| Name: | Class: | Class Register Number: |
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中正中学
CHUNG CHENG HIGH SCHOOL (MAIN)



**PRELIMINARY EXAMINATION 2024**
SECONDARY 4

**MATHEMATICS** **4052/01**

**Paper 1** **Friday 23 August 2024**

**2 hours 15 minutes**

Candidates answer on the Question Paper.

# MARKS SCHEME

This document consists of **20** printed pages.

# *Mathematical Formulae*

*Compound interest*

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

*Mensuration*

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC$ = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where $\theta$ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where $\theta$ is in radians

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

**1** The length of a court is 0.028 km and the width of the court is 16 m. Express the ratio of the length to the width of the court in its simplest form.

$28:16$
$7:4$

*Answer* ..... **:** ..... [1]

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**2** Given that $0 < x < 180$, find the possible values of $x$ for $\sin x° = 0.985$.
Give your answers correct to 1 decimal place.

$\sin x° = 0.985$
$x = \sin^{-1} 0.985$ or $180 - \sin^{-1} 0.985$
$= 80.1$ or $99.9$

*Answer* $x$ = .....or..... [2]

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**3** **(a)** Simplify $\dfrac{\left(4a^4b\right)^3}{10a^3b^7}$, leaving your answer in positive index form.

$$\frac{\left(4a^4b\right)^3}{10a^3b^7} = \frac{64a^{12}b^3}{10a^3b^7}$$
$$= \frac{32a^9}{5b^4}$$

*Answer* ..... [2]

**(b)** Given that $2^x = 3$, $2^y = 7$ and $2^z = \dfrac{9}{49}$, use the laws of indices to find the value of $2^{3x-y+\frac{1}{2}z}$.

$$2^{3x-y+\frac{1}{2}z} = \left(2^x\right)^3 \div 2^y \times \left(2^z\right)^{\frac{1}{2}}$$
$$= 3^3 \div 7 \times \sqrt{\frac{9}{49}}$$
$$= \frac{81}{49}$$

Law of indices:
1: $\left(a^m\right)^n = a^{mn}$
2: $a^m \times a^n = a^{m+n}$
3: $a^m \div a^n = a^{m-n}$
4: $a^{\frac{1}{n}} = \sqrt[n]{a}$

to award M1 if student show understanding of either law of indices

*Answer* ..... [2]

**4** A group of students won \$226 in a competition and shared the amount equally, leaving \$4 leftover. Competing for a second time, the same group of students won \$296. They shared the \$296 along with the \$4 left over from the first competition equally, with no amount remaining.

Assuming that each student received a whole number of dollars in both distributions, find the greatest possible number of students in the group.

| | 222 | 300 |
|---|---|---|
| 2 | 111 | 150 |
| 3 | 37 | 50 |

HCF is 6. There are 6 students.

*Answer* ……students [2]

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**5** **(a)** On 6 June 2020, Elijah invested some money in a bank which pays a simple interest at a rate of 3.5% per annum. He received a total interest of \$680.40 on 6 June 2023.

Find the amount of money Elijah invested in the bank.

$$\text{Amount of interest earn per year} = \$680.40 \div 3$$
$$= \$226.80$$
$$\text{Amount of money invested} = \frac{100}{3.5} \times \$226.80$$
$$= \$6480$$

*Answer* \$…….... [2]

**(b)** Freddy invested \$8000 in another bank that paid compound interest at a rate of 1.75% per annum, compounded quarterly.

Find the total amount Freddy received at the end of 5 years. Give your answer correct to the nearest cent.

$$\text{Total amount} = 8000\left(1 + \frac{1.75 \div 4}{100}\right)^{5\times 4}$$
$$= \$8729.87 \text{ (nearest cent)}$$

*Answer* \$…….... [2]

**6**

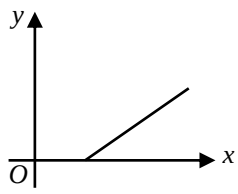


**Diagram 1**

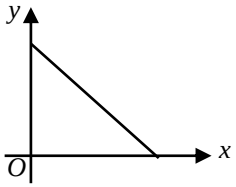


**Diagram 2**

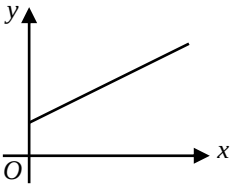


**Diagram 3**

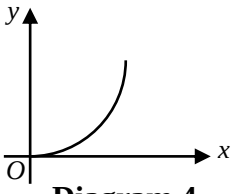


**Diagram 4**

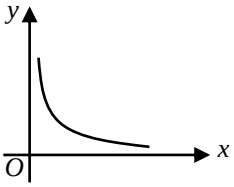


**Diagram 5**

From the diagrams above, select one of them which best illustrates each of the following statements.

