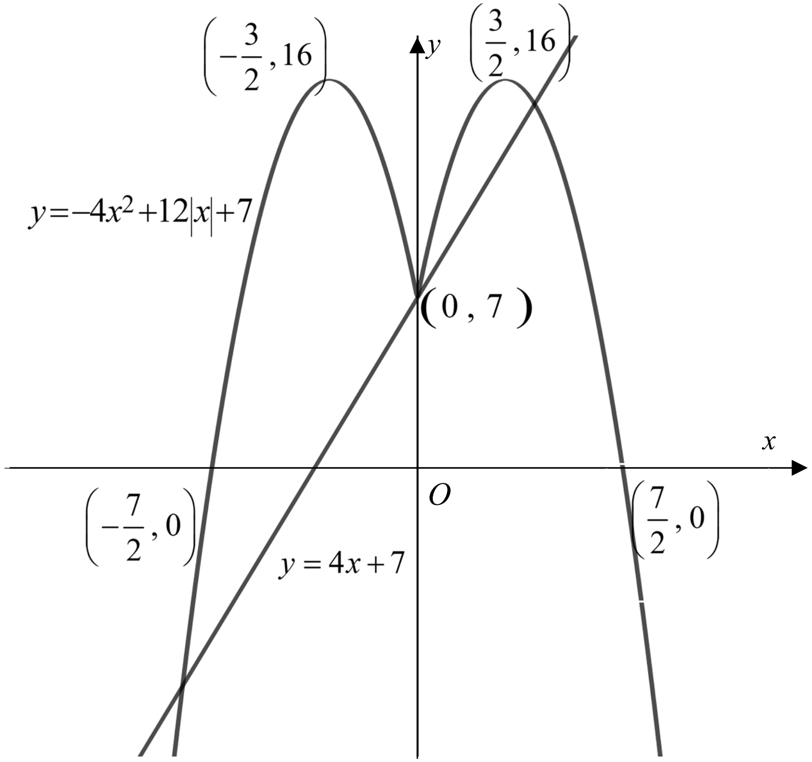
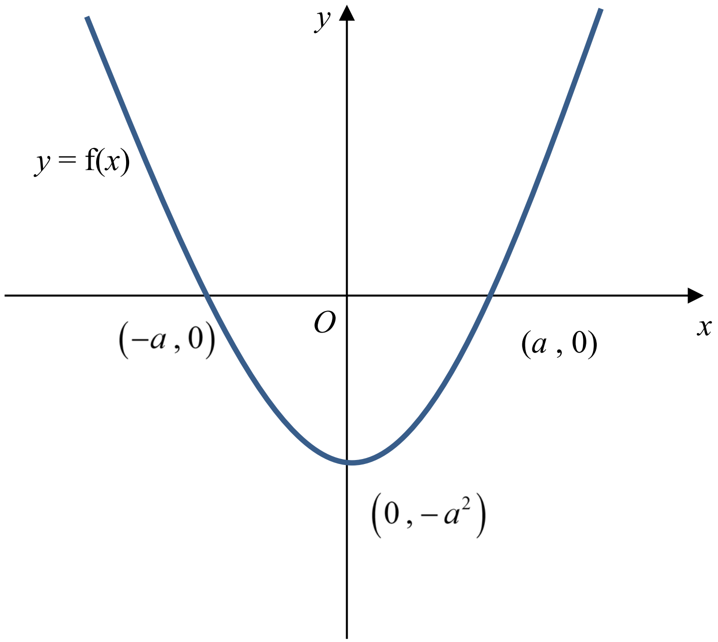
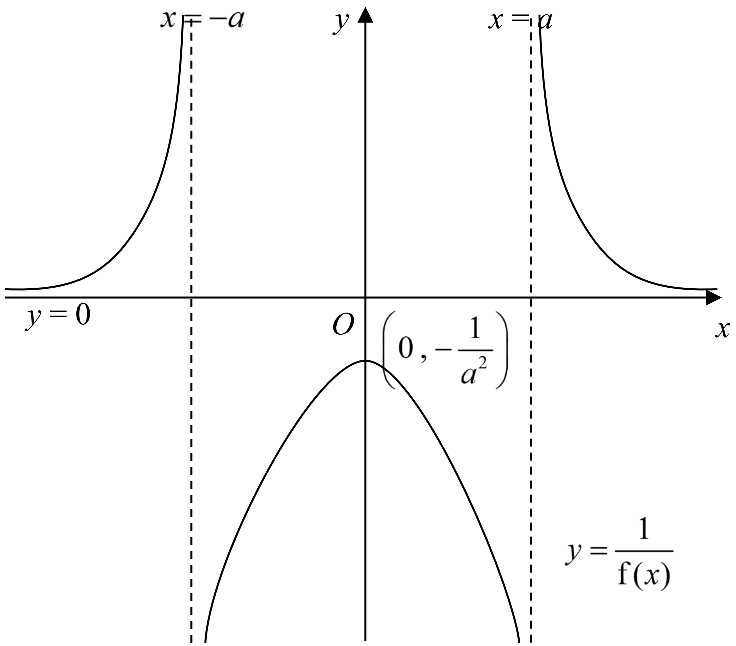
	Solutions
1 (i)	

