



H3 Physics 9814 - Newtonian Mechanics Problems

1 G.C.E. 'A' Level Physics Special Paper 2000 Question 7(c)

Raindrops drip from rest from the edge of a roof at a height h above the ground. The drops leave the roof one after the other at a steady rate of n per unit time, where n is large. In the calculations which follow, neglect air resistance.

- (i) Find an expression, in terms of n , h and the acceleration of free fall g , for the number of drops in the air at any one instant of time. [2]
- (ii) Draw a sketch showing the approximate positions of the raindrops at the instant in (i). [1]
- (iii) Find, in terms of h , the height h_{med} above and below which equal numbers of raindrops are found, (h_{med} : the median height). [4]
- (iv) Find in terms of h , the average height $\langle h \rangle$ of the raindrops above the ground. [3]

[Hint: $1^2 + 2^2 + 3^2 + \dots + N^2 = (N/6)(N+1)(2N+1) \approx N^3/3$ for large N .]

2 G.C.E. 'A' Level Physics Special Paper 2001 Question 1

A student, standing on the platform at Keppel Road Station, Singapore, is watching the arrival of a train from Kuala Lumpur: She notes that the engine and the first carriage take 2.0 s to pass her, and the next two carriages take 2.4 s. The engine, and each of the carriages, is 20 m long. The braking deceleration of the train is constant. When the train comes to a stop, the student finds that the rear part of the last carriage is opposite her.

- (a) Calculate the deceleration of the train. [3]
- (b) Deduce the total number of vehicles (the engine plus the carriages) in the train. [3]



3 G.C.E. 'A' Level Physics Special Paper 2006 Question 10

A small object of density 900 kg m^{-3} is dropped into a very deep tank that contains water of density 1000 kg m^{-3} . The object enters the water with a velocity of -7.00 m s^{-1} , where the upwards direction is positive.

- (a) Assume that the object is not subject to a drag force as it moves through the water.
- (i) Calculate the maximum depth reached by the object and the total time from entering the water before it resurfaces. The hydrostatic upthrust experienced by a body of volume V when immersed in a fluid of density ρ is $V\rho g$, where g is the acceleration of free fall. [6]
 - (ii) Sketch a full-page graph to show how the velocity v of the object depends on time t . Start at the instant the object enters the water and finish when it resurfaces. Mark the axes with important values of v and t . Label this graph **A**. Check your answer in (i) by considering an appropriate area on your graph. [3]
- (b) In fact the object resurfaced in exactly 11 s. This should be shorter than your answer in (a)(i). The discrepancy suggests that a drag force acts on the object. Assume that the drag force F_d is proportional to the velocity v of the object through the water. It is convenient to write this proportionality as

$$F_d = -kmv$$

where k is a constant and m is the mass of the object, which is also a constant.

(Because the object under consideration is of fixed size, there is no need to consider the variation of drag force with the dimensions of the object. Similarly, because the object is falling through water only, there is no variation of drag force with viscosity.)

- (i) Obtain the equation of motion of the object in terms of acceleration $\frac{dv}{dt}$, k , g and appropriate number(s). [2]
- (ii) Hence show that an equation relating the value of v to time t

$$kt = -\ln \frac{(1.09 - kv)}{(1.09 + 7.00k)}$$

where k is measured in s^{-1} , t in s and v in m s^{-1} . [3]

[Hint: $\int \frac{dx}{(a - bx)} = -\frac{1}{b} \ln(a - bx)$]

- (iii) Solution of the equation in (ii) and substitution of the appropriate boundary conditions gives the value $k = 0.092 \text{ s}^{-1}$. (You are not asked to make this deduction.)

Use the value of k to find the times taken for the object,

1. to travel from the surface to the lowest point, [2]
 2. to return from the lowest point to the surface. [1]
- (iv) On the same axes as the graph in (a)(ii), sketch the graph to show how v depends on t when there is this drag force. Again, mark the axes with important values. Label this graph **B**. Comment on any features of areas on this graph compared with areas in the graph of (a)(ii). [3]



4 G.C.E. 'A' Level Physics Special Paper 1999 Question 6

A particle of mass m strikes a horizontal plane with velocity u at an angle θ to the plane. The collision is not perfectly elastic, and after the impact the particle moves off with velocity v at an angle ϕ to the plane as shown in Fig. 6.1.