**(a)** $y = 4\pi x^3$.

**(b)** The cost of a project \$$y$ is a linear function of $x$, where $x$ is the number of man-hours required to complete the project. The project has a fixed cost of \$100.

**(c)** An object is travelling at a constant speed towards a fixed point $O$. The distance $y$ (in metres) represents how far the object is from point $O$ at time $x$ (in minutes).

*Answer* **(a)** Diagram ……4…… [1]

**(b)** Diagram ……3…… [1]

**(c)** Diagram ……2…… [1]

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**7** Write as a single fraction in its simplest form $\dfrac{4-x}{x+2}-\dfrac{2}{3-x}$.

$$\frac{4-x}{x+2}-\frac{2}{3-x}=\frac{(4-x)(3-x)-2(x+2)}{(x+2)(3-x)}$$

$$=\frac{12-7x+x^2-2x-4}{(x+2)(3-x)}$$

$$=\frac{x^2-9x+8}{(x+2)(3-x)}$$

$$=\frac{(x-1)(x-8)}{(x+2)(3-x)}$$

*Answer* ……..... [2]

**8** The table shows the distribution of the weights of 40 students.

| Weight ($W$ kg) | $40 < W \leq 50$ | $50 < W \leq 60$ | $60 < W \leq 70$ | $70 < W \leq 80$ | $80 < W \leq 90$ |
|---|---|---|---|---|---|
| Frequency | 6 | 8 | $a$ | 11 | 3 |

**(a)** Find the value of $a$.

*Answer* $a =$ 12 [1]

**(b)** Calculate an estimate for

**(i)** the mean weight of the students,

*Answer* 64.25 kg [1]

**(ii)** the standard deviation of the weights.

*Answer* 11.7 kg [1]

**(c)** An error in the weighing machine caused the students' weights to be recorded 2 kg more than their actual values.

Explain how the mean and standard deviation will change after the error is rectified.

*Answer*

The mean will decrease by 2kg to 62.25kg while the standard deviation remains unchanged. [1]

**9** Andy bought a limited-edition watch from an online website based in Thailand for 10 460 baht. Andy also paid \$15 in Singapore dollars for shipping and GST of 9% on the cost of the watch. The exchange rate between Thai baht (฿) and Singapore dollars (S\$) was ฿100=S\$$P$. Andy spent a total of \$436 in Singapore dollars. Find the value of $P$, giving your answer correct to 3 significant figures.

$$\text{Cost of watch in Singapore dollars (without GST)} = \frac{10460}{100} \times P$$
$$= 104.6P$$
$$\text{Cost of watch in Singapore dollars (with GST)} = 1.09 \times 104.6P$$
$$= 114.014P$$
$$114.014P + 15 = 436$$
$$P = 3.69 \text{(3sf)}$$

*Answer* $P =$..... [3]

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**10** The gradient of the line joining the points $(-3+2a, 7)$ and $(a+1, 2)$ is $-\frac{2}{3}$. Find the value of $a$.

$$\text{gradient} = \frac{2-7}{a+1-(-3+2a)}$$
$$= \frac{-5}{-a+4}$$
$$\text{Given gradient } = -\frac{2}{3}$$
$$\frac{-5}{-a+4} = -\frac{2}{3}$$
$$-15 = -2(-a+4)$$
$$a = -\frac{7}{2}$$

*Answer* $a =$ ..... [2]

**11** Given that $a^2 - \frac{6a}{b} + \frac{9}{b^2} = 0$, find the value of $ab$.

$$a^2 - \frac{6a}{b} + \frac{9}{b^2} = 0$$
$$a^2 - 2a\left(\frac{3}{b}\right) + \frac{9}{b^2} = 0$$
$$\left(a - \frac{3}{b}\right)^2 = 0$$
$$a - \frac{3}{b} = 0$$
$$a = \frac{3}{b}$$
$$ab = 3$$

