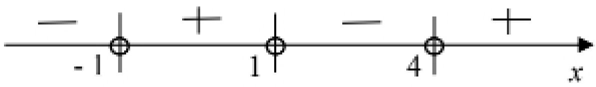
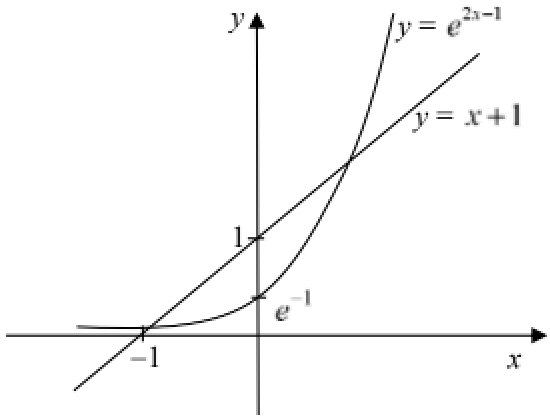
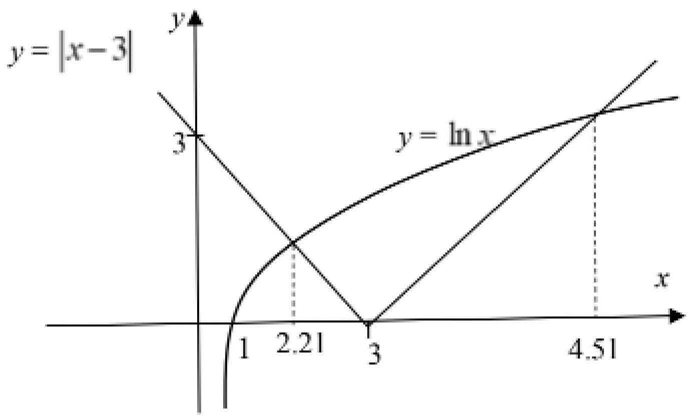
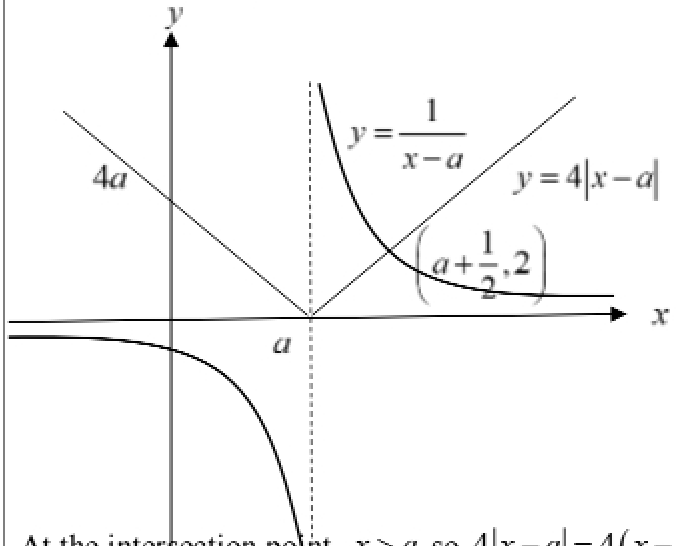
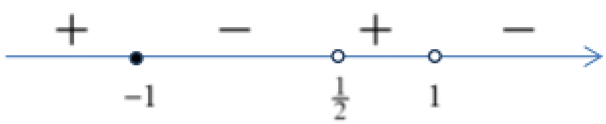
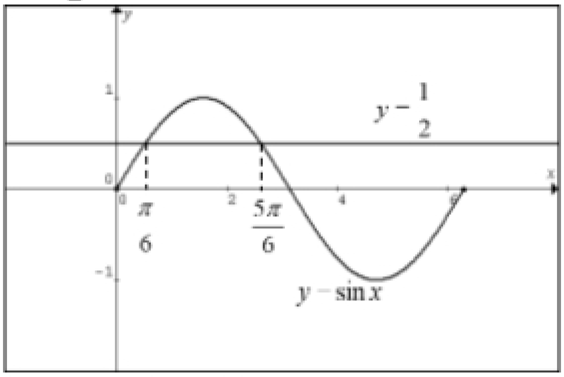
2022 C1 Block Test Revision Package Solutions
Chapter 3 Inequalities and System of Equations

