BEATTY SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2020

O

SUBJECT : Mathematics **LEVEL : Sec 4E/5N**

PAPER : 4048 / 2 **DURATION : 2 hour 30 minutes**

SETTER : Mr Lai Chee Kit
Mrs Samsol **DATE : 26 August 2020**

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use
100

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P(1+\frac{r}{100})^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** It is given that $A = \frac{1}{2}h\left(a^2 - b^2\right)$.

(i) Evaluate A when $a = 4$, $b = 2.5$ and $h = 6$.

Answer $A =$ ..... [1]

(ii) Express b in terms of A, a and h.

Answer $b =$ ..... [2]

(b) Express as a single fraction in its simplest form $\frac{7}{4x^2 - 4x + 1} - \frac{2}{2x - 1}$.

Answer ..... [3]

(c) **(i)** Solve the inequality $\frac{5-4x}{7} \geq \frac{x+4}{2}$.

Answer ……... [2]

(ii) State the greatest integer value of x which satisfies $\frac{5-4x}{7} \geq \frac{x+4}{2}$.

Answer $x =$ ……..... [1]

(d) Solve the equation $\frac{x+7}{2} + \frac{2x}{5} = x$.

Answer $x =$ …….. [2]

2 Ethan went on a journey of 80 km. For the first $\frac{3}{5}$ of the journey, his average speed was x km/h.

(a) Write down an expression, in terms of x, for the time taken in hours for the first part of the journey.

Answer ..... hours [1]

He increased his average speed by 6 km/h for the remaining journey. The time taken for the second part of the journey is 18 minutes less than the first part of the journey.

(b) Write down an equation in x to represent this information and show that it simplifies to

$$3x^2 - 142x - 2880 = 0 .$$

[4]

Answer

(c) Solve the equation $3x^2 - 142x - 2880 = 0$, giving your solutions correct to two decimal places.

Answer x = …… or ……..... [3]

(d) Calculate the time taken for the whole journey. Give your answer in hours and minutes, correct to the nearest minute.

Answer …….. hours…….. minutes [2]

3 **(a)** The following table shows the volume of mail handled by SingPost from 2009 to 2012.

Number of mail	2009	2010	2011	2012
Domestic	876 114	663 497	669 543	655 317
International	126 548	116 851	110 944	109 463

https://data.gov.sg/dataset/volume-of-mail-handled-by-type-annual

(i) Calculate the percentage decrease in the total volume of mail from 2009 to 2012.

Answer …… % [2]

(ii) Calculate the mean number of mail per month for 2012. Give your answer in standard form, correct to 3 significant figures.

Answer …… [2]

(iii) In which two years are the ratio of the number of domestic mail to the number of international mail approximately the same? Justify your answer clearly.

Answer Years …… and

……..... [2]

(b) A bank offers a saving account with an interest rate of 1.5% per year, compounded monthly.
Shawn invests $8500 in this account.

Calculate the total amount of interest he has earned at the end of 3 years.
Give your answer correct to the nearest cent.

Answer $..... [2]

(c) The exchange rate between Singapore dollars ($) and Thai Baht (THB) is $1= THB 22.66.

Mandy visited her friend in Thailand. She bought a watch for THB 5400 and paid using her credit card.
The credit card company converts the price to Singapore dollars.
She was charged a fee of 1.2% for the currency conversion.

Calculate the total amount in Singapore dollars that Mandy has to pay the credit card company. Give your answer correct to the nearest dollar.

Answer $..... [3]

4 The variables x and y are connected by the equation $y = \frac{x^3}{4} - 3x - 1$.

Some corresponding values of x and y are given in the table below.

x	–4	–3	–2	–1	0	1	2	3	4
y	–5	p	3	1.75	–1	–3.75	–5	–3.25	3

(a) Find the value of p.