*Answer* $ab =$..... [2]

**12** Factorise completely $4mn-16n^2-4m^2n+64n^3$.

$$\begin{aligned}4mn-16n^2-4m^2n+64n^3 &= 4n\left(m-4n\right)-4n\left(m^2-16n^2\right)\\ &= 4n\left(m-4n\right)-4n\left(m-4n\right)\left(m+4n\right)\\ &= 4n\left(m-4n\right)\left(1-m-4n\right)\end{aligned}$$

*Answer* ……..... [3]

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**13** A bag contains 4 black coins, 7 red coins and 11 white coins. Two coins are drawn from the bag at random, one after another without replacement.

**(a)** Find the probability that a white coin will be chosen on the second draw.

$$\begin{aligned}\text{Total number of coins} &= 4+7+11\\ &= 22\\ \text{P}\left(\text{second coin is white}\right) &= \text{P}\left(\text{1st coin of any colours, 2nd coin is white}\right)\\ &= 1\times\frac{11}{22}\\ &= \frac{1}{2}\end{aligned}$$

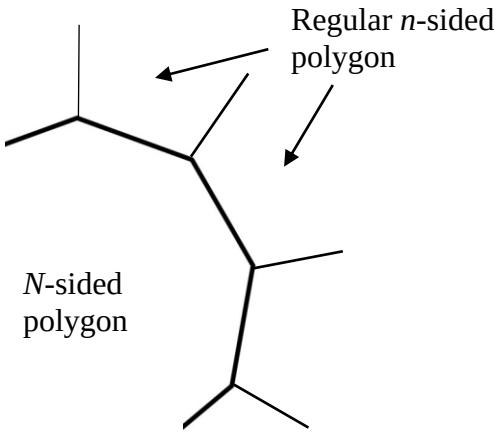
*Answer* ……..... [1]

**(b)** $x$ yellow coins are added to the bag. The probability of picking a black coin in **both** draws is $\frac{1}{50}$. Find the value of $x$.

$$\begin{aligned}&\text{Total number of coins } = 22+x\\ &\text{P}\left(\text{both coins are black}\right) = \frac{4}{22+x}\times\frac{3}{21+x}\\ &\frac{12}{\left(22+x\right)\left(21+x\right)} = \frac{1}{50}\\ &600 = \left(22+x\right)\left(21+x\right)\\ &600 = 462+22x+21x+x^2\\ &x^2+43x-138=0\\ &\left(x+46\right)\left(x-3\right)=0\\ &x=-46 \text{ or } \quad x=3\\ &\text{(rejected)}\end{aligned}$$

*Answer* $x$ = ……..... [3]

**14** A number of regular *n*-sided polygons are placed together in a ring to form a regular *N*-sided polygon as shown in the diagram below.




**(a)** Show that $N = \frac{2n}{n-4}$.

*Answer*

$$\frac{180(n-2)}{n} \times 2 + \frac{180(N-2)}{N} = 360$$
$$2N(n-2) + n(N-2) = 2nN$$
$$2nN - 4N + nN - 2n = 2nN$$
$$2n = nN - 4N$$
$$N(n-4) = 2n$$
$$N = \frac{2n}{n-4}$$

[3]

**(b)** Hence, explain why a regular octagon cannot be formed by placing smaller *n*-sided regular polygons in a ring.

*Answer*

$$\frac{2n}{n-4} = 8$$
$$2n = 8n - 32$$
$$6n = 32$$
$$n = \frac{16}{3}$$

It is not a octagon as *n* is not an integer.

[2]

**15** **(a)** Express $x^2+5x+7$ in the form $(x+a)^2+b$.

$$x^2+5x+7=\left(x+\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2+7$$
$$=\left(x+\frac{5}{2}\right)^2+\frac{3}{4}$$

*Answer* ……..... [2]

**(b)** Hence, explain why the expression will never be negative.

*Answer*

Since $\left(x+\frac{5}{2}\right)^2 \geq 0$ for all $x$, $\left(x+\frac{5}{2}\right)^2+\frac{3}{4} \geq \frac{3}{4}$. Therefore the expression will never be negative.