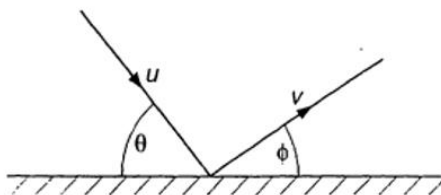


Fig. 6.1

In such a collision, the component of velocity parallel to the plane remains constant, but the magnitude of the component normal to the plane after the impact is e times its magnitude before, where e is a constant less than one.

- (i) Find the fraction F of the kinetic energy of the particle lost in the impact. Give your answer in terms of e and θ . [5]
 - (ii) Describe the main features of a collision, similar to the one shown in Fig. 6.1, but in which the value of the constant e is exactly one. [2]
- (b) A ball bounces inelastically down a flight of steps in a plane perpendicular to the front edges of the steps as shown in Fig. 6.2.

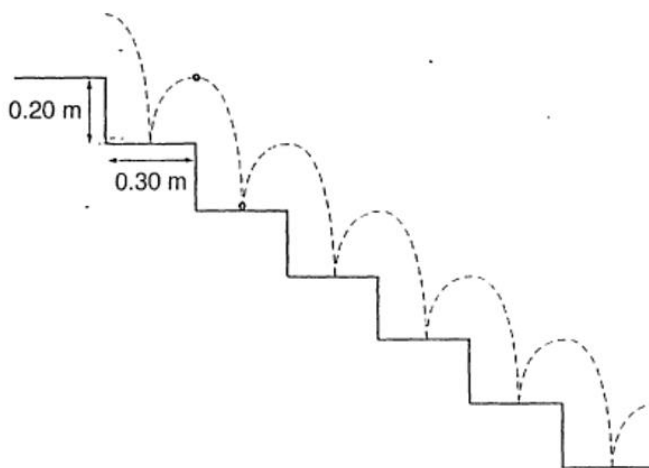


Fig. 6.2

Each step is 0.20 m high and 0.30 m deep. It is observed that the ball always bounces exactly in the middle of each step, and that after each bounce it rises to the height of the previous step. Air resistance can be neglected.

- (i) Show that the value of the constant e for these impacts (i.e., the vertical component of velocity immediately after impact divided by the vertical component of velocity immediately before) is 0.71. [2]
- (ii) Show also that the horizontal component of velocity of the ball is 0.62 m s^{-1} . [4]



- (c) At the bottom of the steps in (b) the ball continues to bounce along a horizontal pavement as shown in Fig. 6.3.

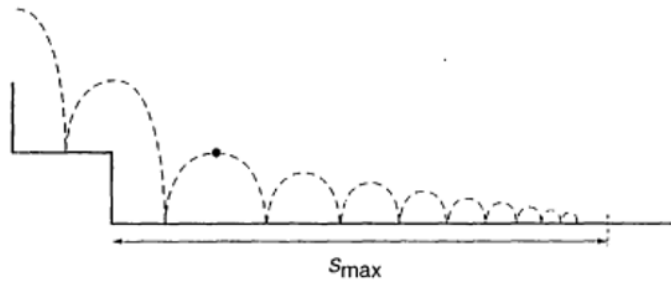


Fig. 6.3

The collisions of the ball with the pavement have the same of e (0.71) as for the impacts with the steps. Show that the ball does bounce beyond a distance from the bottoms of the steps and calculate s_{\max} . [7]

[Hint: if $x < 1$, $1 + x + x^2 + x^3 + \dots = 1 / (1-x)$]



5 G.C.E. 'A' Level Physics Special Paper 1993 Question 1c

The magnitude of the air resistance acting on a cyclist is kv_t^2 , where v_t is the velocity of the air relative to the cyclist and k is a constant. The force acts in the direction of the relative velocity.

