RAFFLES INSTITUTION
H2 Mathematics 9758
2024 Year 6 Term 3 Revision
Mock Paper 2

Section A: Pure Mathematics [40 marks]

1 (i) The function f is defined as follows:

$$f : x \mapsto 3\cos x - 2\sin x,\ x \in \mathbb{R},\ -\pi < x < \pi.$$

Write f(x) as $R\cos(x+\alpha)$, where R and α are constants to be found. Hence, or otherwise, find the range of f and sketch the curve. [4]

(ii) The function g is defined as follows:

$$g : x \mapsto 3\cos x - 2\sin x,\ x \in \mathbb{R},\ -a < x < b.$$

Given that the function g^{-1} exists, write down the largest value of b. Find $g^{-1}(x)$. [3]

2 The first four terms of a sequence of numbers are 3, 1, 1 and 3. S_n is the sum of the first n terms of this sequence.

(i) Explain why S_n cannot be a quadratic polynomial in n. [2]

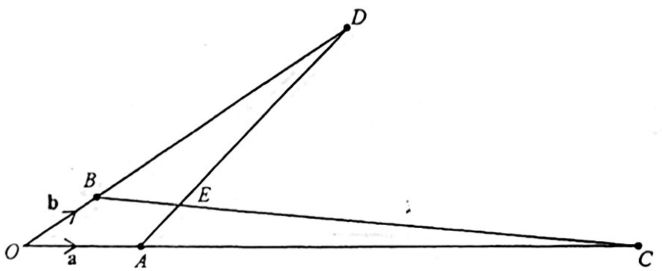
It is given that S_n is a cubic polynomial.

(ii) Find S_n in terms of n. [4]

(iii) Find an expression in terms of n for the nth term of the sequence. [3]

3 (a) The angle between the vectors $3\mathbf{i} - 2\mathbf{j}$ and $6\mathbf{i} + d\mathbf{j} - \sqrt{7}\mathbf{k}$ is $\cos^{-1}\left(\frac{6}{13}\right)$.
Show that $2d^2 - 117d + 333 = 0$. [3]

(b)



With reference to origin O, the points A, B, C and D are such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{AC} = 5\mathbf{a}$ and $\overrightarrow{BD} = 3\mathbf{b}$. The lines AD and BC cross at E (see diagram).

(i) Find $\overrightarrow{OE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [6]

(ii) The point F divides the line CD in the ratio 5 : 3. Show that O, E and F are collinear, and find $OE : OF$. [4]

4 (i) Given that $y = \tan(e^{2x} - 1)$, show that $\frac{dy}{dx} = ke^{2x}(1 + y^2)$, where k is to be found. Hence find the values of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ when $x = 0$. [6]

(ii) Write down the first three non-zero terms in the Maclaurin series for $\tan(e^{2x} - 1)$. [1]

(iii) The first two non-zero terms in the Maclaurin series for $\tan(e^{2x} - 1)$ are equal to the first two non-zero terms in the series expansion of $e^{ax}\ln(1 + nx)$. By using appropriate expansions from the List of Formulae (MF26), find the constants a and n. Hence find the third non-zero term of the series expansion of $e^{ax}\ln(1 + nx)$ for these values of a and n. [4]

Section B: Probability and Statistics [60 marks]

5 This question is about six couples. Each couple consists of a husband and a wife.

The 12 people visit a theatre, and sit in a row of 12 seats.

(i) In how many different ways can the 12 people sit so that each husband and wife in a couple sit next to each other? [2]

(ii) In how many different ways can the 12 people sit so that the 6 wives all sit next to each other, and none of the wives sits next to her own husband? [3]

The group decides to form a committee to arrange future outings. The committee will consist of 3 of the 12 people. At least 1 of the wives will be on the committee but no husband and wife couple will be included.

(iii) In how many ways can the committee be formed? [3]

6 Giant pumpkins are often irregular in shape. In order to account for the different shapes of pumpkins, growers of giant pumpkins measure the size of a pumpkin by a combination of three measurements, called the 'over the top' length. Pumpkin growers keep records so that they can estimate the mass of giant pumpkins while they are still growing. The over the top lengths (d m) and the masses (m kg) of a random sample of 7 giant pumpkins are as follows.

d	2.31	2.9	4.05	5.5	6.7	7.92	9.17
m	11	14	47	104	170	282	449

(i) Draw a scatter diagram of these data, and explain how you know from your diagram that the relationship between m and d should not be modelled by an equation of the form $y = ax + b$. [2]

(ii) Which of the formulae $m = ed^2 + f$ and $m = gd^3 + h$, where e, f, g and h are constants, is the better model for the relationship between m and d? Explain fully how you decided, and find the constants for the better formula. [5]

(iii) Use the formula you chose from part (ii) to estimate the mass of a giant pumpkin with

(a) over the top length 6 m,
(b) over the top length 12 m.