A Inequalities

1	**RVHS11/C1BT/Q1** $x^2-2x+3=(x-1)^2+2\geq 2>0$. Hence x^2-2x+3 is always positive. By observation, $1^3-4(1)^2-1+4=0$ So $x+1$ is one factor. Then we use long division or comparing coefficients to find the other two factors $x+1, x-4$ $\dfrac{x^2-2x+3}{x^3-4x^2-x+4}\geq 0 \Rightarrow \dfrac{x^2-2x+3}{(x-1)(x^2-3x-4)}\geq 0$ $\Rightarrow \dfrac{x^2-2x+3}{(x-1)(x+1)(x-4)}\geq 0$ Since x^2-2x+3 is always positive,  $\Rightarrow (x-1)(x+1)(x-4)>0$ $\Rightarrow -1<x<1$ or $x>4$	The question states without the use of calculator, therefore have to show find the first factor by observation Need to show that the numerator is always positive
2.	**RI10/C1BT/Q2**  The curves intersect at $x=-0.944$ and $x=0.792$ Hence, for $e^{2x-1}>x+1$, we have $x<-0.944$ or $x>0.792$ (to 3 s.f.)	

	$\mathrm{e}^{-(2x+1)} > 1-x \;\Rightarrow\; \mathrm{e}^{2(-x)-1} > -x+1$ Replace x by $-x$: $-x < -0.944$ or $-x > 0.792$ $\Rightarrow x > 0.944$ or $x < -0.792$ (to 3 s.f.)	
3. **(a)**	**RI11/C1BT/Q6**  From the graphs, $0 < x < 2.21$ or $x > 4.51$. $\left\lvert \dfrac{1-3x}{x} \right\rvert + \ln x > 0$ $\Rightarrow \left\lvert \dfrac{1}{x} - 3 \right\rvert > -\ln x$. $\Rightarrow \left\lvert \dfrac{1}{x} - 3 \right\rvert > \ln\left(\dfrac{1}{x}\right)$ replace x with $\dfrac{1}{x}$ For intermediate steps, use more decimal places for better accuracy of the final answer $\Rightarrow 0 < \dfrac{1}{x} < 2.20794$ or $\dfrac{1}{x} > 4.50524$. $\therefore x > 0.453$ or $0 < x < 0.222$.	
(b)	$\dfrac{2x^2 - 7x + 6}{x^2 - x - 2} < 1$ $\Rightarrow \dfrac{2x^2 - 7x + 6 - \left(x^2 - x - 2\right)}{x^2 - x - 2} < 0$	Move all the terms to one side and combine into a single expression

	$\Rightarrow \dfrac{x^2-6x+8}{x^2-x-2}<0$ $\Rightarrow \dfrac{(x-4)(x-2)}{(x-2)(x+1)}<0$ $\Rightarrow \dfrac{(x-4)}{(x+1)}<0$ and $x \neq 2$ $\therefore -1<x<4$ and $x \neq 2$ [Alternative Answer: $-1<x<2$ or $2<x<4$]	
4.	**ACJC10/C1BT/Q3**  At the intersection point, $x>a$ so $4\lvert x-a\rvert=4(x-a)$. We want to solve $4(x-a)=\dfrac{1}{x-a} \Rightarrow (x-a)^2=\dfrac{1}{4} \Rightarrow x-a=\pm\dfrac{1}{2} \Rightarrow x=a+\dfrac{1}{2}$ Therefore, $x<a$ or $x>a+\dfrac{1}{2}$.	Since $x>a$
5(i)	**HCI14/C1BT/Q4** $\dfrac{2x^2+4}{(x-1)(1-2x)} \leq -1 \Rightarrow \dfrac{2x^2+4}{(x-1)(1-2x)}+1 \leq 0$ $\therefore \dfrac{2x^2+4+\left(-2x^2+3x-1\right)}{(x-1)(1-2x)} \leq 0$ $\Rightarrow \dfrac{3(x+1)}{(x-1)(1-2x)} \leq 0$  $\therefore -1 \leq x<\frac{1}{2}$ or $x>1$.	