Figure 1 shows a cyclist with a road speed v . The wind speed is w and it has a direction θ relative to the direction in which the cyclist is moving.

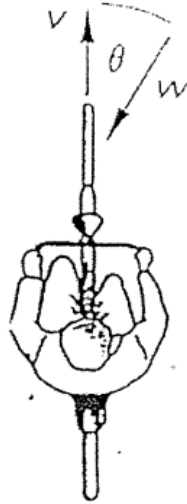


Figure 1

- (i) Show that the power developed by the cyclist in overcoming a head wind ($\theta = 0^\circ$) is $kv(v + w)^2$.
[3]
- (ii) Show that the power developed by the cyclist in overcoming a head wind ($\theta = 90^\circ$) is $kv^2(v^2 + w^2)^{\frac{1}{2}}$.
[6]
- (iii) Hence show that the power required to cycle at speed v in a cross wind of speed v is $\sqrt{2}$ times as great as the power required to cycle at speed v in still air.
[2]



6 G.C.E. 'A' Level Physics Special Paper 1994 Question 4

- (a) A body moving in a circle with constant angular speed also has constant linear speed. Explain how it is that the body nevertheless accelerates. [3]
- (b) A helicopter moves in level flight with uniform speed v along the line Ox, as shown in plan view in Fig. 4.1.

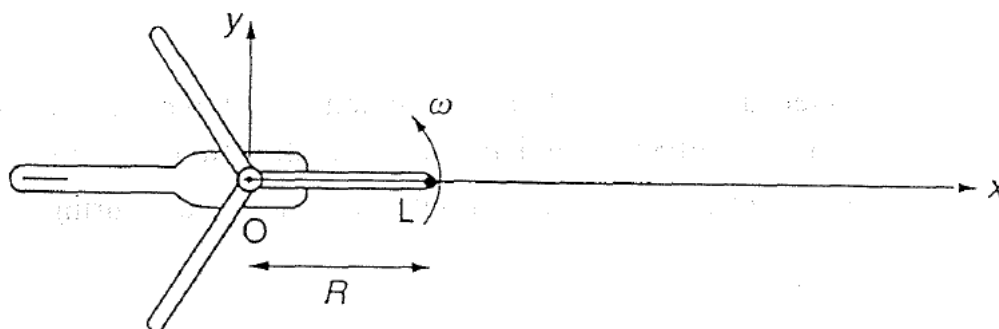


Fig. 4.1

Attached to the end of one of the rotors, at a distance R from the rotor's axle, is a point light source L. The rotor rotates with uniform angular speed ω in an anti-clockwise direction. At time $t = 0$, the axle is at the origin and the light bearing rotor is along Ox, as shown in Fig. 4.1.

- (i) Find the x and y co-ordinates of the light source at time t . [3]
- (ii) Explain how the velocity of the light source at time t could be calculated. [2]
- (iii) What is the acceleration of the light source? [2]
- (iv) Consider the particular case when the helicopter is moving with speed $v = 5\pi \text{ m s}^{-1}$, the rotor has angular speed $\omega = \pi \text{ rad s}^{-1}$, and the length R of the rotor is 5 m (so that $R\omega = v$).

Calculate a series of values of the co-ordinates (x, y) of the light source. Hence, draw a graph to scale showing the pattern traced out by the light source, as seen by an observer on the ground at night. [8]

- (v) At a different rotor speed ω_1 , but with the same flight speed v , the pattern shown in Fig. 4.2 is observed from ground.