[1]

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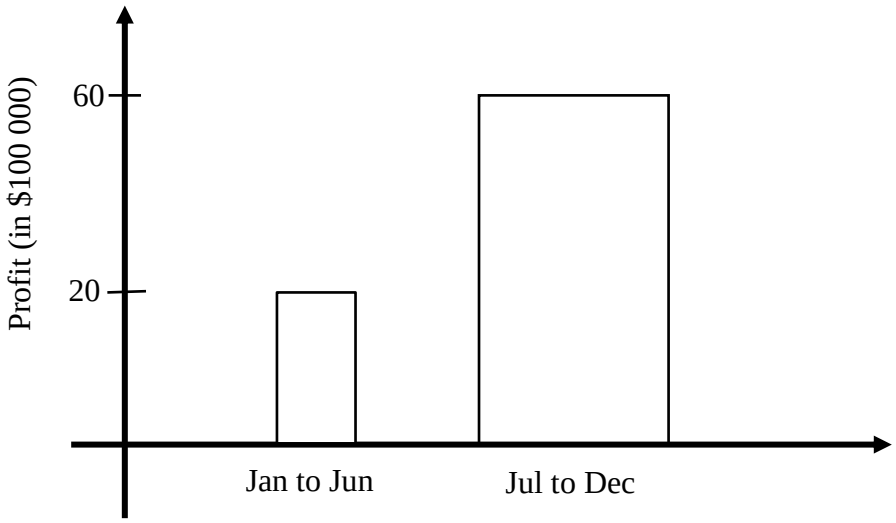
**16** A company presented their 2023 financial report in this graph.




State one aspect of the graph that may be misleading and explain how they may lead to a misinterpretation.

*Answer*

Each bar in the graph should be of the same width. The different widths of each bar will lead readers to misinterpretation such as interpreting a wider bar to mean a profit of a larger proportion that it actually is.

[2]

**17** $\varepsilon = \{\text{integer } x:\ 0 < x \leq 12\}$

$A = \{\text{prime numbers}\}$

$B = \{\text{numbers that have at least 2 distinct factors}\}$

**(a)** Explain why $A$ is a proper subset of $B$ without listing down the elements.

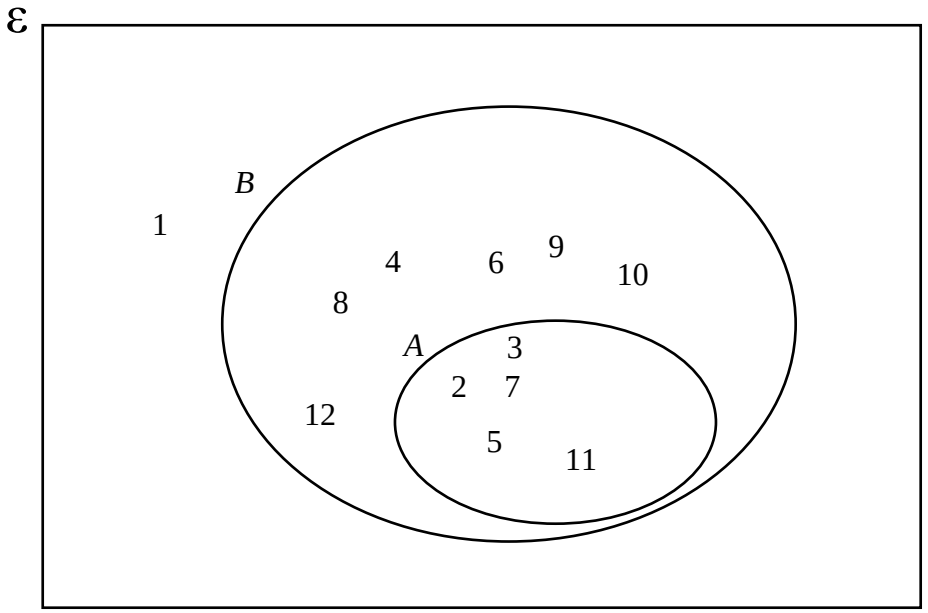
*Answer*

Prime numbers have 2 distinct factors, therefore every element of set $A$ must also be an element of set $B$. However, some elements in $B$ are composites and therefore not in Set $A$. Thus $A$ is a proper subset of $B$.