Explain which of your two estimates is more reliable. [3]

7 'Bings' are sweets that are sold in packets of 6. Each packet is made up of randomly chosen coloured sweets. On average 10% of Bings are yellow.

(i) Explain why a binomial distribution is appropriate for modelling the number of yellow sweets in a packet. Find the probability that a randomly chosen packet of Bings contains no more than one yellow sweet. [3]

(ii) Kev buys 90 randomly chosen packets of Bings. Find the probability that at least 80 of these packets contain no more than one yellow sweet. [2]

On average the proportion of Bings that are red is p. It is known that the modal number of red sweets in a packet is 2.

(iii) Use this information to find exactly the range of values that p can take. [4]

8 A bag contains 3 blue counters, 1 red counter and y yellow counters. Darvina chooses 3 counters at random from the bag, without replacement. The random variable S is the sum of the number of blue counters chosen and twice the number of red counters chosen.

(i) Show that $\mathrm{P}(S=3)=\dfrac{6(3y+1)}{(y+4)(y+3)(y+2)}$. [2]

(ii) Given that $\mathrm{P}(S=3)=\dfrac{7}{20}$, calculate y. Hence find the probability distribution of S. [6]

9 A type of metal bolt is manufactured with a nominal radius of 0.8 cm. In fact, the radii of the bolts, measured in cm, have the distribution $\mathrm{N}(0.8, 0.01^2)$.

(i) Find the percentage of bolts that have a radius between 0.79 cm and 0.82 cm. [1]

Metal washers are manufactured to fit on the bolts. The inside radii of the washers, measured in cm, have the distribution $\mathrm{N}(0.81, 0.012^2)$.

(ii) Write down the distribution of the inside circumference of the washers, in cm, and find the circumference that is exceeded by 5% of the washers. [4]

A bolt and a washer are a 'good fit' if

- the inside radius of the washer is greater than the radius of the bolt and
- the inside radius of the washer is not more than 0.04 cm greater than the radius of the bolt.

(iii) A washer is chosen at random, and a bolt is chosen at random. Find the probability that the washer and bolt are a good fit. [3]

The outside radii of the washers, measured in cm, have the distribution $\mathrm{N}(\mu, \sigma^2)$. It is known that 15% of the washers have an outside radius greater than 1.25 cm and 25% have an outside radius of less than 1.15 cm.

(iv) Find the values of μ and σ. [4]

10 The average time required for the manufacture of a certain type of electronic control panel is 17 hours. An alternative manufacturing process is trialled, and the time taken, t hours, for the manufacture of each of 50 randomly chosen control panels using the alternative process is recorded. The results are summarised as follows.

$$n=50 \qquad \Sigma t=835.7 \qquad \Sigma t^2=14067.17$$

The Production Manager wishes to test whether the average time taken for the manufacture of a control panel is different using the alternative process, by carrying out a hypothesis test.

(i) Explain whether the Production Manager should use a 1-tail test or a 2-tail test. [1]

(ii) Explain why the Production Manager is able to carry out a hypothesis test without knowing anything about the distribution of the times taken to manufacture the control panels. [2]

(iii) Find unbiased estimates of the population mean and variance and carry out the test at the 10% level of significance for the Production Manager. [6]

(iv) Suggest a reason why the Production Manager might be prepared to use an alternative process that takes a longer average time than the original process. [1]

The Finance Manager wishes to test whether the average time taken for the manufacture of a control panel is shorter using the alternative process. The Finance Manager finds that the average time taken for the manufacture of each of 40 randomly chosen control panels, using the alternative process, is 16.7 hours. He carries out a hypothesis test at the 10% level of significance.