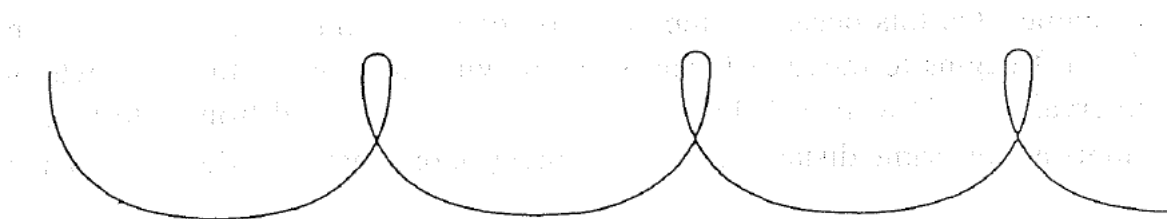


Fig. 4.2

What can you deduce about the value of ω_1 , compared with the original value of $\pi \text{ rad s}^{-1}$? Explain your answer. (A qualitative treatment only is required.) [2]



7 G.C.E. 'A' Level Physics Special Paper 2001 Question 7

- (a) A student correctly states the conditions for equilibrium of a system by means of the following equations:

$$\Sigma(\text{force}) = 0 \quad \text{equation 7.1}$$

$$\Sigma(\text{torque}) = 0 \quad \text{equation 7.2}$$

- (i) Explain the physical meaning of both equations. [2]
- (ii) The student attempts to apply the conditions to a particle in uniform circular motion, and states:

‘Because the particle is moving with uniform angular velocity, no torque is acting. The particle is therefore in equilibrium.’

The student’s statement may be correct in some respects, but is certainly incorrect and incomplete in others. Identify any features of the statement which are correct. In what respects is the statement incorrect and incomplete? [6]

- (b) A semicircular shape of radius R is cut from a thick piece of wood. The wood is of non-uniform density, and its centre of gravity is at the point G shown in Fig. 7.1.

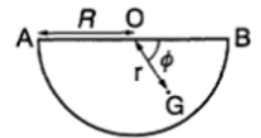


Fig. 7.1

The weight of the shape is W . G is at a distance r from the centre O of the semicircle, along a radius making an angle ϕ with the diameter AOB of the semicircle.

In a series of experiments to locate the centre of gravity, the following observations are made.

1. When placed with its circular face on a horizontal surface, the shape comes to rest with its diameter AOB at an angle θ to the horizontal (Fig. 7.2).

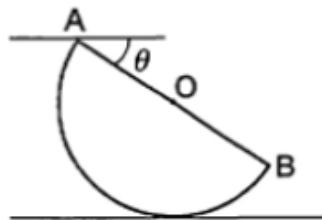


Fig. 7.2

2. When a uniform rod of weight P and length R is placed along the part AO of the diameter, the system comes to rest with AOB exactly horizontal (Fig. 7.3).

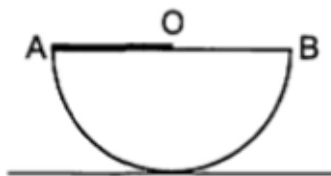


Fig. 7.3



3. When the rod is fixed along the part OB of the diameter, the system comes to rest with AOB at an angle θ to the vertical (Fig. 7.4).

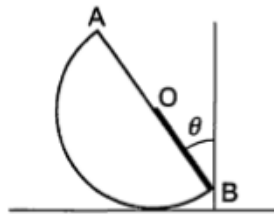


Fig. 7.4

4. When the shape is freely pivoted about a horizontal axis through B, it comes to rest with AOB at an angle θ to the vertical (Fig. 7.5).

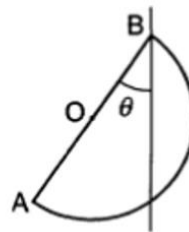


Fig. 7.5

- (i) From each of these observations, obtain a relationship between two or more of the quantities r , R , P , W , θ and ϕ . In each case, explain your reasoning. [8]
- (ii) Use your **four** relationships from (i) to
1. Show that the angle θ is 35.3° ,
 2. Find an expression for P in terms of W ,
 3. Locate the centre of gravity of the shape. [4]

[Hint: $\cos 2\theta = 1 - 2 \sin^2 \theta$.]