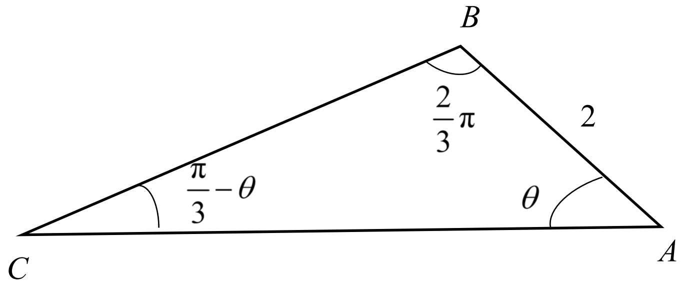
1 (ii)	<u>Method 1: (Graphical Method) – Preferred and efficient method</u> From the graph, for $-4x^2+12\|x\|+7 \le 4x+7$, The intersection points have x-coordinates −4 and 2 respectively. $x \le -4$ or $x=0$ or $x \ge 2$ <u>Method 2: (Algebraic Method) – Not preferred for this question</u> $-4x^2+12\|x\|+7 \le 4x+7$ $12\|x\| \le 4x+4x^2$ $\|x\| \le \frac{x+x^2}{3}$ $-\frac{x+x^2}{3} \le x \le \frac{x+x^2}{3}$ $-(x+x^2) \le 3x$ and $3x \le x+x^2$ $x+x^2+3x \ge 0$ $x^2-2x \ge 0$ $x(x+4) \ge 0$ $x(x-2) \ge 0$ $x \le -4$ or $x \ge 0$ $x \le 0$ or $x \ge 2$ Combining $x \le -4$ or $x \ge 0$ and $x \le 0$ or $x \ge 2$ on a number line, $x \le -4$ or $x=0$ or $x \ge 2$.

2 (i)	
2 (ii)	$(-a\ ,0)$ → vertical asymptote $x=-a$ $(a\ ,0)$ → vertical asymptote $x=a$ Minimum point $\left(0,-a^2\right)$ → Maximum point $\left(0,-\frac{1}{a^2}\right)$

	
2 (iii)	The tangents to the curve of $y=\frac{1}{\mathrm{f}(x)}$ tends to a horizontal line / becomes parallel with the $x-$axis when $x \to \pm\infty$.
3 (i)	Let us consider an arithmetic progression with $a=12$ and $d=2$, where n is the number of odd days. Then $u_n = 12+(n-1)(2)$. Let $u_n = 42$, then $12+(n-1)(2)=42$ gives $n=16$. Hence it requires 16 odd numbered days to complete 42 km. Starting on 1st day, the last day is given by 1 + (16-1)(2) = 31. Therefore Mary completed 42 km on the 31st day of training. (shown)

(ii)	Let us consider a GP, with $a = 12$, $r = \left(1+\frac{x}{100}\right)$ and $n = 31$ for Leo to first run 42 km Using $u_n = ar^{n-1}$ $12\left(1+\frac{x}{100}\right)^{31-1} = 42$ using GC, $x = 4.26$ (2 d.p.)
(iii)	Total distance covered by Mary =2 $\times$ distance covered during 1st 15 odd numbered days + distance covered on 31st day Total distance covered = $\left(2\left[\frac{15}{2}\left(2(12)+(15-1)\times 2\right)\right]+42\right)$ $+\frac{12\left(1-(1+0.042643)^{31}\right)}{1-(1+0.042643)}$ $=822+745.516$ $=1567.516$ $=1570\ (3\text{s.f.})$

4 (i)	Let T be the temperature of a heated body at t min. $\frac{\mathrm{d}T_{in}}{\mathrm{d}t} = 0,\ \frac{\mathrm{d}T_{out}}{\mathrm{d}t} = k(T-\theta),\ k > 0$ $\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\mathrm{d}T_{in}}{\mathrm{d}t} - \frac{\mathrm{d}T_{out}}{\mathrm{d}t} = -k(T-\theta),$ where $k > 0$ $\frac{1}{T-\theta}\frac{\mathrm{d}T}{\mathrm{d}t} = -k$ Integrating with respect to t both sides, $\int \frac{1}{T-\theta}\,\mathrm{d}T = \int -k\ \mathrm{d}t$ $\ln\lvert T-\theta\rvert = -kt + C,$ where C is an arbitrary constant $\ln(T-\theta) = -kt + C,$ since $T > \theta$. $T - \theta = e^{C}e^{-kt}$ $T = Ae^{-kt} + \theta,$ where $A = e^{C}$.
	Given that $\theta = 25$, $T = Ae^{-kt} + 25$. When $t = 0,\ T = 175,$ $175 = Ae^{0} + 25$ $\therefore A = 150$ $\frac{\mathrm{d}T}{\mathrm{d}t} = -150ke^{-kt}$ when $t = 0,\ \frac{\mathrm{d}T}{\mathrm{d}t} = -3,$ $\therefore -150k = -3 \Rightarrow k = \frac{1}{50}$ $\therefore T = 150e^{-0.02t} + 25$ For the apple pie to cool to 40^{0}C, $T = 40$,