(v) Explain, with justification, how the population variance of the times will affect the conclusion made by the Finance Manager. [3]

Raffles Institution
H2 Mathematics (9758)
Solution for 2024 Mock Paper 2

<table>
<tr><th colspan="3">Solution</th></tr>
<tr><td>1(i)</td><td>

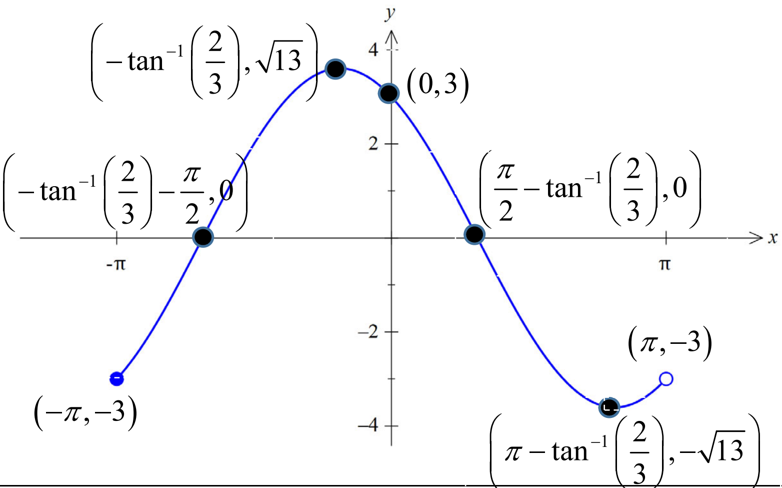
$\mathrm{f}(x) = 3\cos x - 2\sin x = R\cos(x+\alpha) = R\cos x\cos\alpha - R\sin x\sin\alpha$

$R\cos\alpha = 3 \qquad\qquad R\sin\alpha = 2$

$R = \sqrt{(R\cos\alpha)^2 + (R\sin\alpha)^2} = \sqrt{13}$ (or 3.61)

$\tan\alpha = \dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{2}{3} \Rightarrow \alpha = \tan^{-1}\left(\dfrac{2}{3}\right)$ (or 0.588)

$\mathrm{f}(x) = \sqrt{13}\cos(x + \tan^{-1}\dfrac{2}{3})$ and $R_{\mathrm{f}} = \left[-\sqrt{13},\ \sqrt{13}\right]$



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Use R-formula

Note that the question does not require exact values, so you may round your answer to 3sf.

$\mathrm{f}(x)$ can be obtained by translating $\mathrm{f}(x) = \sqrt{13}\cos x$ by $\tan^{-1}\dfrac{2}{3}$ units along negative x direction

Due to the domain of f, the right end-point must be excluded.

</td></tr>
<tr><td>1(ii)</td><td>

For g^{-1} exists, the largest value of b is $\pi - \alpha = \pi - \tan^{-1}\left(\dfrac{2}{3}\right)$

Let $y = \mathrm{g}(x) = 3\cos x - 2\sin x = \sqrt{13}\cos\left(x + \tan^{-1}\left(\dfrac{2}{3}\right)\right)$

$x = \cos^{-1}\dfrac{y}{\sqrt{13}} - \tan^{-1}\left(\dfrac{2}{3}\right)$

Therefore, $\mathrm{g}^{-1}(x) = \cos^{-1}\dfrac{x}{\sqrt{13}} - \tan^{-1}\left(\dfrac{2}{3}\right)$

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Solution	
2 (i)	If S_n is a quadratic polynomial in n, $S_n = an^2 + bn + c$, where a, b and c are constants. $S_1 = a + b + c = 3$ (1) $S_2 = 4a + 2b + c = 3 + 1 = 4$ (2) $S_3 = 9a + 3b + c = 3 + 1 + 1 = 5$ (3) $S_4 = 16a + 4b + c = 3 + 1 + 1 + 3 = 8$ (4) From GC, the system of linear equations has no solution, hence S_n cannot be a quadratic polynomial in n.
2 (ii)	Given S_n is a cubic polynomial in $n \Rightarrow S_n = an^3 + bn^2 + cn + d$ where a, b, c and d are constants $S_1 = a + b + c + d = 3$ $S_2 = 8a + 4b + 2c + d = 4$ $S_3 = 27a + 9b + 3c + d = 5$ $S_4 = 64a + 16b + 4c + d = 8$ Solving using GC, $a = \frac{1}{3},\ b = -2,\ c = \frac{14}{3},\ d = 0$ Therefore, $S_n = \frac{1}{3}n^3 - 2n^2 + \frac{14}{3}n$
2 (iii)	For $n \geq 2$, $u_n = S_n - S_{n-1}$ $= \left(\frac{1}{3}n^3 - 2n^2 + \frac{14}{3}n\right) - \left[\frac{1}{3}(n-1)^3 - 2(n-1)^2 + \frac{14}{3}(n-1)\right]$ $= \left(\frac{1}{3}n^3 - 2n^2 + \frac{14}{3}n\right) - \left[\frac{1}{3}(n^3 - 3n^2 + 3n - 1) - 2(n^2 - 2n + 1) + \frac{14}{3}(n-1)\right]$ $= \left(\frac{1}{3}n^3 - 2n^2 + \frac{14}{3}n\right) - \left[\frac{1}{3}n^3 - n^2 + n - \frac{1}{3} - 2n^2 + 4n - 2 + \frac{14}{3}n - \frac{14}{3}\right]$ $= n^2 - 5n + 7$ When $n = 1$, $u_1 = S_1 = 3 = 1^2 - 5(1) + 7$, which also follows the form $n^2 - 5n + 7$. Hence, the nth term of the sequence is $n^2 - 5n + 7$.