Answer $p =$……..... [1]

(b) On the grid opposite, draw the graph of $y = \frac{x^3}{4} - 3x - 1$ for $-4 \leq x \leq 4$. [3]

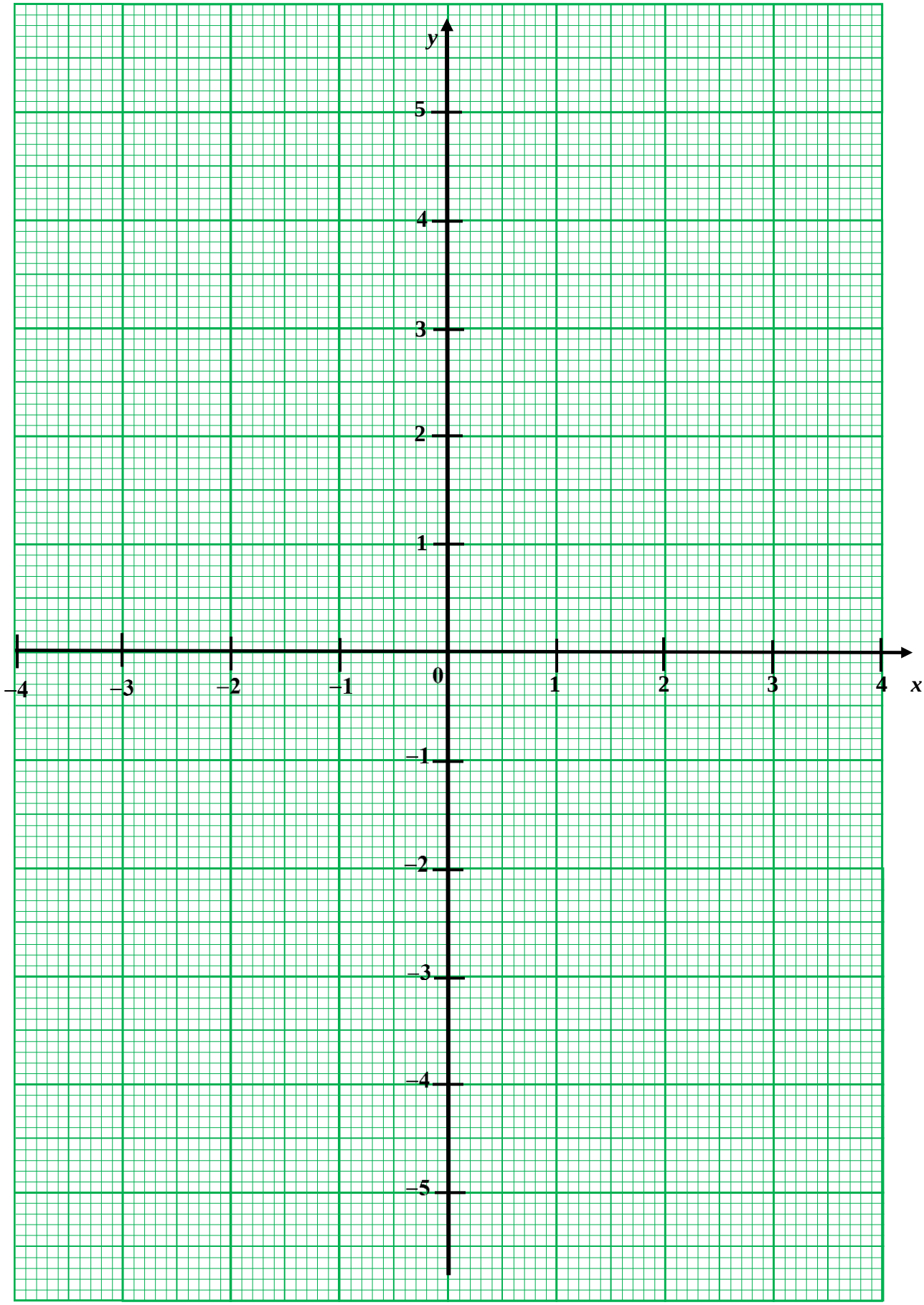
(c) The equation $\frac{x^3}{4} - 3x - 3 = 0$ has three solutions.

Explain how this can be seen from your graph.

Answer [2]

(d) By drawing a tangent, find the gradient of the curve at $(3, -3.25)$.

Answer ……. [2]


y
5
4
3
2
1
–4
–3
–2
–1
0
1
2
3
4
x
–1
–2
–3
–4
–5

(e) **(i)** On the same grid, draw the graph of $2y + x = 6$ for $-4 \leq x \leq 4$. [1]

(ii) Find the equation, in the form $x^3 + ax + b = 0$, which is satisfied by the x-coordinate of the point at which the two graphs meet.