	$40 = 150e^{-0.02t} + 25$ $150e^{-0.02t} = 15$ $e^{-0.02t} = 0.10$ $t = 115.13$ $= 115 \text{ min} \ \ (3 \text{ sf})$
5 (i)	 $A\hat{C}B = \pi - \frac{2}{3}\pi - \theta = \frac{\pi}{3} - \theta$ By Sine Rule, $\frac{BC}{\sin B\hat{A}C} = \frac{AB}{\sin A\hat{C}B}$

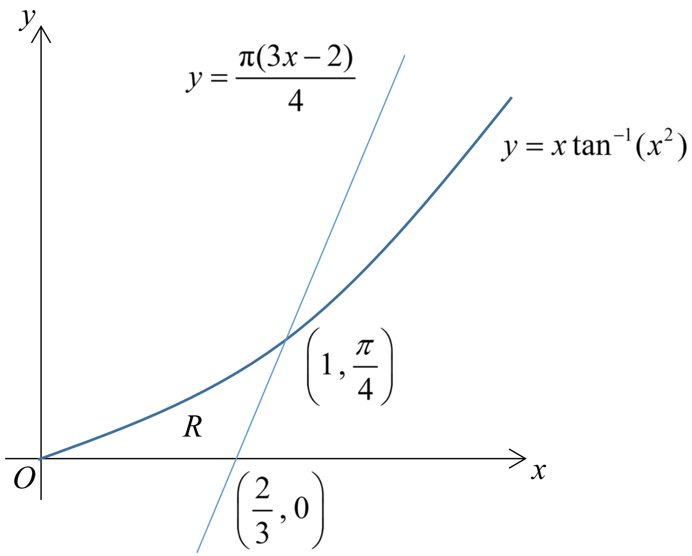
	$BC=\dfrac{AB\sin B\hat{A}C}{\sin A\hat{C}B}$ $=\dfrac{2\sin\theta}{\sin\left(\dfrac{\pi}{3}-\theta\right)}$ $=\dfrac{2\sin\theta}{\sin\dfrac{\pi}{3}\cos\theta-\cos\dfrac{\pi}{3}\sin\theta}$ $=\dfrac{2\sin\theta}{\dfrac{\sqrt{3}}{2}\cos\theta-\dfrac{1}{2}\sin\theta}$ $=\dfrac{4\sin\theta}{\sqrt{3}\cos\theta-\sin\theta}$
(ii)	Given that θ is a sufficiently small angle,

	$BC=\dfrac{4\sin\theta}{\sqrt{3}\cos\theta-\sin\theta}$ $\approx\dfrac{4\theta}{\sqrt{3}\left[1-\dfrac{\theta^2}{2}\right]-\theta}$ $\approx 4\theta\left[\sqrt{3}-\theta\right]^{-1}$ $=\dfrac{4}{\sqrt{3}}\theta\left[1-\dfrac{\theta}{\sqrt{3}}\right]^{-1}$ $=\dfrac{4\sqrt{3}}{3}\theta\left[1+\dfrac{1}{\sqrt{3}}\theta\right]$ $=\dfrac{4\sqrt{3}}{3}\theta+\dfrac{4}{3}\theta^2+\ldots$
(iii)	$\mathrm{e}^{a\theta}\ln(1+\theta)$ $=\left[1+a\theta+\dfrac{(a\theta)^2}{2!}+\ldots\right]\left[\theta-\dfrac{\theta^2}{2}+\ldots\right]$ $=\theta-\dfrac{\theta^2}{2}+a\theta^2+\ldots$ $=\theta+\left(a-\dfrac{1}{2}\right)\theta^2+\ldots$ Given that the term in θ^2 are equal in (ii) & (iii),

	$a-\frac{1}{2}=\frac{4}{3}$ $a=\frac{11}{6}$
6 (i)	$(\sqrt{r+2}+\sqrt{r})(\sqrt{r+2}-\sqrt{r})$ $=\left[\sqrt{(r+2)}\right]^2-\left(\sqrt{r}\right)^2$ $=\left[(r+2)-r\right]$ $=$ 2 (Shown) $\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{4}+\sqrt{6}}+\frac{1}{\sqrt{5}+\sqrt{7}}+...+\frac{1}{\sqrt{n}+\sqrt{n+2}}$ $=\sum_{r=3}^{n}\left(\frac{1}{\sqrt{r}+\sqrt{r+2}}\right)$ $=\sum_{r=3}^{n}\frac{1}{2}(\sqrt{r+2}-\sqrt{r})$, from above