Solution		
3 (a)	Since angle between vectors is $\cos^{-1}\left(\frac{6}{13}\right)$, $\frac{\begin{pmatrix}3\\-2\\0\end{pmatrix}.\begin{pmatrix}6\\d\\\sqrt{7}\end{pmatrix}}{\sqrt{9+4}\sqrt{36+d^2+7}}=\frac{6}{13}$ $\frac{18-2d}{\sqrt{13}\sqrt{43+d^2}}=\frac{6}{13}$ $13^2(18-2d)^2=36(13)(43+d^2)$ $169(324+4d^2-72d)=20124+468d^2$ $208d^2-12168d+34632=0$ $2d^2-117d+333=0$ (shown)	
3 (b)(i)	$\overrightarrow{OE}=\overrightarrow{OA}+\overrightarrow{AE}=\mathbf{a}+\lambda\overrightarrow{AD}=\mathbf{a}+\lambda(4\mathbf{b}-\mathbf{a})$ for some $\lambda\in\mathbb{R}$ Similarly, $\overrightarrow{OE}=\overrightarrow{OB}+\overrightarrow{BE}=\mathbf{b}+\mu\overrightarrow{BC}=\mathbf{b}+\mu(6\mathbf{a}-\mathbf{b})$, for some $\mu\in\mathbb{R}$ Therefore, $\overrightarrow{OE}=\mathbf{a}+\lambda(4\mathbf{b}-\mathbf{a})=\mathbf{b}+\mu(6\mathbf{a}-\mathbf{b})$ $\Rightarrow(1-\lambda-6\mu)\mathbf{a}=(1-4\lambda-\mu)\mathbf{b}$ Since $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors, $1-\lambda-6\mu=0$ ---(1) $1-4\lambda-\mu=0$ ---(2) Solving (1) and (2), $\lambda=\frac{5}{23}$, $\mu=\frac{3}{23}$ $\overrightarrow{OE}=\mathbf{a}+\frac{5}{23}(4\mathbf{b}-\mathbf{a})=\frac{18}{23}\mathbf{a}+\frac{20}{23}\mathbf{b}$	Alternatively, write down equation of lines AD and BC and use them to solve for $\overrightarrow{OE}$ since E lies on both lines. Alternatively, make use of uniqueness of linear combination of two non-zero and non-parallel vectors
3 (b)(ii)	$\overrightarrow{OE}=\frac{18}{23}\mathbf{a}+\frac{20}{23}\mathbf{b}$	

	Using ratio theorem, $\overrightarrow{OF}=\frac{5\overrightarrow{OD}+3\overrightarrow{OC}}{3+5}=\frac{5(4\mathbf{b})+3(6\mathbf{a})}{3+5}=\frac{18\mathbf{a}+20\mathbf{b}}{8}=\frac{23}{8}\overrightarrow{OE}$ Since $\overrightarrow{OE}$ is parallel to $\overrightarrow{OF}$ with common point O, O, E and F are collinear $OE : OF$ = 8: 23	

<table>
<tr><th colspan="3">Solution</th></tr>
<tr><td>4 (i)</td><td>

$y=\tan\left(\mathrm{e}^{2x}-1\right)$

$\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{2x}\sec^2\left(\mathrm{e}^{2x}-1\right)=2\mathrm{e}^{2x}\left[1+\tan^2\left(\mathrm{e}^{2x}-1\right)\right]=2\mathrm{e}^{2x}\left(1+y^2\right)$

Differentiating with respect to x,

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\mathrm{e}^{2x}\left(2y\frac{\mathrm{d}y}{\mathrm{d}x}\right)+4\mathrm{e}^{2x}\left(1+y^2\right)$

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=4\mathrm{e}^{2x}\left(1+y\frac{\mathrm{d}y}{\mathrm{d}x}+y^2\right)$

$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=8\mathrm{e}^{2x}\left(1+y\frac{\mathrm{d}y}{\mathrm{d}x}+y^2\right)+4\mathrm{e}^{2x}\left(y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left[\frac{\mathrm{d}y}{\mathrm{d}x}\right]^2+2y\frac{\mathrm{d}y}{\mathrm{d}x}\right)$