<table>
<tr>
<td>(ii)</td>
<td>Replace x by $\sin x$, so the solution is
$-1 \le \sin x < \frac{1}{2}$ or $\sin x > 1$ (rej)


Hence the set of solution is
$\left\{x \in \mathbb{R} : 0 \le x < \frac{\pi}{6} \text{ or } \frac{5\pi}{6} < x \le 2\pi\right\}$.</td>
<td>Sketch the curve of $\sin x$</td>
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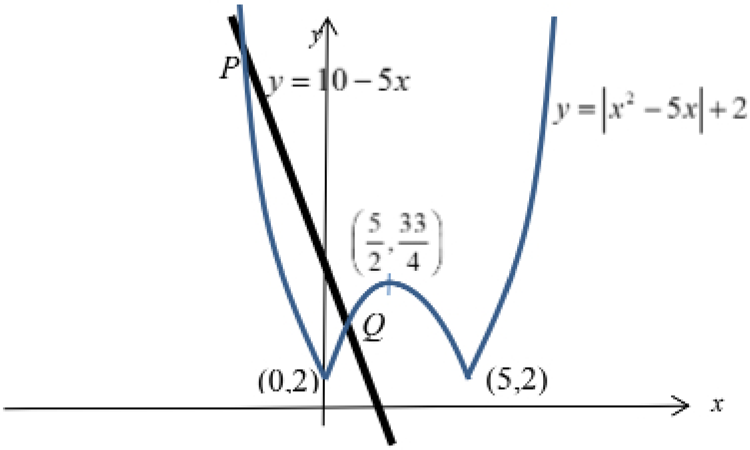
<td>6.</td>
<td>AJC16/C1BT/Q8


$\left|x^2-5x\right| > 8-5x \Rightarrow \left|x^2-5x\right|+2 > 10-5x$
At point P: $(x^2-5x)+2=10-5x$
$x^2=8 \Rightarrow x=\pm\sqrt{8}=\pm 2\sqrt{2}$
Since it is at second quadrant, $x=-2\sqrt{2}$

At point Q: $-(x^2-5x)+2=10-5x$
$x^2-10x+8=0$
$\Rightarrow \quad x=\frac{10\pm\sqrt{100-4(8)}}{2}=5\pm\sqrt{17}$
Since $x<5, x=5-\sqrt{17}$
From the graph, the solution for $\left|x^2-5x\right| > 8-5x$ is
$x<-2\sqrt{2}$ or $x>5-\sqrt{17}$.</td>
<td>Question states "answer in exact form", will need to resolve the modulus and solve for the intersection. Use the graph to find the correct inequality</td>
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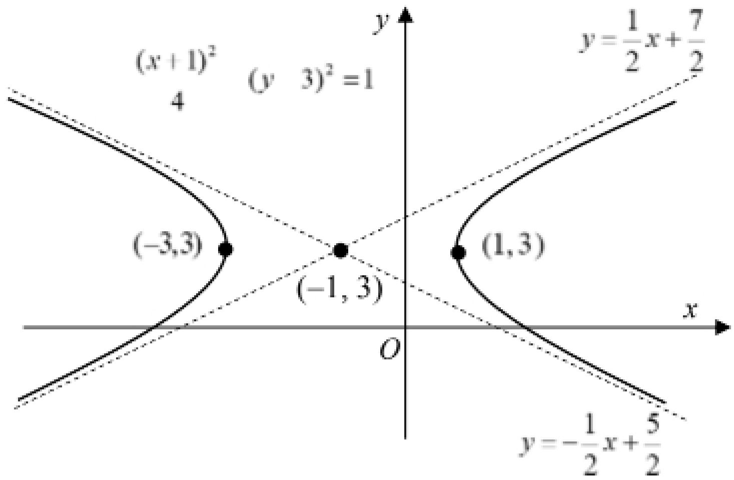
7.	**CJC16/C1BT/Q2**	Remember to label the centre of the hyperbola



(i) $\dfrac{(x+1)^2}{4} - (y-3)^2 = 1$

Hyperbola, centre $(-1,3)$.