	$$=\frac{1}{2}\times\left\{\begin{matrix}\sqrt{5}-\sqrt{3}\\+\sqrt{6}-\sqrt{4}\\+\sqrt{7}-\sqrt{5}\\\vdots\\+\sqrt{n}-\sqrt{n-2}\\+\sqrt{n+1}-\sqrt{n-1}\\+\sqrt{n+2}-\sqrt{n}\end{matrix}\right\}$$ $$=\frac{1}{2}(\sqrt{n+2}+\sqrt{n+1}-\sqrt{4}-\sqrt{3})$$ $$=\frac{1}{2}(\sqrt{n+2}+\sqrt{n+1}-2-\sqrt{3})$$
(ii)	$$\frac{1}{\sqrt{a}+\sqrt{a+2}}+\ldots+\frac{1}{\sqrt{98}+\sqrt{100}}=1+\frac{3}{2}(\sqrt{11}-\sqrt{7}).$$ $$\sum_{r=a}^{98}\frac{1}{\sqrt{r}+\sqrt{r+2}}=1+\frac{3}{2}(\sqrt{11}-\sqrt{7})$$ $$\sum_{r=3}^{98}\frac{1}{\sqrt{r}+\sqrt{r+2}}-\sum_{r=3}^{a-1}\frac{1}{\sqrt{r}+\sqrt{r+2}}=1+\frac{3}{2}(\sqrt{11}-\sqrt{7})$$ $$\frac{1}{2}(\sqrt{98+2}+\sqrt{98+1}-2-\sqrt{3})-\frac{1}{2}(\sqrt{a-1+2}+\sqrt{a-1+1}-2-\sqrt{3})$$

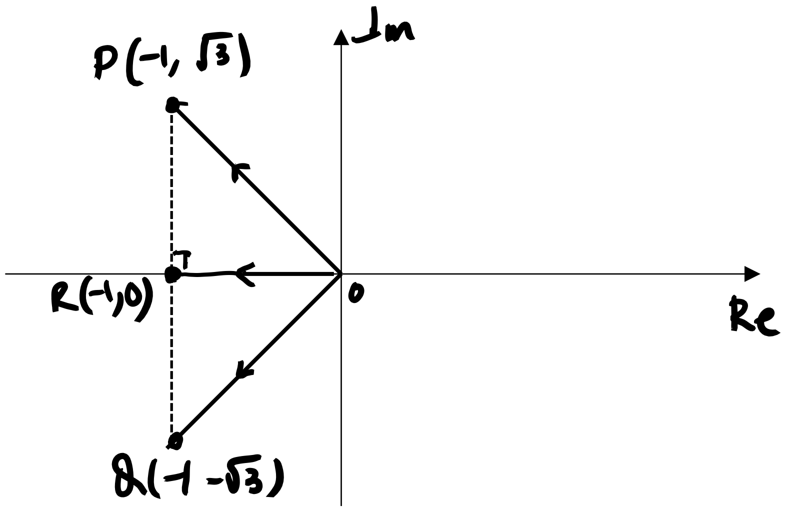
	$=1+\frac{3}{2}(\sqrt{11}-\sqrt{7})$ $\frac{1}{2}(\sqrt{100}+\sqrt{99}-2-\sqrt{3})-\frac{1}{2}(\sqrt{a+1}+\sqrt{a}-2-\sqrt{3})$ $=1+\frac{3}{2}\left(\sqrt{11}-\sqrt{7}\right)$ $(10+3\sqrt{11}-2-\sqrt{3})-(\sqrt{a+1}+\sqrt{a}-2-\sqrt{3})=2+3(\sqrt{11}-\sqrt{7})$ $8+3\sqrt{11}-\sqrt{3}-\sqrt{a+1}-\sqrt{a}+2+\sqrt{3}=2+3\sqrt{11}-3\sqrt{7}$ $\sqrt{a+1}+\sqrt{a}=8+3\sqrt{7}$ Using GC, $a=63$.
7 (i)	Let $u=\tan^{-1}(x^2)$, $\frac{dv}{dx}=x$ Then $\frac{du}{dx}=\frac{2x}{1+(x^2)^2}$, $v=\frac{x^2}{2}$

	$\int x\tan^{-1}(x^2)\,\mathrm{d}x$ $= \frac{x^2}{2}\tan^{-1}(x^2) - \int \frac{x^3}{1+x^4}\,\mathrm{d}x$ $= \frac{x^2}{2}\tan^{-1}(x^2) - \frac{1}{4}\int \frac{4x^3}{1+x^4}\,\mathrm{d}x$ $= \frac{x^2}{2}\tan^{-1}(x^2) - \frac{1}{4}\ln(1+x^4) + C, \quad 1+x^4>0$ where C is an arbitrary constant $\mathrm{f}(x) = 1+x^4$ $\mathrm{f}'(x) = 4x^3$
(ii)	 The two curves intersect at $x = 1$

	Area of R $= \int_0^1 x\tan^{-1}\left(x^2\right)\mathrm{d}x -$ Area of triangle $= \int_0^1 x\tan^{-1}\left(x^2\right)\mathrm{d}x - \frac{1}{2}\left(\frac{1}{3}\times\frac{\pi}{4}\right)$ $= \left[\frac{x^2}{2}\tan^{-1}\left(x^2\right) - \frac{1}{4}\ln(1+x^4)\right]_0^1 - \frac{\pi}{24}$ $= \frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{4}\ln 2 - \frac{\pi}{24}$ $= \left(\frac{\pi}{12} - \frac{1}{4}\ln 2\right)$ units^2 <u>ALT</u>