When $x=0$, $y=0$, $\frac{\mathrm{d}y}{\mathrm{d}x}=2$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=4$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=24$

</td><td>Try to express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y so that implicit differentiation can be carried out. Tedious to apply direct differentiation here.</td></tr>
<tr><td>4 (ii)</td><td>

$\tan\left(\mathrm{e}^{2x}-1\right)\approx 0+2x+\frac{4}{2}x^2+\frac{24}{6}x^3+\ldots$

$=2x+2x^2+4x^3+\ldots$

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<tr><td>4 (iii)</td><td colspan="2">

$\mathrm{e}^{ax}\ln\left(1+nx\right)=\left(1+ax+\frac{(ax)^2}{2!}+\frac{(ax)^3}{3!}+\ldots\right)\left(nx-\frac{(nx)^2}{2}+\frac{(nx)^3}{3}+\ldots\right)$

$=nx+\left[(ax)(nx)-\frac{(nx)^2}{2}\right]+\left[(ax)\left(-\frac{(nx)^2}{2}\right)+\frac{(nx)^3}{3}+(nx)\left(\frac{(ax)^2}{2!}\right)\right]+\ldots$

$=nx+\left(an-\frac{n^2}{2}\right)x^2+\left(-\frac{an^2}{2}+\frac{n^3}{3}+\frac{a^2n}{2}\right)x^3$

$\tan\left(\mathrm{e}^{2x}-1\right)=2x+2x^2+4x^3+\ldots$

Comparing the coefficients of x and x^2,

$n=2$

$2a-\frac{(2)^2}{2}=2\Rightarrow a=2$

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	Coefficient of $x^3 = -\frac{2(2^2)}{2} + \frac{2^3}{3} + \frac{2^2(2)}{2!} = \frac{8}{3}$ Hence, the third non-zero term is $\frac{8}{3}x^3$

<table>
<tr><th colspan="3">Solution</th></tr>
<tr><td>5 (i)</td><td>Number of ways of arranging the 6 couples = 6!

Number of ways of arranging the husband and wife within each of the 6 couples = 2^6

Required number of ways = $=6!\times 2^6=46080$</td><td></td></tr>
<tr><td>5 (ii)</td><td>Case 1 : wives sit together at one end and husbands sit at the other end

Number of ways to arrange the 6 wives within the unit $=6!$

Number of ends to sit the unit = 2

Number of choices for the husband on the end of the unit = 5

Number of ways to sit the remaining 5 men = $5!$

Number of ways $=6!\times 5\times 5!\times 2=864000$

Case 2 : wives sit together, but not at any ends of the row

A unit then consists of 2 husbands and 6 wives, with each husband on the extreme ends of the unit.

Number of ways to arrange the 6 wives within the unit $=6!$

Number of ways to sit the 2 husbands beside the wives

$=(1\times 5)+(4\times 4)=21$

<table><tr><td>1</td><td>W_a W_b W_c W_d W_e W_f</td><td>5</td></tr><tr><td>H_f</td><td></td><td></td></tr></table>
<table><tr><td>4</td><td>W_a W_b W_c W_d W_e W_f</td><td>4</td></tr><tr><td>not H_f</td><td></td><td></td></tr></table>
Number of ways to arrange this unit and the remaining 4 men

$=(4+1)!$

Number of ways $=6!\times 21\times 5!=1814400$

Required number of ways $=864000+1814400=2678400$</td><td>If the man sitting to the left of the wives is the husband to the right-most wife, all the remaining 5 men can be chosen to sit on the right of the wives.