To find asymptotes:

$$\frac{(x+1)^2}{4} - (y-3)^2 = 0$$

$$\Rightarrow y - 3 = \pm\frac{x+1}{2}$$

$$\Rightarrow y = \frac{1}{2}x + \frac{7}{2} \qquad y = -\frac{1}{2}x + \frac{5}{2}$$

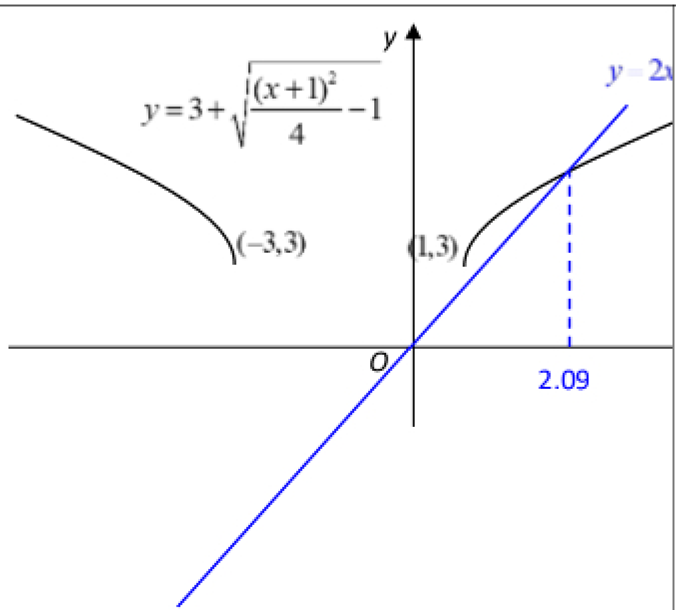
(ii)

$$\frac{(x+1)^2}{4} - (y-3)^2 = 1$$

$$\Rightarrow (y-3)^2 = \frac{(x+1)^2}{4} - 1$$

$$\Rightarrow y = \underbrace{3 + \sqrt{\frac{(x+1)^2}{4} - 1}}_{\text{upper portion of hyperbola in (i)}} \quad \text{or} \quad y = \underbrace{3 - \sqrt{\frac{(x+1)^2}{4} - 1}}_{\text{lower portion of hyperbola in (i)}}$$

To solve $3 + \sqrt{\dfrac{(x+1)^2}{4} - 1} > 2x$, we need to Sketch $y = 2x$.



From graph, $x \le -3$ or $1 \le x < 2.09$.

8. **IJC16/C1BT/Q5**

(i) Method 1:

$$3x^2 - 3x + 1 = 3\left(x - \frac{1}{2}\right)^2 + \frac{1}{4}$$

Since $\left(x - \frac{1}{2}\right)^2 \ge 0$, $3\left(x - \frac{1}{2}\right)^2 + \frac{1}{4}$ is always positive

Method 2:

Discriminant $= (-3)^2 - 4(3)(1) = -3$

Discriminant < 0 **and** coefficient of x^2 is positive, so $3x^2 - 3x + 1$ is always positive

(ii)