	Area of R $= \int_0^{\frac{2}{3}} x\tan^{-1}\left(x^2\right)\mathrm{d}x + \int_{\frac{2}{3}}^{1}\left[x\tan^{-1}\left(x^2\right) - \frac{\pi\left(3x-2\right)}{4}\right]\mathrm{d}x$ $= \left[\frac{x^2}{2}\tan^{-1}\left(x^2\right) - \frac{1}{4}\ln(1+x^4)\right]_0^{\frac{2}{3}}$ $+ \left[\frac{x^2}{2}\tan^{-1}\left(x^2\right) - \frac{1}{4}\ln(1+x^4) - \frac{\pi}{4}\left(\frac{3x^2}{2} - 2x\right)\right]_{\frac{2}{3}}^{1}$ $= \left[\frac{2}{9}\tan^{-1}\left(\frac{4}{9}\right) - \frac{1}{4}\ln\left(\frac{97}{81}\right)\right]$ $+ \left[\begin{array}{l}\left[\frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{4}\ln 2 - \frac{\pi}{4}\left(-\frac{1}{2}\right)\right] \\ -\left[\frac{2}{9}\tan^{-1}\left(\frac{4}{9}\right) - \frac{1}{4}\ln\left(\frac{97}{81}\right) - \frac{\pi}{4}\left(\frac{2}{3} - \frac{4}{3}\right)\right]\end{array}\right]$ $= \frac{\pi}{8} - \frac{1}{4}\ln 2 + \frac{\pi}{8} + \frac{\pi}{4}\left(-\frac{2}{3}\right)$ $= \left(\frac{\pi}{12} - \frac{1}{4}\ln 2\right)$ units^2
(iii)	Required volume $= \pi\int_0^1 \left[x\tan^{-1}\left(x^2\right)\right]^2 \mathrm{d}x - \frac{1}{3}\pi\left(\frac{\pi}{4}\right)^2\left(\frac{1}{3}\right)$ $= 0.109$ units^3 <u>ALT</u> Required volume

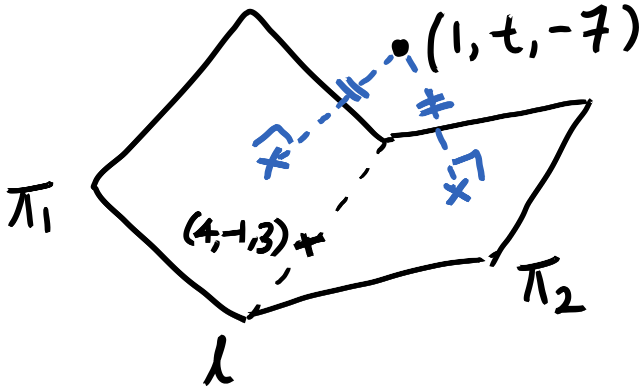
	$=\pi\int_0^{\frac{2}{3}}\left[x\tan^{-1}\left(x^2\right)\right]^2 \mathrm{d}x+\pi\int_{\frac{2}{3}}^{1}\left[x\tan^{-1}\left(x^2\right)\right]^2-\left[\frac{\pi\left(3x-2\right)}{4}\right]^2 \mathrm{d}x$ $=0.109\ \text{units}^3$
8 (a) (i)	Since the coefficients of the polynomial equation are all real, complex roots occurs in complex conjugate pairs. Since $z_1=-1+\sqrt{3}\mathrm{i}$ is a complex root, its complex conjugate $z_2=-1-\sqrt{3}\mathrm{i}$ is also a root of the equation. The third root is a real root. Since $\left[z-\left(-1+\sqrt{3}\mathrm{i}\right)\right]\left[z-\left(-1-\sqrt{3}\mathrm{i}\right)\right]$ $=\left[\left(z+1\right)-\sqrt{3}\mathrm{i}\right]\left[\left(z+1\right)+\sqrt{3}\mathrm{i}\right]$ $=(z+1)^2-\left(\sqrt{3}\mathrm{i}\right)^2=z^2+2z+4$ Let the third root be $z=k, k\in\mathbb{R}$ $z^3+3z^2+az+b=(z-k)(z^2+2z+4)$. Comparing coefficient of z: $a=-2k+4$ Comparing coefficient of z^0: $b=-4k$ Comparing coefficient of z^2: $3=2-k\Rightarrow k=2-3=-1$ $\therefore b=4, a=6$ Hence the other roots are $z_2=-1-\sqrt{3}\mathrm{i}$ and $z_3=-1$.