In all other cases, only 4 men can sit on the right of the wives (excluding the husband of the right-most wife).</td></tr>
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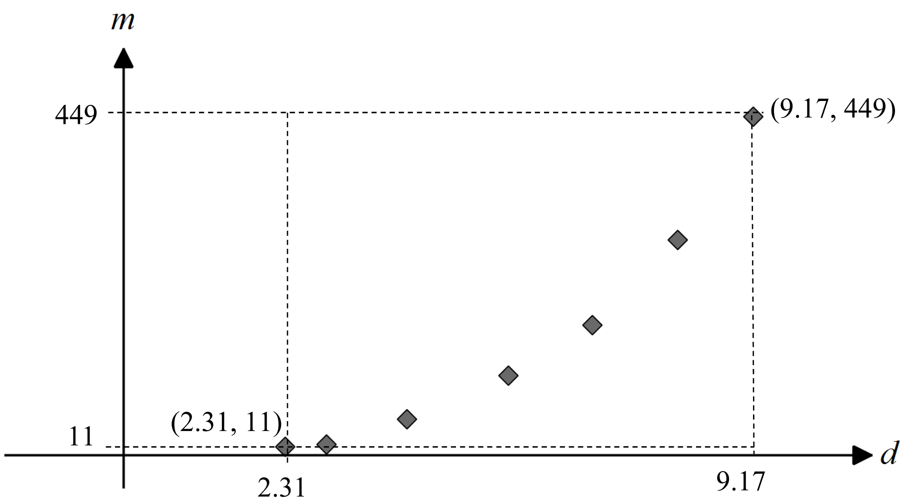
	Alternative method (Complement): Consider the 6 wives as a unit. Number of ways with wives sitting together, and left-most wife sitting with her husband $= 6!\times(5+1)! = 518400$ ------- (A) Number of ways with wives sitting together, and right-most wife sitting with her husband $= 6!\times(5+1)! = 518400$ -------- (B) Number of ways with wives together, and <u>both</u> left- and right-most wife sitting with their husbands $= 6!\times(4+1)! = 86400$ -------- (C) Number of ways with wives sitting together, and <u>either</u> left- or right-most wife sitting with her husband $= \text{A}+\text{B}-\text{C} = 518400+518400-86400 = 950400$ Number of ways with wives sitting together $= 6!\times(6+1)! = 3628800$ Required number of ways $= 3628800-950400 = 2678400$	(C) is included in both (A) and (B), so we have to remove a copy of it.
5 (iii)	**Case 1** : Exactly 1 wife in the committee Number of ways $= {}^6C_1 \times {}^5C_2 = 60$ **Case 2** : Exactly 2 wives in the committee Number of ways $= {}^6C_2 \times {}^4C_1 = 60$ **Case 3** : 3 wives in the committee Number of ways $= {}^6C_3 = 20$ Required number of ways = $60+60+20 = 140$ **Alternative method (Complement):** Number of ways without restriction $= {}^{12}C_3 = 220$ Number of ways such that there are no wives $= {}^6C_3 = 20$ Number of ways such that there is exactly a couple included $= {}^6C_1 \times {}^{10}C_1 = 60$ Required number of ways = $220-20-60 = 140$	Note that the case with no wives and the case with a couple are mutually exclusive

Solution		
6 (i)	 Since m increases at an increasing rate as d increases, the relationship between m and d should not be modelled by a linear equation of the form $y = ax + b$.	
6 (ii)	For the formula: $m = ed^2 + f$, $r = 0.98889$ (5 s.f.) where $m = 5.4324d^2 + 41.790$ (5 s.f.) For the formulae : $m = gd^3 + h$, $r = 0.99951$ (5 s.f.) where $m = 0.57165d^3 + 3.7431$ (5 s.f.) Since the magnitude of r for the formula: $m = gd^3 + h$ is closer to 1 than the other formula, the model using the formula: $m = gd^3 + h$ is a better model. Therefore, $m = 0.572d^3 + 3.74$ (3 s.f.)	
6 (iii)	**(a)** The mass of a giant pumpkin with over the top length 6 m = 127.22 = 127 (3 s.f.) **(b)** The mass of a giant pumpkin with over the top length 12 m = 991.55 = 992 (3 s.f.) Since 6 lies within the range of data values of d but 12 does not, the estimate in **(a)** is more reliable.	