Answer ..... [2]

5 **(a)** The table shows the grade of students in classes S1 and S2 in a Mathematics Test.

	Grade *A*	Grade *B*	Grade *C*
S1	11	10	5
S2	*n*	14	4

A student is selected at random from these two classes.

The probability that the student selected scores a Grade *A* is $\frac{8}{19}$.

Find the number of students in class S2.

Answer …… [3]

(b) A box contains 6 blue cards, 10 red cards and 2 yellow cards.

Two cards are taken out of the box at random without replacement.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

Answer

(ii) Find, as a fraction in its simplest form, the probability that

(a) 2 blue cards are selected,

Answer …… [1]

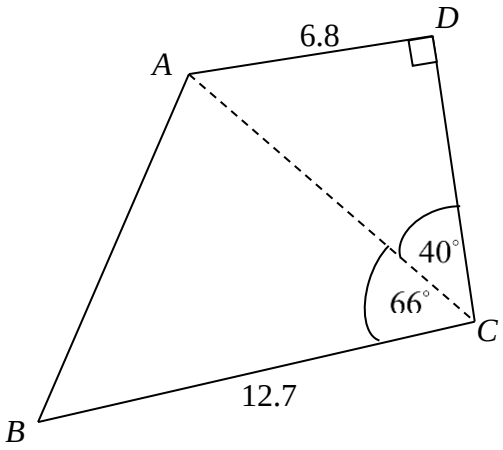
(b) a blue card and a red card are selected,

Answer ……. [2]

(c) at least one card is red.

Answer …….. [2]

6



The diagram shows a garden *ABCD* on horizontal ground with a straight walking path *AC*. It is given that *AD* = 6.8 m, *BC* = 12.7 m, $\angle DCA = 40^\circ$, $\angle BCA = 66^\circ$ and $\angle ADC = 90^\circ$.

(a) Show that *AC* = 10.58 km, correct to two decimal places.

Answer [2]

(b) Find

(i) the length of *AB*,

Answer ..... m [2]

(ii) area of *ABCD*.

Answer ……. m^2 [3]

(c) A vertical lamp post is standing at *A* such that its angle of elevation from *B* is 8°.
Find the angle of depression of *C* from the top of the lamp post.

Answer …… [3]

7 In order to travel to their work place, the employees of a company can take public transport from the nearest MRT station. They can either travel by public bus, LRT or express bus. The fares of taking a public bus, LRT and express bus are \$0.47, \$0.75 and \$1.27 respectively.

(a) Express the fares in a 3×1 matrix, **F**.

[1]

Answer

(b) The employees work in three shifts.
The following table shows how they travel to the company for work from the nearest MRT station.

	Public bus	LRT	Express bus
Shift 1	16	16	18
Shift 2	20	21	8
Shift 3	20	17	0

This information can be represented by the following matrix,

$$\mathbf{S} = \begin{pmatrix} 16 & 16 & 18 \\ 20 & 21 & 8 \\ 20 & 17 & 0 \end{pmatrix}$$

Evaluate the matrix $\mathbf{T} = \mathbf{SF}$.

Answer [1]

(c) Explain what each element in matrix **T** represents.

Answer

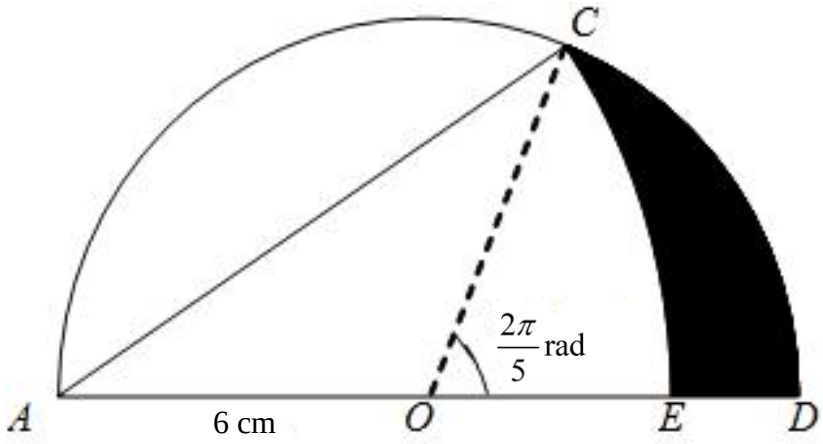
[1]

(d) Write down a 1×3 matrix, **N**, such that the element of **NT** represent the total fare paid by the employees traveling to the company from the MRT station each day.
Hence, find the total fare.