(a) (ii)	$z_1 = -1+\sqrt{3}\mathrm{i} = 2\left[\cos\left(\frac{2\pi}{3}\right)+\sin\left(\frac{2\pi}{3}\right)\right]$ $z_2 = -1-\sqrt{3}\mathrm{i} = 2\left[\cos\left(-\frac{2\pi}{3}\right)+\sin\left(-\frac{2\pi}{3}\right)\right]$ $z_3 = -1 = 1(\cos\pi + \mathrm{i}\sin\pi)$ Let *P*, *Q* and *R* represent the complex numbers z_1, z_2 and z_3 respectively. 
(b)	$w = \left(\frac{1}{2+\mathrm{i}\alpha}\right)\cdot\left(\frac{2-\mathrm{i}\alpha}{2-\mathrm{i}\alpha}\right) = \frac{2-\mathrm{i}\alpha}{4+\alpha^2}$ $w^* = \frac{2+\mathrm{i}\alpha}{4+\alpha^2}$

	$ww^* = \|w\|^2 = \dfrac{4+\alpha^2}{\left(4+\alpha^2\right)^2} = \dfrac{1}{4+\alpha^2}$ ---(1) $w + w^* = \dfrac{4}{4+\alpha^2} = 4\,ww^*$ (Proved). --- (2) From equation (2), $w = x + y\mathrm{i}$, where $x, y \in \mathbb{R}$, $\left(x+y\mathrm{i}\right)+\left(x-y\mathrm{i}\right)=4\left(x^2+y^2\right)$ $2x = 4\,(x^2 + y^2)$ --- (3) $\Rightarrow 2x^2 + 2y^2 - x = 0$ $\Rightarrow 2\left(x^2 - \frac{1}{2}x\right) + 2y^2 = 0$ $\Rightarrow \left(x - \frac{1}{4}\right)^2 - \frac{1}{16} + y^2 = 0$ $\Rightarrow \left(x - \frac{1}{4}\right)^2 + y^2 = \left(\frac{1}{4}\right)^2$ ---(Cartesian Equation) Hence w lies on a circle with centre at $\left(\frac{1}{4}, 0\right)$ and radius $\frac{1}{4}$ units.
9 (i)	$\pi_1 : \mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \lambda_1 \begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix} + \mu_1 \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$ $\pi_2 : \mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix} + \mu_2 \begin{pmatrix} 1 \\ -3 \\ 12 \end{pmatrix}$

	By referring to the equations of π_1 and π_2, we observe that a common point is (4, –1, 3) and the vector $\begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix}$ is parallel to both planes. Hence a point on l is (4, –1, 3) and a vector parallel to l is $\begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix}$. Equation of l : $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix}, \beta \in \mathbb{R}$
(ii)	$\pi : \mathbf{r} \cdot \begin{pmatrix} k \\ 3k+7 \\ 1 \end{pmatrix} = k-4$ $l : \mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 4+3\beta \\ -1-\beta \\ 3+7\beta \end{pmatrix}, \beta \in \mathbb{R}$ Since $\begin{pmatrix} 4+3\beta \\ -1-\beta \\ 3+7\beta \end{pmatrix} \cdot \begin{pmatrix} k \\ 3k+7 \\ 1 \end{pmatrix}$ $= 4k + 3k\beta - 3k - 7 - 3k\beta - 7\beta + 3 + 7\beta$ $= k - 4,$

	any point that lies on l also lies on the plane, so l lies in π for any constant k. Alternative solution : $\begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} k \\ 3k+7 \\ 1 \end{pmatrix} = 3k - 3k - 7 + 7 = 0$ Since the normal is perpendicular to the line l, line l is parallel to π. $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} k \\ 3k+7 \\ 1 \end{pmatrix} = k - 4$ Hence the point $(4,-1,3)$ lies on π .Hence line l lies on π. Let π_3 be π for a particular value of k. Since $C(3, -2, 7)$ lies on π_3, $\begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} \bullet \begin{pmatrix} k \\ 3k+7 \\ 1 \end{pmatrix} = k - 4$ $3k - 6k - 14 + 7 = k - 4$ $k = -\frac{3}{4}$

	$\pi_3 : \mathbf{r}\bullet\begin{pmatrix} -\frac{3}{4} \\ 3\left(-\frac{3}{4}\right)+7 \\ 1 \end{pmatrix} = -\frac{3}{4}-4$ $\mathbf{r}\bullet\begin{pmatrix} -\frac{3}{4} \\ \frac{19}{4} \\ 1 \end{pmatrix} = -\frac{19}{4} \Rightarrow \mathbf{r}\bullet\begin{pmatrix} -3 \\ 19 \\ 4 \end{pmatrix} = -19$ 3
(iii)	

Normal of π_1 is parallel to $\begin{pmatrix}3\\-1\\7\end{pmatrix}\times\begin{pmatrix}3\\-1\\4\end{pmatrix}=3\begin{pmatrix}1\\3\\0\end{pmatrix}$

Normal of $\pi_2=\begin{pmatrix}3\\-1\\7\end{pmatrix}\times\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}0\\7\\1\end{pmatrix}$