Solution		
7 (i)	There are only two outcomes for a sweet, either "yellow" or "not yellow" and there is a fixed number of 6 sweets in each packet. Since the sweets are randomly chosen and the probability of choosing a yellow sweet is 0.1 on average, we may assume this probability to be constant, and that randomly chosen sweets being yellow are independent to one another. Let X be the number of sweets out of 6 in the packet that are yellow. $X \sim \mathrm{B}(6, 0.1)$ $\mathrm{P}(X \le 1) = 0.885735$	This question is <u>not</u> asking for assumption. So, there is a need to write out all the conditions for it to be a binomial distribution. The final answer is exact.
7 (ii)	Let Y be the number of packets out of 90 that contain no more than one yellow sweet. $Y \sim \mathrm{B}(90, 0.885735)$ $\mathrm{P}(Y \ge 80) = 1 - \mathrm{P}(Y \le 79) = 0.546$ (3 s.f.)	
7 (iii)	Let R be the number of sweets out of 6 sweets in the packet that are red, then $R \sim \mathrm{B}(6, p)$ Since mode of R is 2, $\mathrm{P}(R = 2) > \mathrm{P}(R = 3)$ and $\mathrm{P}(R = 2) > \mathrm{P}(R = 1)$ $\mathrm{P}(R = 2) > \mathrm{P}(R = 3) \Rightarrow \binom{6}{2} p^2 (1-p)^4 > \binom{6}{3} p^3 (1-p)^3$ $\Rightarrow 15p^2(1-p)^4 > 20p^3(1-p)^3$ $\Rightarrow (1-p) > \frac{4}{3}p$ since $p > 0$ and $(1-p) > 0$ $\Rightarrow \left(\frac{4}{3}+1\right)p < 1$ $\Rightarrow p < \frac{3}{7}$ $\mathrm{P}(R = 2) > \mathrm{P}(R = 1) \Rightarrow \binom{6}{2} p^2 (1-p)^4 > \binom{6}{1} p(1-p)^5$	As the question requires the exact range of values, all workings must be shown. Do not use the table of values from GC as working. $0 < p < 1 \Rightarrow 1 - p > 0$

	$\Rightarrow 15p^2(1-p)^4 > 6p(1-p)^5$ $\Rightarrow \frac{5}{2}p > (1-p)$ since $p>0$ and $(1-p)>0$ $\Rightarrow \left(\frac{5}{2}+1\right)p > 1$ $\Rightarrow p > \frac{2}{7}$ Therefore, $\frac{2}{7} < p < \frac{3}{7}$	

Solution		
8(i)	For S to be 3, either 1 blue, 1 red and 1 yellow are chosen or 3 blues are chosen $\mathrm{P}(1R,1B,1Y) = \frac{3}{y+4}\times\frac{1}{y+3}\times\frac{y}{y+2}\times 3!$ $= \frac{18y}{(y+4)(y+3)(y+2)}$ $\mathrm{P}(3B) = \frac{3}{y+4}\times\frac{2}{y+3}\times\frac{1}{y+2} = \frac{6}{(y+4)(y+3)(y+2)}$ $\mathrm{P}(S=3) = \frac{18y}{(y+4)(y+3)(y+2)} + \frac{6}{(y+4)(y+3)(y+2)}$ $= \frac{18y+6}{(y+4)(y+3)(y+2)}$ $= \frac{6(3y+1)}{(y+4)(y+3)(y+2)}$ (Shown)	Note that this expression works even if there are no yellow counters.
8(ii)	$\mathrm{P}(S=3) = \frac{6(3y+1)}{(y+4)(y+3)(y+2)} = \frac{7}{20}$ Using GC, since y is a positive integer, $y = 2$.	Key $\mathrm{Y}_1 = \frac{6(3x+1)}{(x+4)(x+3)(x+2)}$

$$\mathrm{P}(S=1)=\mathrm{P}(1B,2Y)=\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}\times\frac{3!}{2!}=\frac{18}{120}=\frac{3}{20}$$

$$\mathrm{P}(S=2)=\mathrm{P}(2B,1Y)+\mathrm{P}(1R,2Y)$$
$$=\frac{3}{6}\times\frac{2}{5}\times\frac{2}{4}\times\frac{3!}{2!}+\frac{1}{6}\times\frac{2}{5}\times\frac{1}{4}\times\frac{3!}{2!}=\frac{7}{20}$$

$$\mathrm{P}(S=4)=\mathrm{P}(2B,1R)=\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}\times\frac{3!}{2!}=\frac{18}{120}=\frac{3}{20}$$

s	1	2	3	4
$\mathrm{P}(S=s)$	$\frac{3}{20}$	$\frac{7}{20}$	$\frac{7}{20}$	$\frac{3}{20}$