Numerical Answers:

1 (i) $n\sqrt{\frac{2h}{g}}$; (iii) $\frac{3}{4}h$; (iv) $\frac{2}{3}h$

2 (a) 1.52 m s^{-2} ; (b) 8

3 (a)(i) 22.5 m ; 12.8 s ; (b)(i) $dv/dt = 1.09 - kv$; b(iii)(1) 5.05 s (2) 5.95 s

4 (a)(i) $(1 - e^2)\sin^2\theta$ (c) 1.0 m

7 (b)(ii) $2P = 0.688W$; (b)(ii) $3r = 0.578R$

Suggested solutions – H3 Newtonian Mechanics Problems 2024

1	(i)	<p>Let there be N number of raindrops in the air at any one instant of time, t. The rate of falling raindrops is n, which can be expressed as N/t.</p> <p>Time to fall through the height h is</p> $\left(s = ut + \frac{1}{2}at^2 \right)$ $h = 0 + \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h}{g}}$ <p>There are N raindrops, which gives $N = nt = n\sqrt{\frac{2h}{g}}$</p>
	(ii)	<p>Follow 1, 4, 9, 16, pattern</p> <p style="text-align: center;">.</p> <p style="text-align: center;">.</p> <p style="text-align: center;">.</p> <p style="text-align: center;">.</p>
	(iii)	<p>There should be $N/2$ raindrops above the h_{median} and $N/2$ raindrops below the h_{median}.</p> $\frac{N}{2} = n\sqrt{\frac{2h_{fall}}{g}}$ $\frac{n\sqrt{\frac{2h}{g}}}{2} = n\sqrt{\frac{2h_{fall}}{g}} \rightarrow h_{fall} = \frac{h}{4}$ $h_{median} = h - h_{fall} = h - \frac{h}{4} = \frac{3h}{4}$
	(iv)	$\langle h \rangle_{fall} = \frac{h_1 + h_2 + \dots + h_N}{N}$ <p>At t_1, there is 1 raindrop, at t_2, there are 2 raindrops etc.</p> $t_1 = \frac{at}{N} \text{ where } a = 1, 2, \dots, N. \text{ (at } t_N \text{ there are } N \text{ raindrops)}$

$$h_1 = \frac{1}{2} g t_1^2 = \frac{1}{2} g \left(\frac{1 t}{N} \right)^2 = \frac{1}{2} \frac{g t^2}{N^2} 1^2$$

$$h_2 = \frac{1}{2} g t_2^2 = \frac{1}{2} g \left(\frac{2 t}{N} \right)^2 = \frac{1}{2} \frac{g t^2}{N^2} 2^2$$

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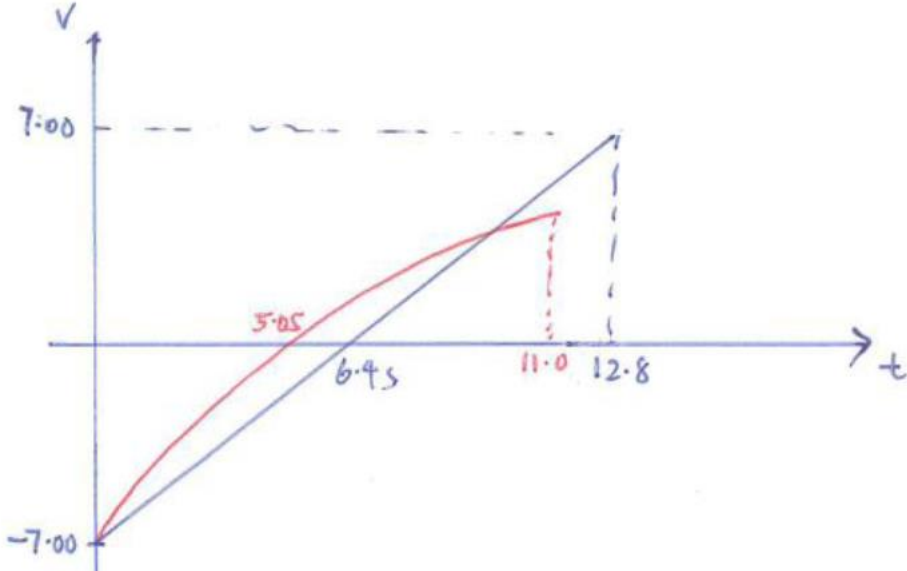
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$$h_N = \frac{1}{2} g t_N^2 = \frac{1}{2} g \left(\frac{N t}{N} \right)^2 = \frac{1}{2} \frac{g t^2}{N^2} N^2$$