Shortest distance of $B\ (1, t, -7\ )$ from π_1 is

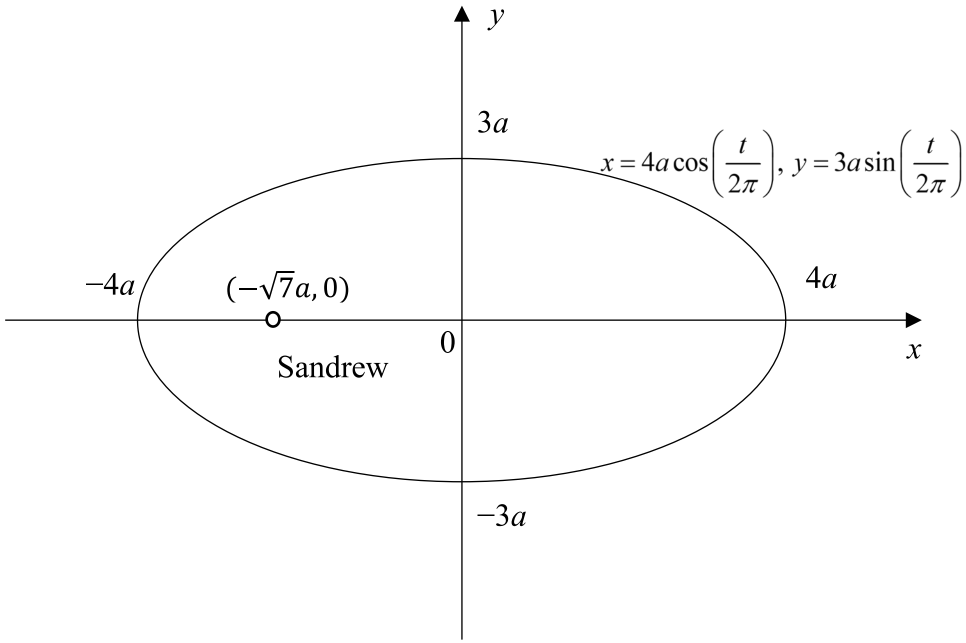
$$\frac{\left|\left[\begin{pmatrix}1\\t\\-7\end{pmatrix}-\begin{pmatrix}4\\-1\\3\end{pmatrix}\right]\bullet\begin{pmatrix}1\\3\\0\end{pmatrix}\right|}{\sqrt{1+9}}=\frac{\left|\begin{pmatrix}-3\\t+1\\-10\end{pmatrix}\bullet\begin{pmatrix}1\\3\\0\end{pmatrix}\right|}{\sqrt{10}}=\frac{1}{\sqrt{10}}|3t|$$

Shortest distance of $B\ (1, t, -7\ )$ from π_2 is

$$\frac{\left|\left[\begin{pmatrix}1\\t\\-7\end{pmatrix}-\begin{pmatrix}4\\-1\\3\end{pmatrix}\right]\bullet\begin{pmatrix}0\\7\\1\end{pmatrix}\right|}{\sqrt{50}}=\frac{\left|\begin{pmatrix}-3\\t+1\\-10\end{pmatrix}\bullet\begin{pmatrix}0\\7\\1\end{pmatrix}\right|}{\sqrt{50}}=\frac{1}{\sqrt{50}}|7t-3|$$

Given $\frac{1}{\sqrt{10}}|3t|=\frac{1}{\sqrt{50}}|7t-3|$

From GC, t = 0.219 or 10.281 (3 decimal places)

10(i)	
(ii)	Area $= \int_{-4a}^{4a} y_1 \ \mathrm{d}x - \int_{-4a}^{4a} y_2 \ \mathrm{d}x$ At $x = 4a,\ t = 0$ or $4\pi^2$. At $x = -4a,\ t = 2\pi^2$

	$=\int_{-4a}^{4a} y\ \mathrm{d}x - \int_{-4a}^{4a} y\ \mathrm{d}x$ $=\int_{2\pi^2}^{0} 3a\sin\left(\frac{t}{2\pi}\right)\left(-\frac{2}{\pi}a\sin\left(\frac{t}{2\pi}\right)\right)\ \mathrm{d}t$ $-\int_{2\pi^2}^{4\pi^2} 3a\sin\left(\frac{t}{2\pi}\right)\left(-\frac{2}{\pi}a\sin\left(\frac{t}{2\pi}\right)\right)\ \mathrm{d}t$ $=\frac{12}{2\pi}a^2\int_{0}^{2\pi^2} \sin^2\left(\frac{t}{2\pi}\right)\ \mathrm{d}t + \frac{12}{2\pi}a^2\int_{2\pi^2}^{4\pi^2} \sin^2\left(\frac{t}{2\pi}\right)\ \mathrm{d}t$ $=\frac{3}{\pi}a^2\int_{0}^{4\pi^2} 2\sin^2\left(\frac{t}{2\pi}\right)\ \mathrm{d}t$ $=\frac{3}{\pi}a^2\int_{0}^{4\pi^2} \left[1-\cos\left(\frac{t}{\pi}\right)\right]\ \mathrm{d}t$ $=\frac{3}{\pi}a^2\left[t-\pi\sin\left(\frac{t}{\pi}\right)\right]_0^{4\pi^2}$ $=\frac{3}{\pi}a^2\left[4\pi^2-\pi\sin\left(\frac{4\pi^2}{\pi}\right)\right]$ $=12a^2\pi$
(iii)	$\frac{\mathrm{d}x}{\mathrm{d}t}=-4a\left(\frac{1}{2\pi}\right)\sin\left(\frac{t}{2\pi}\right)$ $\qquad$ $\frac{\mathrm{d}y}{\mathrm{d}\theta}=3a\left(\frac{1}{2\pi}\right)\cos\left(\frac{t}{2\pi}\right)$ $=-\frac{2a}{\pi}\sin\left(\frac{t}{2\pi}\right)$ $\qquad$ $=\frac{3a}{2\pi}\cos\left(\frac{t}{2\pi}\right)$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$= \frac{\frac{3}{2\pi} a \cos\left(\frac{t}{2\pi}\right)}{-\frac{2}{\pi} a \sin\left(\frac{t}{2\pi}\right)} = -\frac{3}{4} \cot\left(\frac{t}{2\pi}\right)$$

When $t = 2\pi p$,

$x = 4a \cos p,\ y = 3a \sin p$

Gradient of the normal to the orbit is $\frac{4}{3} \tan p$.