[2]

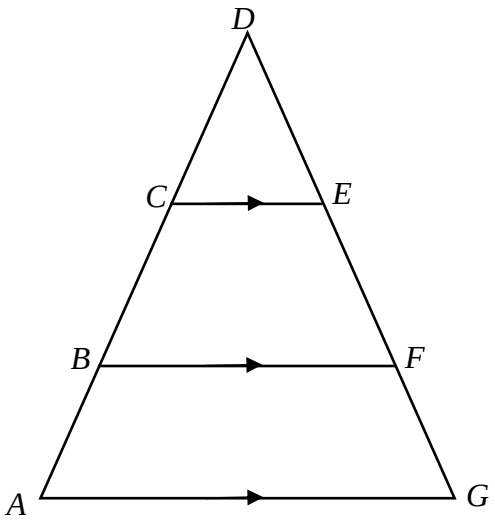
**(b)** Draw a Venn diagram and list down the elements to illustrate the above information.

*Answer*




[2]

**18** The diagram shows a triangle $AGD$ with the points $C$ and $B$ lying on $AD$ and points $E$ and $F$ lying on $GD$.




It is given that $CE//BF//AG$, $BF = 2CE$ and $AG = 3CE$.

**(a)** Show that triangle $DCE$ is similar to triangle $DBF$.

*Answer*

$\angle DCE = \angle DBF$ (corresponding angles)
$\angle CDE = \angle BDE$ (common angle)
$\therefore$ Triangle $DCE$ is similar to triangle $DBF$

[2]

**(b)** Given that the area of trapezium $BFEC = 15 \text{ cm}^2$, find the area of trapezium $AGFB$.

$$\frac{\text{Area of } \Delta DCE}{\text{Area of } \Delta DBF} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\frac{\text{Area of } \Delta DCE}{\text{Area of } \Delta DAG} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

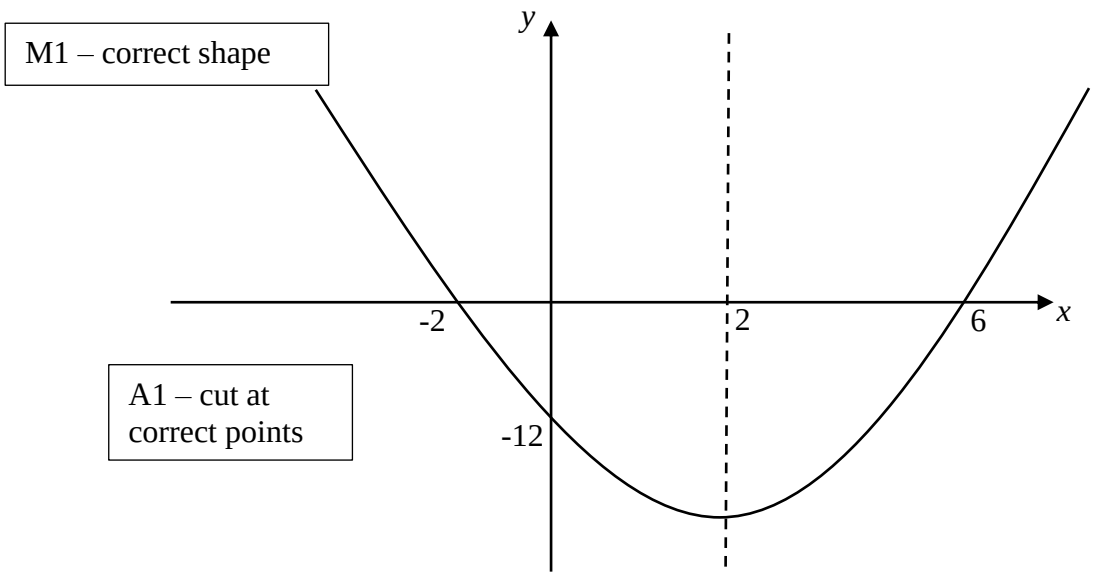
$$\frac{\text{area of trapezium } AGFB}{\text{area of trapezium } BFEC} = \frac{9-4}{4-1} = \frac{5}{3}$$

$$\frac{\text{area of trapezium } AGFB}{15} = \frac{5}{3}$$

area of trapezium $AGFB = 25 \text{ cm}^2$

*Answer* …….....$\text{cm}^2$ [3]

**19** Sketch the graph of $y=(2+x)(x-6)$ on the axes below. Indicate clearly the values where the graph crosses the axes and write down the equation of the line of symmetry.