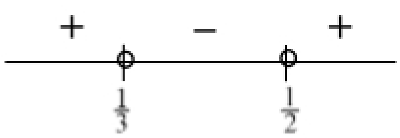
$$\frac{x}{2x-1} \le \frac{1}{3x-1}$$

$$\Rightarrow \frac{x(3x-1) - (2x-1)}{(2x-1)(3x-1)} \le 0$$

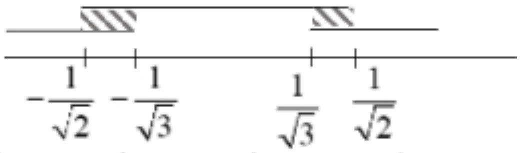
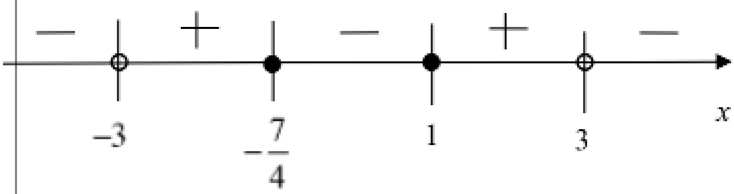
$$\Rightarrow \frac{3x^2 - 3x + 1}{(2x-1)(3x-1)} \le 0$$

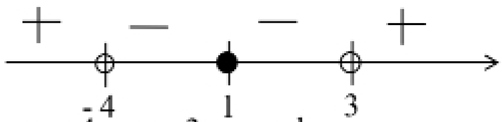
Since $3x^2 - 3x + 1$ is always positive, it suffices to solve:

$$\frac{1}{(2x-1)(3x-1)} \le 0$$



$\therefore$ solution is $\frac{1}{3} < x < \frac{1}{2}$

	(iii) Replacing "x" by "x^2" in the solution for (ii): $\frac{1}{3} < x^2 < \frac{1}{2}$ $\Rightarrow x^2 < \frac{1}{\sqrt{2}}$ and $x^2 > \frac{1}{\sqrt{3}}$ Find the intersection between the 2 inequalities $\Rightarrow -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ and $x > \frac{1}{\sqrt{3}}$ or $x < -\frac{1}{\sqrt{3}}$  $\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}} < x < -\frac{1}{\sqrt{3}}$	
9	**NJC16/C1BT/Q4** $\frac{29+3x}{9-x^2} \geq 4$ $\Rightarrow \frac{29+3x-4(9-x^2)}{9-x^2} \geq 0$ $\Rightarrow \frac{(4x+7)(x-1)}{(3-x)(3+x)} \geq 0$  Hence, $-3 < x \leq -\frac{7}{4}$ or $1 \leq x < 3$. $\frac{3\lvert x\rvert-29}{x^2-9} \geq 4 \Rightarrow \frac{29-3\lvert x\rvert}{9-x^2} \geq 4$ Replace x with $-\lvert x\rvert$. $-3 < -\lvert x\rvert \leq -\frac{7}{4}$ or $1 \leq -\lvert x\rvert < 3$.(No solution $\because -\lvert x\rvert < 0$). $\Rightarrow \frac{7}{4} \leq \lvert x\rvert < 3$ Hence $-3 < x \leq -\frac{7}{4}$ or $\frac{7}{4} \leq x < 3$.	
10	**SRJC16/C1BT/Q6**	$(x-1)^2$ gives a repeated root. Therefore there is no change in sign in the number line before and after $x=1$

	$\dfrac{3x-13}{x^2+x-12} \le 1$ $\Rightarrow \dfrac{3x-13}{x^2+x-12} - 1 \le 0$ $\Rightarrow \dfrac{-x^2+2x-1}{x^2+x-12} \le 0$ $\Rightarrow \dfrac{-(x-1)^2}{(x-3)(x+4)} \le 0$ $\Rightarrow \dfrac{(x-1)^2}{(x-3)(x+4)} \ge 0$  $\therefore x < -4$ or $x > 3$ or $x = 1$ $\dfrac{\ln x^3 - 13}{(\ln x)^2 + \ln x - 12} \le 1 \Rightarrow \dfrac{3\ln x - 13}{(\ln x)^2 + \ln x - 12} \le 1$ By replacing x in $\dfrac{3x-13}{x^2+x-12} \le 1$ by $\ln x$, $\therefore \ln x < -4$ or $\ln x > 3$ or $\ln x = 1$ $\Rightarrow 0 < x < e^{-4}$ or $x > e^3$ or $x = e$		

