ideal voltmeter has infinite resistance, there is no current flowing through it, and no current flowing between A and B. It is an open circuit.
- There is no potential difference across $r$ since there is no current flowing through it. The voltmeter reading is 1.5 V, which is the e.m.f. of the cell.




voltmeter reading = terminal p.d. = 1.5 V

**Fig. 18** Ideal voltmeter reading the terminal p.d. of 1.5 V cell in an open circuit

- Fig. 19 shows a closed circuit. Since current flows through the whole circuit (but not through the ideal voltmeter), there is a potential difference across $r$.

- The voltmeter is still reading the terminal p.d. across the cell. However, the voltmeter reading is less than 1.5 V, i.e. less than the e.m.f. of the cell.

voltmeter reading = terminal p.d. < 1.5 V

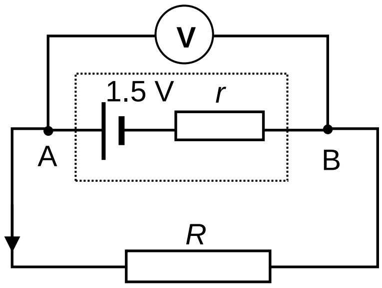



**Fig. 19** Ideal voltmeter reading the terminal p.d. of 1.5 V cell in a closed circuit

- In the circuit shown in Fig. 20, the same current $I$ flows through both $R$ and $r$.

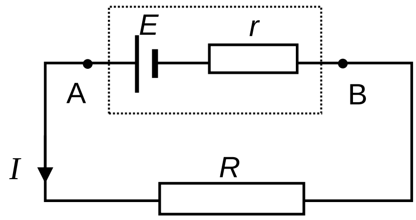



**Fig. 20** Cell (e.m.f. $E$, internal resistance $r$) connected to external resistance $R$

- The e.m.f. $E$ is equal to the sum of the p.d. across $R$ and the p.d. across $r$.

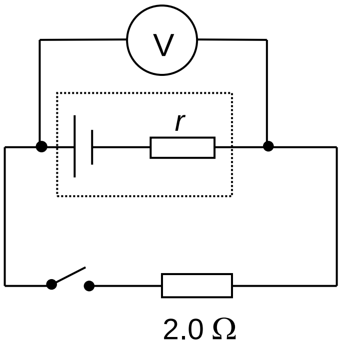
$$\boxed{E = IR + Ir \quad \text{or} \quad E = I\,(R + r)}$$

- **NOTE:** The terminal p.d. across the cell is equal to the p.d. across AB. This is also equal to the p.d. across the external resistance $R$.

- terminal p.d. across the cell = p.d. across $R$ = $IR = E - Ir$ → $V = E - Ir$

**<u>Example 13</u>**

A battery is connected in series with a 2.0 Ω resistor and a switch as shown. A voltmeter connected across the battery reads 12.0 V when the switch is open but 8.0 V when it is closed.
Calculate the internal resistance $r$ of the battery.




**Solution:**

e.m.f., $E$ = 12.0 V
p.d. across resistor, $V$ = 8.0 V

Current through resistor, $I = \frac{V}{R} = \frac{8.0}{2.0} = 4.0$ A

Same current also flows through cell, so using $E = V + Ir$ → $r = \frac{E - V}{I} = \frac{12.0 - 8.0}{4.0} = 1.0\ \Omega$

**Example 14**

A battery of e.m.f. 1.50 V has a terminal p.d. of 1.25 V when a resistor of 25 Ω is connected to it.

(a) Calculate the internal resistance *r.*

(b) Calculate the current flowing and the terminal p.d. when a resistor of 10 Ω replaces the 25 Ω resistor.

**Solution:**

(a) Initial terminal p.d. = p.d. across 25 Ω resistor

$V = 1.25$ V

Current through 25 Ω resistor, $I = \frac{V}{R} = \frac{1.25}{25} = 0.050$ A

Same current also flows through battery.

Internal resistance, $r = \frac{E - V}{I} = \frac{1.50 - 1.25}{0.050}$ ( Using $E = V + Ir$ )

$= 5.0\ \Omega$

(b) Current through 10 Ω resistor, $I_1 = \frac{E}{R + r} = \frac{1.50}{10 + 5.0}$ ( Using $E = I\,(R + r)$ )

$= 0.10$ A

New terminal p.d. = p.d. across 10 Ω resistor

$V_1 = I_1 R$

$= (0.10)(10)$

$= 1.0$ V

### 4.2.2 Effect of internal resistance on output power and efficiency

- For Fig. 20, by conservation of energy:

  Power generated by cell = Power transferred to *R* + Power transferred to *r*

  $$\boxed{EI = I^2R + I^2r}$$

- Output power of the cell = power transferred to *R* (or any other external circuit components present).

- The internal resistance of a cell affects
  - the output power of the cell, and
  - the efficiency of transfer of power from the cell to the external circuit components.

- $$\boxed{\text{Efficiency of transfer of power} = \frac{\text{power transferred to external circuit components}}{\text{power generated by cell}}}$$

**Example 15**

An electrical source with internal resistance $r$ is used to operate a lamp of resistance $R$.