X	Y1
0	$\frac{1}{4}$
1	$\frac{2}{5}$
2	$\frac{7}{20}$
3	$\frac{2}{7}$

The solution is clearer when you tabulate the probabilities

Solution		
9(i)	Let X be the radius of a randomly chosen metal bolt in cm. $X \sim \mathrm{N}\left(0.8,\ 0.01^2\right)$ $\mathrm{P}\left(0.79 < X < 0.82\right) = 0.81859$ (5 s.f.) Required percentage is 81.9%. (3 s.f.)	
9(ii)	Let Y be the inside radius of a randomly chosen washer in cm $Y \sim \mathrm{N}\left(0.81,\ 0.012^2\right)$ Let C be the inside circumference of a randomly chosen washer in cm Since $C = 2\pi Y$, $C \sim \mathrm{N}\left(2\pi(0.81),\ (2\pi(0.012))^2\right)$ Therefore, $C \sim \mathrm{N}\left(5.09,\ 0.00568\right)$ (3 s.f.) Let c be the circumference that is exceeded by 5% of the washers. $\mathrm{P}\left(C > c\right) = 0.05$ Using GC, $c = 5.21$ (3 s.f.)	Use the exact parameters of C for better accuracy.
9(iii)	$Y - X \sim \mathrm{N}\left(0.81 - 0.8,\ 0.012^2 + 0.01^2\right)$ $Y - X \sim \mathrm{N}\left(0.01,\ 0.000244\right)$ $\mathrm{P}\left(0 < Y - X \le 0.04\right) = 0.71158$ (5 s.f.) $= 0.712$ (3 s.f.)	
9(iv)	Let T be the the outside radius of a randomly chosen washer in cm. $T \sim \mathrm{N}\left(\mu,\ \sigma^2\right)$. $\mathrm{P}\left(T > 1.25\right) = 0.15$ $\mathrm{P}\left(Z > \frac{1.25-\mu}{\sigma}\right) = 0.15$ Using GC, $\frac{1.25-\mu}{\sigma} = 1.0364$ $\mu + 1.0364\sigma = 1.25 \quad ---(1)$ $\mathrm{P}\left(T < 1.15\right) = 0.25$ $\mathrm{P}\left(Z < \frac{1.15-\mu}{\sigma}\right) = 0.25$ Using GC, $\frac{1.15-\mu}{\sigma} = -0.67449$ $\mu - 0.67449\sigma = 1.15 \quad ---(2)$ Using GC, $\mu = 1.19$, $\sigma = 0.0584$ (3 s.f.)	

Solution		
10(i)	Since the Production Manager is looking for change in the average time which can be more or less than the current one, he should use a 2-tail test.	
10(ii)	Since the sample size of 50 is large, by Central Limit Theorem, and the average time taken will follow a normal distribution approximately. Furthermore, unbiased estimates of the population mean and variance can be found using the sample data.	
10(iii)	Let T be the time taken to manufacture of a randomly chosen electronic control panel. The unbiased estimate for the population mean is $\bar{t} = \frac{835.7}{50} = 16.714$ The unbiased estimate for the population variance is $s^2 = \frac{1}{50-1}\left(14067.17 - \frac{(835.7)^2}{50}\right) = 2.0261 \text{ (5 s.f.)} = 2.03 \text{ (3 s.f.)}$ $\mathrm{H}_0 : \mu = 17 \qquad \mathrm{H}_1 : \mu \neq 17$ Perform a two-tail test at 10% level of significance Under H_0, since n = 50 is large, by Central Limit Theorem, $\bar{T} \sim \mathrm{N}\left(17, \frac{2.0261}{50}\right)$ approximately. Using a z-test, p-value = $2\mathrm{P}\left(\bar{T} < 16.714\right) = 0.155389 > 0.10$ Hence, we do not reject H_0 at 10% level of significance and there is insufficient evidence to conclude that the population average time taken in hours for the manufacture of a control panel has changed.	Keep more decimal places for better accuracy.
10(iv)	The Production Manager might be prepared to use an alternative process which might be more resource-efficient (or produce electronic control panels of better quality).	

10(v)	Let the population variance of the time taken to manufacture a randomly chosen control panel be σ^2. $\mathrm{H_0}: \mu = 17 \qquad \mathrm{H_1}: \mu < 17$ Perform a one-tail test at 10% level of significance Under $\mathrm{H_0}$, since $n = 40$ is large, by Central Limit Theorem, $\bar{T} \sim \mathrm{N}\left(17, \dfrac{\sigma^2}{40}\right)$ approximately where $\bar{t} = 16.7$ To reject $\mathrm{H_0}$, p-value = $\mathrm{P}\left(\bar{T} < 16.7\right) \leq 0.10$ $\Rightarrow \mathrm{P}\left(Z \leq \dfrac{16.7-17}{\frac{\sigma}{\sqrt{40}}}\right) \leq 0.1$ Since $\mathrm{P}\left(Z \leq -1.2816\right) = 0.1$, $\dfrac{16.7-17}{\frac{\sigma}{\sqrt{40}}} \leq -1.2816$ $\sigma \leq 1.4805$ $\sigma^2 \leq 2.1918 = 2.19 \quad$ (3 s.f.) If the population variance of the times is less than 2.19 hours, the Finance Manager will conclude that there is sufficient evidence that the average time using the alternative process is shorter.	Note that the question asks for population variance, not its standard deviation.