11(i)	**RI 2020 C1 BT Q8** $\dfrac{x-2}{x^2-x} \ge 1$, $x \ne 0, 1$ $\dfrac{(x-2)-(x^2-x)}{x^2-x} \ge 0$ $\dfrac{-x^2+2x-2}{x^2-x} \ge 0$ $-x^2+2x-2 = -(x-1)^2 - 1 < 0 \quad \forall x \in \mathbb{R}$, since $(x-1)^2 \ge 0 \quad \forall x \in \mathbb{R}$. Therefore, $x^2 - x < 0$ $x(x-1) < 0$  $0 < x < 1$		
(ii) **(a)**	$\dfrac{2-e^x}{e^x - e^{2x}} \ge 1$ $\dfrac{e^x - 2}{e^{2x} - e^x} \ge 1$ Replace x with e^x.		

From **(i)**, $0 < e^x < 1$,

That is, $x < 0$.

(ii) (b)

$$\frac{x-3}{x^2-3x+2} \le 1$$

$$\frac{(x-1)-2}{(x-1)^2-(x-1)} \le 1$$

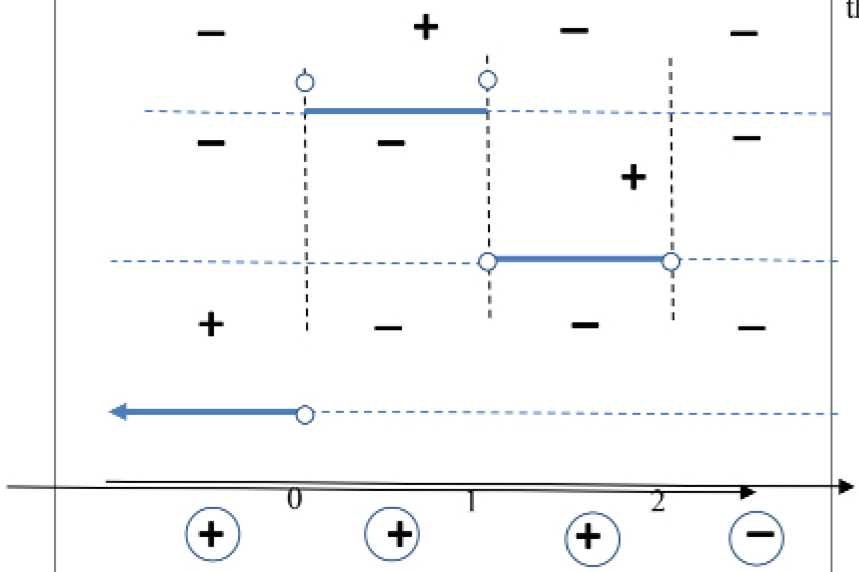
Replace x with $(x-1)$.

From **(i)**, $x-1<0$ or $x-1>1$

That is, $x<1$ or $x>2$

(iii) A way to *visualise* :

Note that 0 and 1 cannot be included in the solution.



Another way to *visualise* :

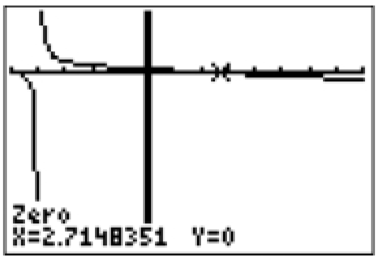
0 1 2

$\frac{2-e^x}{e^x-e^{2x}}-1$	+	−	−	−
$\frac{x-3}{x^2-3x+2}-1$	−	−	+	−
$\frac{x-2}{x^2-x}-1$	−	+	−	−
	+	+	+	−