M1 – correct shape

A1 – cut at correct points

*Answer* line of symmetry: ……. $x=2$

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**20** It is given that $y$ is inversely proportional to the square root of $x$.
Find the percentage change in $x$ when the value of $y$ decreases by 50%.

$$y=\frac{k}{\sqrt{x}}$$

$$\sqrt{x}=\frac{k}{y}$$

$$x=\frac{k^2}{y^2}$$

$$\text{new } x=\frac{k^2}{(0.5y^2)}$$

$$=\frac{4k^2}{y^2}$$

$$\text{Percentage change in } x=\frac{\frac{4k^2}{y^2}-\frac{k^2}{y^2}}{\frac{k^2}{y^2}}\times 100\%$$

$$=300\%$$

*Answer* ……..... [2]

**21** The table shows the prices of movie tickets categorised by different days of the week and the various age groups.

| | Monday to Thursday | Friday | Saturday and Sunday |
|---|---|---|---|
| Child | \$6.00 | \$8.50 | \$9.00 |
| Adult | \$7.00 | \$10.00 | \$12.50 |
| Senior Citizen | \$6.50 | \$9.00 | \$10.00 |

**(a)** Write down a $3\times3$ matrix **P** to represent the above information.

$$\begin{pmatrix} 6 & 8.5 & 9 \\ 7 & 10 & 12.5 \\ 6.5 & 9 & 10 \end{pmatrix}$$

*Answer* **P**=……..... [1]

**(b) (i)** The Ng family comprises a child, two adults and a senior citizen while the Tan family comprises three children and two adults. Represent this information as a $2\times3$ matrix **Q**.

$$\mathbf{Q}=\begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 0 \end{pmatrix}$$

*Answer* **Q** =…….....[1]

**(ii)** Evaluate **QP** and explain what the elements represent.

*Answer*

$$\mathbf{QP}=\begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 0 \end{pmatrix}\begin{pmatrix} 6 & 8.5 & 9 \\ 7 & 10 & 12.5 \\ 6.5 & 9 & 10 \end{pmatrix}$$

$$=\begin{pmatrix} 26.5 & 37.5 & 44 \\ 32 & 45.5 & 52 \end{pmatrix}$$

……

The elements in **QP** represent the respective total cost of movie tickets for the Ng family of 1 child, 2 adults and 1 senior citizen and the Tan Family of 3 children and 2 adults on the different days of the week.

…… [2]

**22** Sam's daily morning routine involves getting breakfast before going to work. He travels by car. On a particular day, Sam leaves home at 8:00 am and arrives at the breakfast place at 8:15 am. He leaves the breakfast place at 8:30 am. He arrives at his office at 9:00 am. Sam's average speed for the whole journey is 42 km/h.

His average speed from home to the breakfast place is 10 km/h faster than his average speed from the breakfast place to the office.

Find the distance between the breakfast place and office.

Total time taken = 1 hour

Total distance travelled = 42 km

Let $x$ be the distance between the breakfast place to office.

Distance between home to breakfast place = $42 - x$

$$V_1 = \frac{42 - x}{\left(\frac{15}{60}\right)} = 4(42 - x)$$

$$V_2 = \frac{x}{\left(\frac{1}{2}\right)} = 2x$$

$$4(42 - x) - 2x = 10$$

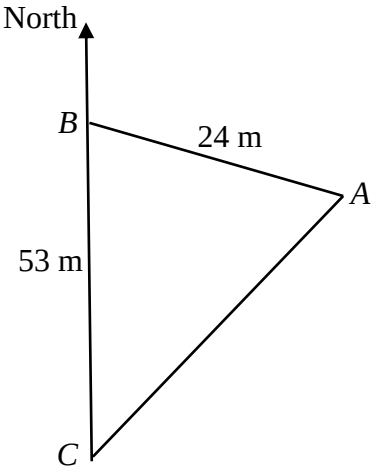
$$168 - 4x - 2x = 10$$

$$6x = 158$$

$$x = 26\frac{1}{3}$$

*Answer* …….....km [4]

**23** Three points *A*, *B* and *C* lie on a horizontal ground are such that $AB = 24$ m and $BC = 53$ m. Point *B* is due north of *C*. The bearing of *A* from *B* is $122°$.