$$\langle h \rangle_{\text{fall}} = \frac{1}{2} \frac{g t^2}{N^2} \left(\frac{1^2 + 2^2 + \dots + N^2}{N} \right) = \frac{h}{N^3} \left(\frac{N^3}{3} \right) = \frac{h}{3}$$

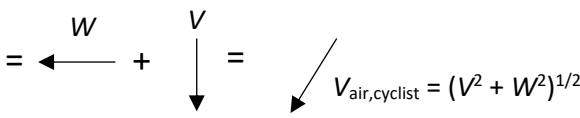
The average height above the ground is then, $\langle h \rangle = h - \frac{h}{3} = \frac{2h}{3}$.

2	(a)	$\left(s = ut + \frac{1}{2}at^2 \right)$ $40 = u(2.0) + \frac{1}{2}(-a)(2.0)^2 \quad (1)$ $40 + 40 = u(4.4) + \frac{1}{2}(-a)(4.4)^2 \quad (2)$ <p>Solving simultaneous equations (1) and (2) gives, $a = -1.52 \text{ m s}^{-2}$</p>
	(b)	$(v^2 = u^2 + 2as)$ $0^2 = 21.52^2 + 2(-1.52)s$ $s = 152.8 \text{ m}$ $N = \frac{s}{20} = 7.6$ <p>So, there should be 8 vehicles.</p>

3	(a)(i)	<p>By Newton's 2nd Law,</p> $F_{net} = U - W = ma$ $a = \frac{U - W}{m} = \frac{\rho_w Vg - \rho_o Vg}{\rho_o V} = \frac{(\rho_w - \rho_o)g}{\rho_o} = \frac{g}{9}$ $v^2 = u^2 - 2ah_{max} \rightarrow v = 0 \rightarrow h_{max} = \frac{7.00^2}{2 \frac{g}{9}} = 22.5 \text{ m}$ $v = u - at \rightarrow t = \frac{7.00}{g/9} = 6.42 \text{ s. Hence, time to resurface is } 12.8 \text{ s.}$
	(a)(ii)	
	(b)(i)	$F_{net} = m \frac{dv}{dt} = W - U - F_d$ $m \frac{dv}{dt} = m \frac{g}{9} - kmv$ $\frac{dv}{dt} = 1.09 - kv$
	(b)(ii)	$\frac{dv}{dt} = 1.09 - kv$ $\frac{dv}{1.09 - kv} = dt \rightarrow \int_{-7.00}^v \frac{dv}{1.09 - kv} = \int_0^t dt$ <p>Use the hint given in the question to solve the integration,</p>

		$-\frac{1}{k} \ln(1.09 - kv) \Big _{-7.00}^v = t$ $-\frac{1}{k} \left(\frac{\ln(1.09 - kv)}{\ln(1.09 + 7.00k)} \right) = t$ $kt = - \left(\frac{\ln(1.09 - kv)}{\ln(1.09 + 7.00k)} \right)$
	(b)(iii)	<p>1. $v = 0 \rightarrow 0.092t = - \left(\frac{\ln(1.09 - 0.092(0))}{\ln(1.09 + 7.00(0.092))} \right) \rightarrow t = 5.05 \text{ s}$</p> <p>2. $11 - 5.05 = 5.95 \text{ s}$</p>