Answer **N** = ……..

Total fare = $ …… [2]

8 **(a)** In the following diagram, *OACD* is a semicircle, with centre *O*. *ACE* is a sector with centre *A*. It is given that $\angle COE = \frac{2\pi}{5}\text{rad}$.



(i) Explain why $\angle OAC$ is $\frac{\pi}{5}$ rad.

Answer

[1]

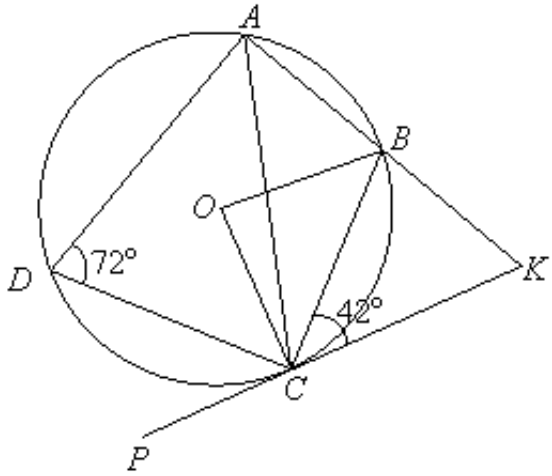
(ii) Find the length of line *AC*.

Answer ……..…… cm [2]

(iii) Find the area of the shaded region.

Answer ……..…… cm^2 [4]

8 **(b)** In the following diagram, O is the centre of the circle and PK is a tangent at C which meets the line AB produced. It is given that $\angle ADC = 72°$ and $\angle BCK = 42°$.



Find the following angles, stating the reasons clearly.

(i) $\angle BAC$,

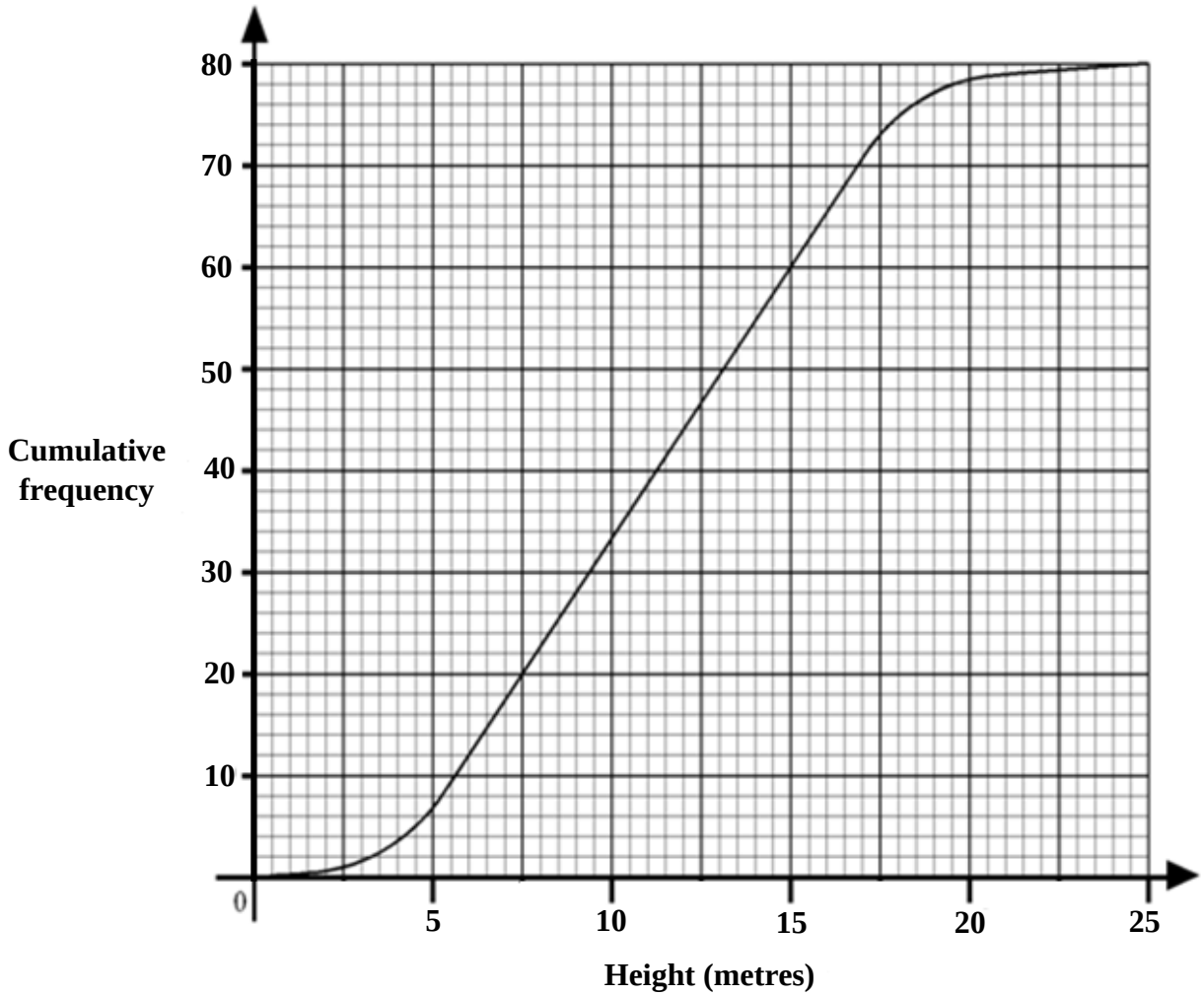
Answer ……..…… ° [2]