Equation of the normal at $(4a \cos p,\ 3a \sin p)$ is

$$y - 3a \sin p = \frac{4}{3} \tan p (x - 4a \cos p)$$

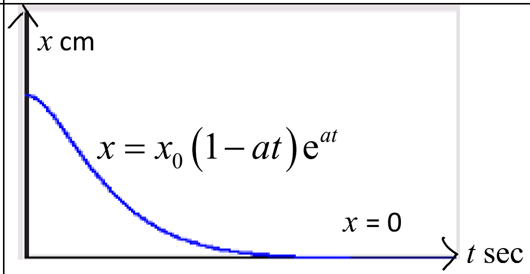
$$y = \frac{4}{3}(\tan p)x - \frac{16}{3} a \sin p + 3a \sin p$$

$$y = \frac{4}{3}(\tan p)x - \frac{7}{3} a \sin p$$

	For the space probe to land on the planet, $x=-\sqrt{7}a, y=0$ $\frac{4}{3}(\tan p)\left(-\sqrt{7}a\right)-\frac{7}{3}a\sin p=0$ $-\frac{a}{3}\left(4\sqrt{7}\tan p+7\sin p\right)=0$ $4\sqrt{7}\tan p+7\sin p=0$ $\sin p\left(\frac{4\sqrt{7}}{\cos p}+7\right)=0$ $\sin p=0,$ or $\cos p=-\frac{4\sqrt{7}}{7}<-1$ (no solution) $p=0,\ \pi$ or 2π Hence $t=0,\ 2\pi^2$ or $4\pi^2$ Hence there is no solution for *p*. Hence the space probe will land on the planet Sandrew if the space probe is launched at $t=0$ or $t=2\pi^2$or $t=4\pi^2$.
11(i)	For $Mr^2+cr+k=0$ to have real and repeated roots, Discriminant = 0 $c^2-4Mk=0$ $c^2=4Mk$ $c=2\sqrt{Mk}$, since $c>0$

	$r=\dfrac{-c\pm\sqrt{c^2-4Mk}}{2M}$ $=-\dfrac{c}{2M}$, since $c^2-4Mk=0$ $=-\dfrac{2\sqrt{Mk}}{2M}$ $=-\dfrac{\sqrt{Mk}}{M}$ Hence, the real and repeated root is $r=-\dfrac{c}{2M}=-\dfrac{\sqrt{Mk}}{M}$
(ii)	When $x=x_0$ when $t=0$, $c_1=x_0$ $x=\left(x_0+c_2t\right)\mathrm{e}^{at}$ ---(1) Differentiating (1) with respect to t: $\dfrac{\mathrm{d}x}{\mathrm{d}t}=\left(x_0+c_2t\right)\left(a\right)\mathrm{e}^{at}+c_2\mathrm{e}^{at}$ $=\mathrm{e}^{at}\left[\left(x_0+c_2t\right)a+c_2\right]$ $=\mathrm{e}^{at}\left[ax_0+c_2\left(1+at\right)\right]---(1)$ Since the particle was released from rest, $t=0$ and $\dfrac{\mathrm{d}x}{\mathrm{d}t}=0$

	$0=(x_0)(a)+c_2$ $c_2=-ax_0$
(iii)	From Eq (1) in (ii), $x=(x_0-ax_0t)\mathrm{e}^{at}$ $\frac{\mathrm{d}x}{\mathrm{d}t}=\mathrm{e}^{at}\left[ax_0-ax_0(1+at)\right]$ $=ax_0\mathrm{e}^{at}[1-1-at]$ $=-a^2x_0t\mathrm{e}^{at}$ --(2) For stationary value of x, $\frac{\mathrm{d}x}{\mathrm{d}t}=0$ For $x_0>0$, $\mathrm{e}^{at}>0$ for $t\geq 0$ $-a^2x_0t\mathrm{e}^{at}=0$ $\Rightarrow t=0$. Hence there is only one stationary value of x for $t\geq 0$. For $t>0$, $x_0>0$, $\mathrm{e}^{at}>0, \frac{\mathrm{d}x}{\mathrm{d}t}<0$ For $t>0$, $x_0>0$, $\mathrm{e}^{at}>0, \frac{\mathrm{d}x}{\mathrm{d}t}<0$

	Since $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ at $t = 0$ and $\frac{\mathrm{d}x}{\mathrm{d}t} < 0$ for t > 0, , x is a decreasing function for all $t \geq 0$.
(iv)	
(v)	No. From the graph in (iv), x, the displacement of the object, can never be negative.