Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) 9740/02

Paper 2 **23 September 2015**

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Total
Score													
Max Score	**10**	**10**	**10**	**10**	**4**	**5**	**7**	**7**	**8**	**8**	**10**	**11**	**100**

Section A: Pure Mathematics [40 marks]

1 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_1 = \frac{1}{4}$ and

$$u_{n+1} = u_n - \frac{4}{n^2(n+1)(n+2)^2}, \quad \text{for all } n \geq 1.$$

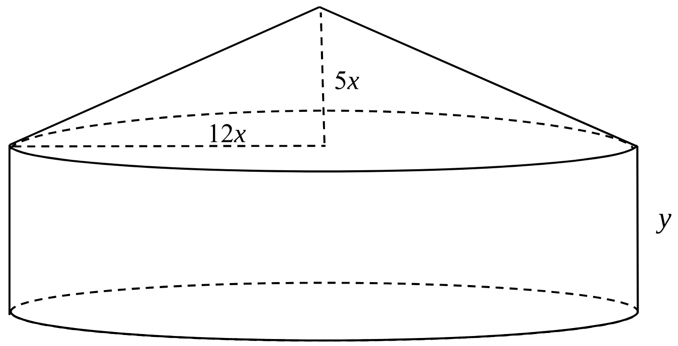
(i) Use the method of mathematical induction to prove that $u_n = \dfrac{1}{n^2(n+1)^2}$. [4]

(ii) Hence find $\displaystyle\sum_{n=1}^{N} \frac{1}{n^2(n+1)(n+2)^2}$. [2]

(iii) Give a reason why the series in part **(ii)** is convergent and state the sum to infinity. [2]

(iv) Use your answer to part **(ii)** to find $\displaystyle\sum_{n=2}^{N} \frac{1}{n(n^2-1)^2}$. [2]

2 [It is given that a right circular cone with base radius r and height h has volume $\frac{1}{3}\pi r^2 h$, and the curved surface area is $\pi r l$, where l is the slanted height of the cone.]



The model of a silo is made up of three parts.

- The roof is modelled by the curved surface of a right circular cone with base radius $12x$ cm and height $5x$ cm.
- The walls are modelled by the curved surface of a cylinder of radius $12x$ cm and height y cm.
- The floor is modelled by a circular disc of radius $12x$ cm.

The three parts are joined together as shown in the diagram. The model is made of material of negligible thickness.

The curved surface of the cone is made of material A at the cost of \$0.05 per cm^2. The curved surface of the cylinder and the base of the cylinder are made of material B at the cost of \$0.02 per cm^2.

Given that it costs \$100 to make the model, show that the volume V cm^3 of the model is given by

$$V = 30000x - 2964\pi x^3.$$

Hence find, using differentiation, the values of x and y which give a model of maximum volume. [10]

3 **(i)** Given that the complex number z satisfies the equation $z=\dfrac{4}{z^*}$, show that $|z|=2$. [1]

(ii) Express $1-\mathrm{i}\sqrt{3}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$. [1]

(iii) On a single Argand diagram, sketch the loci

(a) $z=\dfrac{4}{z^*}$,

(b) $|z+2|=|z-1+\mathrm{i}\sqrt{3}|$. [3]

(iv) The complex numbers z_1 and z_2 satisfy the equations $z=\dfrac{4}{z^*}$ and $|z+2|=|z-1+\mathrm{i}\sqrt{3}|$, where $\arg(z_1)>\arg(z_2)$. Find the exact values of z_1 and z_2, giving your answers in the form $x+\mathrm{i}y$. [4]

(v) Another complex number w satisfies $w=\dfrac{4}{w^*}$ and $\frac{1}{2}\pi<\arg w<\pi$.

Explain why $\arg(z_1-w)-\arg(z_2-w)=\frac{1}{2}\pi$. [1]

4 A farmer owns a parcel of land that is partially covered with weed. The area that is covered with weed increases by 80 m^2 each week. The farmer decides to start weeding. At the start of the first week of weeding, 500 m^2 of the land is covered with weed.

(i) In option 1, the farmer removes weed from 10% of the area covered with weed at the end of each week.

(a) Find the area covered with weed at the end of the second week. [2]

(b) Show that the area covered with weed at the end of the nth week is given by $(0.9^n(500)+k(1-0.9^n))\text{ m}^2$, where k is a constant to be determined. [3]

(c) Find the area covered with weed at the end of a week in the long run. [1]

(ii) In option 2, the farmer removes weed from an area of 50 m^2 at the end of the first week. He removes weed from an additional area of 10 m^2 at the end of each subsequent week. Thus he removes weed from an area of 60 m^2 at the end of the second week, and 70 m^2 at the end of the third week, and so on.

(a) Show that the change in the area covered with weed in the nth week is given by $(40-10n)\text{ m}^2$. [1]

(b) Hence, or otherwise, find the area covered with weed at the end of the nth week in terms of n. [3]

Section B: Statistics [60 marks]

5 A student decides to conduct a survey in his secondary school. His school consists of four levels, with 400 students in each level.

