2022 H2 MA JC1 Test 1 Solution

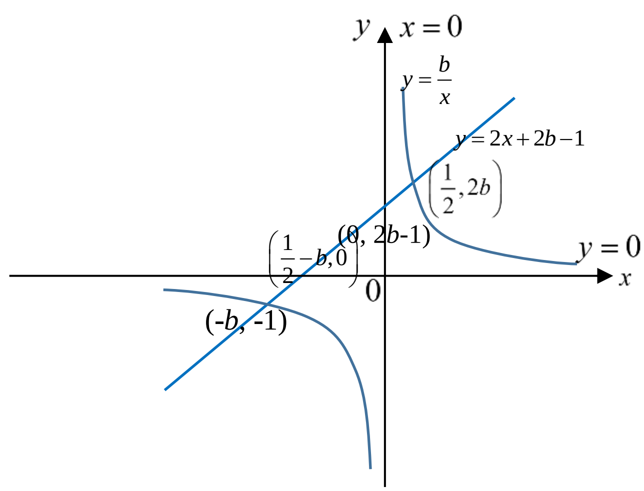
	Q1 Solution	MS
(i)	Let \$$x$ and \$$y$ be the price of each daily adult ticket and each student ticket respectively. Booth X : $5nx + ny = 5976$ -----(1) Booth Y : $357x + 51y = 5763$ -----(2) Booth Z : $7nx + 2ny = 8712$ -----(3) $5x + y - 5976\left(\frac{1}{n}\right) = 0 \quad \cdots\cdots \quad (1)$ $357x + 51y + 0\left(\frac{1}{n}\right) = 5763 \quad \cdots\cdots \quad (2)$ $7x + 2y - 8712\left(\frac{1}{n}\right) = 0 \quad \cdots\cdots \quad (3)$ From GC : $x = 15$, $y = 8$, $\frac{1}{n} = \frac{1}{72}$ Each adult ticket costs \$15, and each student ticket costs \$8 and $n = 72$.	
	Alternatively: $\frac{5x + y}{7x + 2y} = \frac{5976}{8712} \Rightarrow 1728x - 3240y = 0$ (And solve together with (2) from above)	

	Q2 Solution	**MS**
(i)	$\dfrac{7x-15}{x^2+x-6} \geq 1$ $\dfrac{7x-15}{x^2+x-6} - 1 \geq 0$ $\dfrac{7x-15-\left(x^2+x-6\right)}{x^2+x-6} \geq 0$ $\dfrac{7x-15-x^2-x+6}{x^2+x-6} \geq 0$ $\dfrac{-x^2+6x-9}{x^2+x-6} \geq 0$ $\dfrac{x^2-6x+9}{x^2+x-6} \leq 0$ $\dfrac{\left(x-3\right)^2}{(x-2)(x+3)} \leq 0$ Since $\left(x-3\right)^2 \geq 0$, $\dfrac{1}{(x-2)(x+3)} < 0$  For $\left(x-2\right)^2, x=3$ Therefore, $-3 < x < 2$ or $x = 3$	
(ii)	$-3 < \ln x < 2$ $\dfrac{1}{e^3} < x < e^2$	

<table>
<tr><th></th><th>Q3 Solution</th><th>MS</th></tr>
<tr><td>(i)</td><td>Sketch the graphs of $y = 2x + 2b - 1$ and $y = \frac{b}{x}$:

When $x = 0, y = 2b - 1$
$y = 0, x = \frac{1}{2} - b$

$2x + 2b - 1 = \frac{b}{x}$
$2x^2 + 2bx - x = b$
$2x^2 + (2b - 1)x - b = 0$
$(2x - 1)(x + b) = 0$
$x = \frac{1}{2}$ or $x = -b$
$\therefore y = 2b$ or $y = -1$


</td><td></td></tr>
<tr><td>(ii)</td><td>By observation, the 2 graphs intersect at $x = \frac{1}{2}$ and $x = \frac{1}{2} - b$.

Hence, for $\frac{b}{x} > 2x + 2b - 1$ (*), $0 < x < \frac{1}{2}$ or $x < -b$</td><td></td></tr>
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(iii)	Replace x in (*) by $x+b$: $\frac{b}{x+b} > 2(x+b)+2b-1$ $\frac{b}{x+b} > 2x+2b+2b-1$ $\frac{b}{x+b} > 2x+4b-1$ $\therefore 0 < x+b < \frac{1}{2} \Rightarrow -b < x < \frac{1}{2}-b$ Or $x+b<-b \Rightarrow x<-2b$	

	Q4 Solution	**MS**
(i)	$\frac{1}{\sqrt[3]{27+12x}}$ $=\left(27+12x\right)^{-\frac{1}{3}}$ $=\left(27\right)^{-\frac{1}{3}}\left(1+\frac{4}{9}x\right)^{-\frac{1}{3}}$ $=\frac{1}{3}\left(1+\left(-\frac{1}{3}\right)\left(\frac{4}{9}x\right)+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}\left(\frac{4}{9}x\right)^2+\cdots\right)$ $=\frac{1}{3}\left(1-\frac{4}{27}x+\frac{32}{729}x^2+\cdots\right)$ $\approx\frac{1}{3}-\frac{4}{81}x+\frac{32}{2187}x^2$ ....(*) Valid range: $\left\lvert\frac{4}{9}x\right\rvert<1$ $-\frac{9}{4}<x<\frac{9}{4}$	

(ii)	Let $x = \frac{1}{8}$ Since expansion is valid for $-\frac{9}{4} < x < \frac{9}{4}$, LHS: $\left(27 + 12\left(\frac{1}{8}\right)\right)^{-\frac{1}{3}}$ $= \frac{1}{\sqrt[3]{28.5}}$ $= \frac{1}{\sqrt[3]{19}\sqrt[3]{1.5}}$ $\frac{1}{\sqrt[3]{19}\sqrt[3]{1.5}} \approx \frac{1}{3} - \frac{4}{81}\left(\frac{1}{8}\right) + \frac{32}{2187}\left(\frac{1}{8}\right)^2$ (*) $= \frac{716}{2187}$ $\frac{1}{\sqrt[3]{19}\sqrt[3]{1.5}} \approx \frac{716}{2187}$ $\sqrt[3]{19}\sqrt[3]{1.5} \approx 3.05447$ $\sqrt[3]{1.5} \approx 1.145$	