(ii) $\angle BKC$.

Answer ……..…… ° [2]

9 An environmentalist measured the heights of 80 trees in forest A.

The cumulative frequency graph shows the distribution of these heights.



(a) Complete the following frequency table. [1]

Height, *h* (metres)	**Frequency**
$0 < h \leq 5$	7
$5 < h \leq 10$	
$10 < h \leq 15$	
$15 < h \leq 20$	18
$20 < h \leq 25$	2

(b) Use the curve to estimate the

(i) median height,

Answer ……..…… m [1]

(ii) interquartile range,

Answer ……..…… m [2]

(iii) 35^{th} percentile.

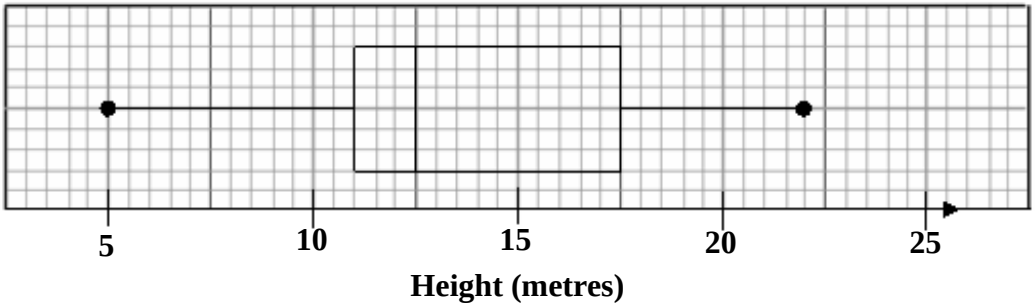
Answer ……..…… m [1]

(c) Estimate the percentage of trees which are above 18 metres.

Answer ……..…… % [2]

(d) The environmentalist measured the heights of 80 trees in forest B.

The box-and-whisker plot shows the distribution of their heights.



It is given that the median height is 12.5 metres

(i) Find the interquartile range.

Answer ……..…… m [2]

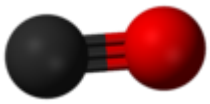
(ii) Make two comparisons between the heights of the trees in the two forests.

Answer [2]

10 Carbon monoxide (CO) is a colourless, odourless and tasteless gas.
It is produced in normal animal metabolism in low quantities, and is thought to have some normal biological functions. However, it can poison people who breathe it when it exceeds a safety level. Once inside the lungs, CO molecules replace the oxygen molecules, thus depriving the organs of oxygen.