(i) The student randomly surveys 10 students from level 1, 20 students from level 2, 30 students from level 3 and 40 students from level 4. Explain whether this method is stratified sampling. [1]

(ii) Describe how a quota sample of size 40 might be obtained, and state one disadvantage of quota sampling. [3]

6 The Amazing Hair Salon offers hairstyling services to both male and female customers. The times spent (in minutes) at the hair salon by male and female customers, denoted by M and F, respectively, are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard Deviation
M	μ	18
F	71	35

(i) Given that $P(M < 32) = P(M > 67)$, state the value of μ. [1]

(ii) Find the probability that the average time spent by five randomly chosen female customers is more than 3 times the time spent by a randomly chosen male customer. State clearly the mean and variance of any normal distribution you use in your calculation. [3]

(iii) A statistician commented that the normal distribution given above for the times spent by female customers is not an appropriate model. What could be his reason for making this comment? [1]

7 Data is transmitted in bytes, where each byte consists of 8 bits. The probability of a bit being corrupted during its transmission is 0.03. A byte is considered ‘corrupted’ if it contains at least 2 corrupted bits. Assume that all bits are not corrupted prior to their transmission.

(i) Show that the probability that a randomly chosen byte is corrupted during its transmission is 0.0223. [1]

(ii) Given that a randomly chosen byte is not corrupted during its transmission, find the probability that it contains no corrupted bits. [3]

(iii) Using a suitable approximation, find the probability that between 5 and 10 bytes are corrupted during the transmission of 100 bytes. State the parameter(s) of the distribution you use. [3]

8 A family of seven, consisting of two sisters, three brothers and their parents, is queuing up in a line outside a restaurant.

Find the number of ways in which the family can queue up if

(i) one parent is standing at the front and one parent is standing at the back of the queue, [1]

(ii) the person standing in between the parents is one of the brothers. [3]

Upon entering the restaurant, the family is seated at a round table.

(iii) Find the number of ways they can be seated such that no two brothers are next to each other. [3]

9 For independent events A and B, it is given that $\mathrm{P}(A \cap B) = 0.05$ and $\mathrm{P}(A \cup B) = 0.55$.

(i) Find $\mathrm{P}(A)$ and $\mathrm{P}(B)$ if $\mathrm{P}(A) < \mathrm{P}(B)$. [4]

For a third event C, $\mathrm{P}(C) = 0.5$ and $\mathrm{P}(A' \cup B' \mid C) = 0.95$.

(ii) Find $\mathrm{P}(A \cap B \cap C)$. [3]

(iii) State, with reason, whether the events A and C are mutually exclusive. [1]

10 In an experiment, different quantities of fertilizer, x ml, were given to seven radish plants of the same height. The heights, y mm, of the plants were measured after ten days. The results are given in the table.

x	10	15	20	25	30	35	40
y	407	412	420	434	450	465	490

(i) Draw a scatter diagram to illustrate the data. [1]

It is suggested that the height y can be modelled by one of the formulae

$$y = a + bx \qquad \text{or} \qquad y = c + dx^2$$

where a, b, c and d are constants.

(ii) Find, correct to 4 decimal places, the value of the product moment correlation coefficient between

(a) x and y,

(b) x^2 and y. [2]

(iii) Use your answers to parts **(i)** and **(ii)** to explain which of $y = a + bx$ or $y = c + dx^2$ is the better model. [1]

(iv) It is desired to estimate the value of x for which $y = 440$. Explain why neither the regression line of x on y nor the regression line of x^2 on y should be used. [1]

(v) By finding the equation of a suitable regression line, find the required estimate in part **(iv)**. Comment on the reliability of your estimate. [3]

11 A call centre has two hotlines: one for enquiries and one for complaints. Over a long period of time, it is found that the average numbers of calls received by the enquiry and complaint hotlines in a minute are 2 and 3 respectively.

(i) State, in this context, two conditions that must be met for the numbers of calls received by each hotline to be well modelled by Poisson distributions. [2]

For the remainder of this question, assume that these conditions are met. You should assume that the two hotlines receive calls independently of each other.

(ii) The probability that no more than 1 call is received by the complaint hotline in t minutes is 1×10^{-5}. Find an equation for t. Hence find the value of t, giving your answer to the nearest whole number. [3]

(iii) Find the probability that the total number of calls received by both hotlines in a randomly chosen period of 10 minutes is more than 40. [2]

(iv) Use a suitable approximation to find the probability that, in a randomly chosen period of 10 minutes, the number of calls received by the complaint hotline exceed the number of calls received by the enquiry hotline. State the parameter(s) of the distribution you use. [3]

12 In 2012, the National Health Board (NHB) of a certain country reported a mean cholesterol level of 199 mg/dL (milligrams per decilitre) for people living in its cities. An employee of the NHB, Abbey, carried out a study in 2015 to test if the mean cholesterol level of city-dwellers has increased. The cholesterol levels, x mg/dL, of a random sample of 90 city-dwellers are measured and the data are summarised by

$$\sum(x-120)=7380, \quad \sum(x-120)^2=629982.$$

(i) Find the unbiased estimates of the population mean and variance. [2]

(ii) Carry out Abbey's test, at the 5% significance level. [3]

Another NHB employee, Betty, was tasked to investigate the cholesterol level of people living in the countryside. She collected a random sample of 20 to test, at the 10% significance level, whether people living in the countryside has the same mean cholesterol level as that reported by NHB in 2012 for city-dwellers. The sample mean is $\bar{y}$ mg/dL and the sample variance is $281.5\ (\text{mg/dL})^2$.

(iii) Use an algebraic method to calculate the set of values of $\bar{y}$ for which the null hypothesis would not be rejected for Betty's test. (Answers obtained by trial and improvement from a calculator will obtain no marks.) [4]

State an assumption about the population that is necessary for the test conducted by Betty to be valid. Explain why, for Abbey's test to be carried out, the same assumption need not be made about the population being studied by her. [2]