**(a)** Find the distance *AC*.

$$AC = \sqrt{53^2 + 24^2 - 2(53)(24)\cos 58}$$
$$= 45.13186...$$
$$= 45.1 \text{ (3sf)}$$

*Answer* ……......m [2]

**(b)** Find the bearing of *C* from *A*.

By sine rule,

$$\frac{\sin \angle BAC}{53} = \frac{\sin \angle ABC}{45.13186...}$$
$$\sin \angle BAC = 53 \times \frac{\sin 58°}{45.13186...}$$
$$\angle BAC = 84.8060...$$
Bearing of $C$ from $A = 360° - 58° - 84.8060...°$
$$= 217.2°$$

*Answer* ……..... ° [3]

**(c)** Find the area of the triangular plot *ABC*.

$$\text{Area of } \Delta ABC = \frac{1}{2}(53)(24)\sin 58°$$
$$= 539.35...$$
$$= 539$$

*Answer* ……..$m^2$ [2]

**24** The first four terms in a sequence of numbers are

$$3+k, 1+k, -1+k, -3+k, ...$$

where $k$ is a constant.

**(a)** Find an expression in terms of $n$ and $k$, for the $n$th term in this sequence.

$$\begin{aligned} T_n &= 3+k+(n-1)(-2) \\ &= 3+k-2n+2 \\ &= 5-2n+k \end{aligned}$$

*Answer* ……..... [1]

**(a)** State two conditions on $k$ such that 39 is a term of the sequence.

*Answer*

$5-2n+k=39$

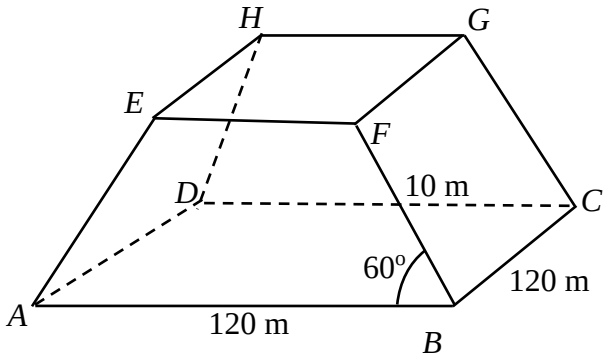
$k-2n=34$

$k=34+2n$

Since $n \geq 1$, $34+2n \geq 36$, therefore $k \geq 36$ and $k$ must be an even number.

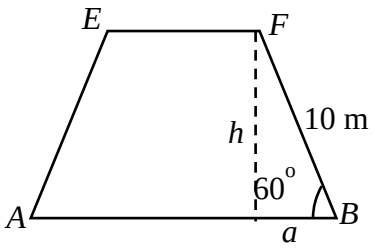
[2]

**25** The diagram below shows an indoor adventure park in the shape of a trapezoidal prism with a square base *ABCD*. The indoor adventure park is positioned on horizontal ground and the walls *ADEH* and *BCFG* are slanted while the walls *ABFE* and *DCGH* are vertically upright.




The top of the prism, *EFGH*, is the ceiling of the adventure park which is also horizontal. *EFGH* is a square and the centre of *EFGH* lies vertically above the centre of *ABCD*.
$AB = 120$ m, $AE = BF = CG = DH = 10$ m and $\angle ABF = 60°$.

**(a)** Find the area of *ABFE*.




Since $AE = BF$, *ABFE* is an isosceles trapezium.

$\sin 60° = \dfrac{h}{10}$

$h = 10\sin 60°$

$= 8.660254...$

$\cos 60° = \dfrac{a}{10}$

$a = 10\cos 60°$

$= 5$

Thus $EF = 12 - 5 - 5 = 2$

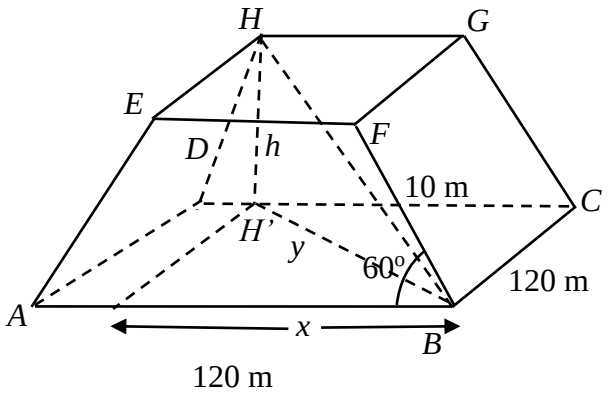
Area of trapezium $ABFE = \dfrac{1}{2}(120 + 110)(8.660254...)$

$= 996$

*Answer* ……$m^2$ [4]

**(b)** The owner of the indoor park wants to build a flying fox feature from point $H$ to point $B$. The angle of depression from point $H$ to point $B$ must not exceed $5°$, in order to meet the safety requirements. Explain with mathematical workings whether the flying fox feature can be built.