Calculate the fraction of the total power delivered to the lamp.

**Solution:**

Power transferred to external circuit = $I^2R$

Power generated by cell = $EI = I^2R + I^2r$

$$\text{Efficiency } \eta = \frac{\text{power transferred to external circuit components}}{\text{power generated by cell}}$$

$$\eta = \frac{I^2R}{I^2R + I^2r}$$

$$\boxed{\eta = \frac{R}{R+r}}$$

**Example 16**

What conditions are necessary if a battery is to deliver maximum power to a resistor connected across its terminals?

Show that, at maximum power, the efficiency of the battery is 50%.

**Solution:**

Let the e.m.f. and internal resistance of the battery be $E$ and $r$ respectively.

$$P = I^2R = \frac{E^2R}{(R+r)^2} \qquad \left(\text{since } I = \frac{E}{R+r}\right)$$

Differentiating $P$ with respect to $R$,

$$\frac{dP}{dR} = \frac{E^2}{(R+r)^2} - \frac{2E^2R}{(R+r)^3}$$

To deliver maximum power $P$ to the resistor,

$$\frac{dP}{dR} = 0$$

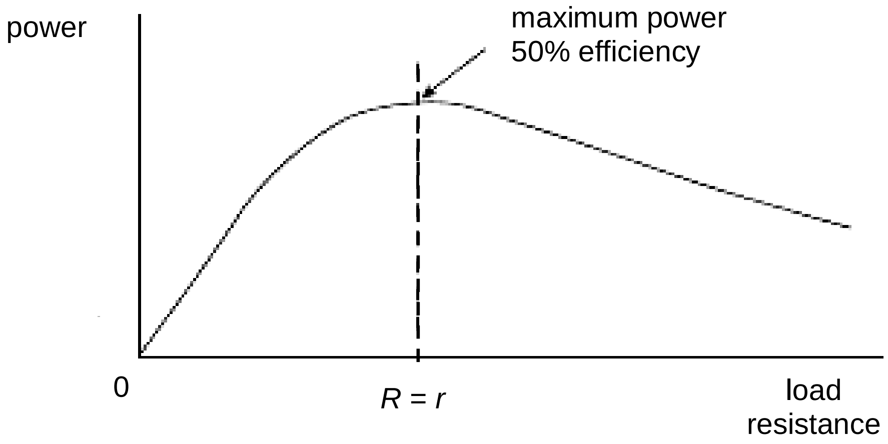
Therefore, $$\frac{E^2}{(R+r)^2} - \frac{2E^2R}{(R+r)^3} = 0$$

Solving the equation, $$R = r$$

Therefore efficiency $\eta = \dfrac{R}{R+r} = \dfrac{R}{2R} = 0.5$ (i.e. 50%)

### 4.2.3 Maximum Power Theorem

- To obtain maximum external power from a source with a **finite** internal resistance, the resistance of the external load must be equal to the internal resistance of the source.




## Appendix

**Ohm's Law**

***Ohm's law states that the current flowing in a metallic conductor is directly proportional to the potential difference applied across it, provided that the physical conditions (such as temperature, stress, etc) are constant.***

To investigate Ohm's law, an ammeter and a rheostat (variable resistor) are connected in series with a conductor as shown in Fig. 1. A voltmeter is connected across the conductor. The current $I$ through the circuit is varied by adjusting the rheostat, and at each value of $I$, the p.d. $V$ is measured. On plotting $I$ vs $V$, we get a straight line through the origin, as shown in Fig. 2.

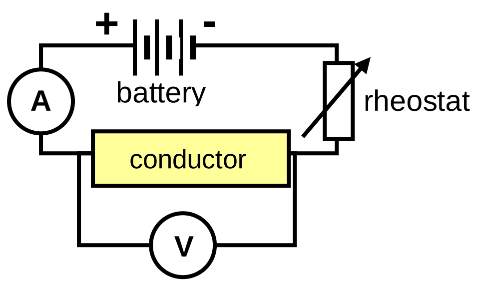



**Fig. 1**

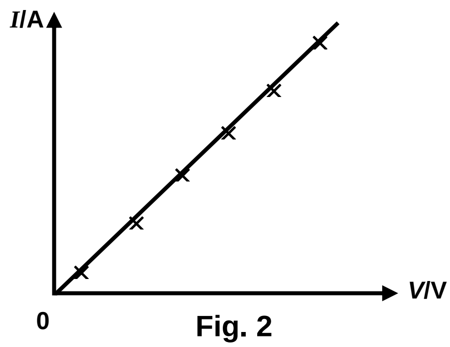



**Fig. 2**

**Ohmic conductors**

Ohmic conductors are those that obeys Ohm’s law. In this type of conductor, the current $I$ is reversed in direction when the p.d. $V$ is reversed but the magnitude of $I$ is unchanged. The $I$-$V$ characteristic is thus a straight line passing through the origin.

**Non-ohmic conductors**

Non-ohmic conductors are those that do not obey Ohm’s law. Many useful components in the electrical industry are non-ohmic, for example, inductors or capacitors. A non-ohmic $I$-$V$ characteristic may be a straight line that does not pass through the origin, or a curve.