$\therefore$The set of values of x required :

$$(-\infty, 2) \setminus \{0,1\}$$

B System of Linear Equations

12 (i)	**MJC13/Promo/Q3** At A, $b+c=a+d$. At B, $a+b+c=48$. At C, $a+c=2b$. At D, $d=b+2a$. After simplifying, $-a+b+c-d=0$. $a+b+c=48$. $a-2b+c=0$. $2a+b-d=0$. Using GC, $a=8, b=16, c=24$ and $d=32$.	
12 (ii)	Total amount collected $= \$0.50(2c+b)$ $= \$0.50(48+16)$ $= \$32$	
13. (i)	**HCI11/C1BT/Q4** $b+\frac{c}{4}=160000$ ….. (1) $a+b+\frac{c}{5}=198000$ …..(2) $2a+b+\frac{c}{6}=240000$ …..(3) $\Rightarrow a=50000, \quad b=100000, \quad c=240000$	
(ii)	**<u>Method 1:</u>**  $-50000t+100000+\frac{240000}{t+4}=0$ $\Rightarrow t=2.71$ The profit first becomes zero in 2003.	
	<u>Method 2:</u> $-50000t+100000+\frac{240000}{t+4}=0$ $\Rightarrow -50000t^2-100000t+640000=0$ $\Rightarrow t=2.71$ or -4.71 (rej $\because t>0$) The profit first becomes zero in 2003.	

Method 3:

X	Y1
0	160000
1	98000
2	40000
3	-15714
4	-70000
5	-1.2E5
6	-1.8E5

X=3

$$-50000t+100000+\frac{240000}{t+4}=0$$

From GC, $t=3$.

The profit first becomes zero in 2003.

14. RI11/C1BT/Q1

Let P, G, and M be the prices ($) of 1 PineApple, Googol and Macrohard shares respectively.

$$10P+50G+300M=40040$$

$$G=P+10M \Rightarrow P-G+10M=0$$

$$(0.1)(10P)+(0.15)(50G)+(0.2)(300M)=6227$$

From GC, $P=\$326, \mathrm{G}=\$582, \mathrm{M}=\$25.60$

15. HCI14/C1BT/Q3

Differentiate implicitly w.r.t x:

$$2Ax+2By\frac{\mathrm{d}y}{\mathrm{d}x}+C+D\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{2Ax+C}{2By+D}$$

Since the point $(1,-1)$ is on the curve,

$$A+B+C-D \quad =-13 \text{ -------- (1)}$$

$\frac{\mathrm{d}y}{\mathrm{d}x}$ at $(1,-1)$ is zero. From (i) $\Rightarrow 2Ax+C=0$:

$$2A \qquad +C \qquad =0 \text{ ----------- (2)}$$

The point $(3,-2)$ is on the curve:

$$9A+4B+3C-2D=-13 \text{ ------- (3)}$$

Tangent at $(3,-2)$ // to y-axis. From (i) $\Rightarrow$ $2By+D=0$:

$$-4B \qquad +D=0 \text{ --------- (4)}$$

Using GC, $A=1, B=4, C=-2, D=16$

16. VJC11/C1BT/Q1

Let c, l, m be the ERP rates for cars, lorries & motorcycles respectively in dollars

	$123c+91l+210m=788.5$ $175c+98l+210m=910$ $154c+103l+190m=850.5$ $\therefore c=2, \quad l=2.50, \quad m=1.50$ New rate for lorries (in \$) $=2.50\times1.2=3$ $\therefore$ Day 3's revenue (in \$) $=154(2)+103(3)+190(1.5)$ $=902$	
17	**ASRJC/2019C1BT1/2** Let x, y and z be the usual prices of a box of chocolates, a box of biscuits and a packet of nuts respectively. $x+y+z=73.4$ ---- (1) $0.85(3x)+x+6y-15+3z=297.97 \Rightarrow 3.55x+6y+3z=312.97$ ---- (2) $0.85(6x)+5y-10+2z=322.39 \Rightarrow 5.1x+5y+2z=332.39$ ---- (3) From GC, $x=36.4$, $y=24.25$, $z=12.75$. The price of a box of chocolate is \$36.40; the price of a box of biscuits is \$24.25; the price of a packet of nuts is \$12.75. Total savings made by Mary = 0.15(3 × 36.40) + 15 = <u>\$31.38</u>.	