*Answer*




$x = 120 - 5$

$= 115$

$y = \sqrt{120^2 + 115^2}$

$= \sqrt{27625}$

$\tan \angle HBH' = \dfrac{8.660254...}{\sqrt{27625}}$

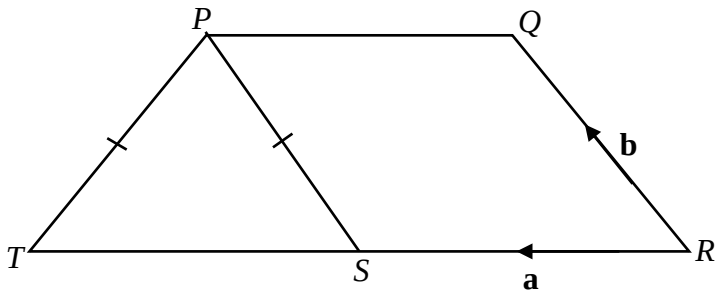
$\angle HBH' = 2.9826...°$

$= 3.0°$

By alternate angle, the angle of depression is $3.0°$ which is less than $5°$, therefore the flying fox feature can be built.

[3]

**26** In the diagram below, *PQRST* is a trapezium made up of a parallelogram *PQRS* and an isosceles triangle *PST* where $PT = PS$. It is given that $\overrightarrow{RS} = \mathbf{a}, \overrightarrow{RQ} = \mathbf{b}$ and $SR = ST$.




**(a)** Show that $\overrightarrow{QS} = \overrightarrow{PT}$.

*Answer*

$$\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS}$$
$$= -\mathbf{b} + \mathbf{a}$$
$$\overrightarrow{PT} = \overrightarrow{PS} + \overrightarrow{ST}$$
$$= -\mathbf{b} + \mathbf{a}$$

Therefore $\overrightarrow{QS} = \overrightarrow{PT}$.

[2]

**(b)** Hence or otherwise, prove that the trapezium is made up of three isosceles triangles *PST*.

*Answer*

Since *PQRS* is a parallelogram,
$PS // QR, PQ // SR$

$PS = QR$ and $PQ = SR$

From (a), since $\overrightarrow{QS} = \overrightarrow{PT}$, thus $PT = QS$.

$PT = QS = PS = QR$ and $PQ = SR = TS$.

By SSS congruency test, triangle *PTS*, triangle *PQS* and triangle *QSR* are congruent, therefore the trapezium is made up of three isosceles triangles *PST*.

[3]

**End of Paper**

**Alternative solutions**

| | |
|---|---|
| **9** | Total cost excluding shipping in Singapore dollars $= \$436 - \$15$<br>$= \$421$<br>Cost of watch excluding GST in Singapore dollars $= \frac{100}{109} \times \$421$<br>$= \$386.238532$<br>$\frac{386.2385321}{P} \times 100 = 10460$<br>$386.2385321 = 104.60P$<br>$P = 3.69$ (3sf) |
| **11** | $a^2b^2 - 6ab + 9 = 0$<br>$(ab)^2 - 2(ab)(3) + 3^2 = 0$<br>$(ab-3)^2 = 0$<br>$ab - 3 = 0$<br>$ab = 3$ |
| **14(a)** | Exterior $\angle$ of $N$-sided polygon $= \frac{360}{N}$<br>Interior $\angle$ of $n$-sided polygon $= \frac{180(n-2)}{n}$<br>$180 + \frac{360}{N} = \frac{2 \times 180(n-2)}{n}$<br>$1 + \frac{2}{N} = \frac{2(n-2)}{n}$<br>$\frac{2}{N} = \frac{2n-4}{n} - 1$<br>$\frac{2}{N} = \frac{2n-4-n}{n}$<br>$\frac{N}{2} = \frac{n}{n-4}$<br>$N = \frac{2n}{n-4}$ |