Useful Information

- Dangerous CO level: more than 35 ppm (parts per million) for one-hour exposure
- 1 ppm is equivalent to 0.811 mg/m^3
- Mass of a CO molecule: 4.65×10^{-26} kg
- Density of CO: 1.14 kg/m^3



In an enclosed space, it is estimated that 1.83×10^{20} CO molecules are produced per second.

(a) Find the total mass of CO produced per hour in kilograms.
Leave your answer in standard form.

Answer ……..…… kg [3]

(b) Susan claims that the volume of air in the enclosed space in an hour is approximately 4500 m^3 and that it is dangerous for a person to be exposed to CO in the enclosed space for an hour.
Do you agree? Support your answer with calculations.
[1 kg = 10^6 mg]

Answer [4]

(c) Find the volume of air in the enclosed space when the presence of CO is considered dangerous.
State an assumption used.
Answer [3]

End of Paper

Answers

1(a)(i) 29.25 (ii) $a = \pm\sqrt{\dfrac{a^2h - 2A}{h}}$ 1(b) $\dfrac{9-4x}{(2x-1)^2}$ 1(c) (i) $x \le -1\dfrac{1}{5}$ 1(c)(ii) $x = -2$

1(d) 35

2(a) $\dfrac{48}{x}$ 2(c) 62.66 or −15.32 2(d) 1 hour 14 minutes

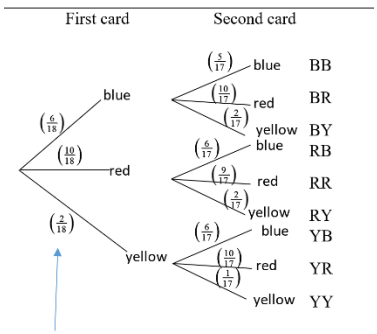
3(a)(i) 23.7% 3(a)(ii) 6.37×10^4 3(a)(iii) Years 2011 and 2012

3(b) $ 390.99 3(c) $241

4(a) 1.25 4(c) Draw the line y = 2. The curve intersects the line at 3 intersection points.

4(d) gradient = 3.75 [Accept 3 to 4] 4(e)(ii) $x^3 - 10x - 16 = 0$

5(a) 31 5(b)(i)



5(b)(ii)(a) $\dfrac{5}{51}$

5(b)(ii)(b) $\dfrac{20}{51}$

5(b)(ii)(c) $\dfrac{125}{153}$

6(a) 10.58 m 6(b)(i) 12.8 m 6(b)(ii) 88.9 m^2 6(c) 9.7°

7(a) $\begin{pmatrix} 0.47 \\ 0.75 \\ 1.27 \end{pmatrix}$ 7(b) $\begin{pmatrix} 42.38 \\ 35.31 \\ 22.15 \end{pmatrix}$

7(c) 42.38 represents the total transport cost of all the employees in shift 1 when they come for work from the nearest MRT station.
35.31 represents the total transport cost of all the employees in shift 2 when they come for work from the nearest MRT station.
22.15 represents the total transport cost of all the employees in shift 3 when they come for work from the nearest MRT station.

7(d) $99.84

8(a) (i) $\angle OAC = \dfrac{2\pi}{5} \div 2 = \dfrac{\pi}{5}$ rad [Angle at centre = 2 × angle at circumference]

8(a)(ii) 9.71 cm 8(a)(iii) 10.1 cm^2

8(b)(i) $\angle BAC = 42°$ (angle at centre = 2 × angle at circumference)

8(b)(ii) $\angle BKC = 66°$ (exterior angle of triangle is sum of the 2 opp interior angles)

9(a)

$5 < h \leq 10$	26
$10 < h \leq 15$	27

9(b)(i) 11.3 m 9(b)(ii) 7.5 cm 9(b)(iii) 9 m 9(c) 6.25%

9(d)(i) 6.5 m 9(d)(ii) Forest B has taller trees on the average as the median height is larger than the median height of Forest A.
Forest B has lesser spread as the interquartile range is smaller than the interquartile range of Forest A.

10(a) 3.06342×10^{-2} kg 10(b) 8.39 ppm < 35 ppm, I do not agree

10(c) 1080 m^3.
The assumption is that the total number of CO molecules produced in an hour remains unchanged.