4	(a)(i)	$e(u \sin \theta) = v \sin \phi$ $v = \sqrt{(u \cos \theta)^2 + (eu \sin \theta)^2}$ $F = \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \frac{\frac{1}{2}mu^2 - \frac{1}{2}m((u \cos \theta)^2 + (eu \sin \theta)^2)}{\frac{1}{2}mu^2}$ $F = \frac{u^2(1 - \cos^2 \theta - e^2 \sin^2 \theta)}{u^2} = (1 - e^2) \sin^2 \theta$
	(a)(ii)	<p>If $e = 1$, no loss in energy takes place in this collision, which indicates an elastic collision. It also implies that $u = v$ and $\theta = \phi$.</p>
	(b)(i)	$v^2 = u^2 + 2as, \quad v = v_{\text{before impact}}, \quad u = 0 \text{ (at max height)}, \quad a = -g$ $v_{\text{before impact}} = \sqrt{2(9.81)(0.40)}$ $v^2 = u^2 + 2as, \quad u = u_{\text{after impact}}, \quad v = 0 \text{ (at max height)}, \quad a = -g$ $u_{\text{after impact}} = \sqrt{2(9.81)(0.20)}$ $e = \frac{u_{\text{after impact}}}{v_{\text{before impact}}} = \frac{\sqrt{2(9.81)(0.20)}}{\sqrt{2(9.81)(0.40)}} = 0.71$
	(b)(ii)	$u_{\text{after impact}} = gt_1 \rightarrow t_1 = \frac{\sqrt{2(9.81)(0.20)}}{9.81} = 0.202 \text{ s}$ $0.40 = \frac{1}{2}gt_2^2 \rightarrow t_2 = \sqrt{\frac{0.80}{9.81}} = 0.286 \text{ s}$ $\sum t = 0.202 + 0.286 = 0.488 \text{ s}$ $d_{\text{hor}} = u_{\text{hor}} \sum t \rightarrow u_{\text{hor}} = \frac{0.30}{0.488} = 0.62 \text{ m s}^{-1}$
	(c)	$s_{\text{max}} = 0.15 + s_1 + s_2 + \dots + s_n$ $s_1 = u_{\text{hor}} t_{\text{flight},1}, \quad s_2 = u_{\text{hor}} t_{\text{flight},2} \text{ etc.}$ $t_{\text{flight},1} = 2 \frac{\sqrt{2(9.81)(0.20)}}{9.81}, \quad t_{\text{flight},2} = (e) \times 2 \frac{\sqrt{2(9.81)(0.20)}}{9.81}, \quad t_{\text{flight},3} = (e)^2 \times 2 \frac{\sqrt{2(9.81)(0.20)}}{9.81}$ $s_1 + s_2 + \dots = u_{\text{hor}} 2 \frac{\sqrt{2(9.81)(0.20)}}{9.81} + u_{\text{hor}} (e) \times 2 \frac{\sqrt{2(9.81)(0.20)}}{9.81} + u_{\text{hor}} (e)^2 \times 2 \frac{\sqrt{2(9.81)(0.20)}}{9.81} + \dots$ $= u_{\text{hor}} 2 \frac{\sqrt{2(9.81)(0.20)}}{9.81} (1 + e + e^2 + \dots) = u_{\text{hor}} 2 \frac{\sqrt{2(9.81)(0.20)}}{9.81} \left(\frac{1}{1 - e} \right)$ $s_{\text{max}} = 0.15 + (0.62) 2 \frac{\sqrt{2(9.81)(0.20)}}{9.81} \left(\frac{1}{1 - 0.71} \right) = 1.01 \text{ m}$

5	(i)	<p>Relative speed is $v + w$.</p> <p>Power developed by the cyclist to overcome the head wind</p> $P = F_{cyclist} v = F_{air} v = k(v + w)^2 v$ <p>Please take note that $F_{cyclist}$ and F_{air} are parallel.</p>
	(ii)	<p>The relative velocity is $v_{air,cyclist} = v_{air,Earth} + v_{Earth,cyclist}$</p> <div style="text-align: center;">  </div> <p>Power developed by the cyclist to overcome the head wind at 90°</p> $P = F_{cyclist} v = (F_{air} \cos \alpha) v = k(v^2 + w^2)(\cos \alpha) v = k(v^2 + w^2) \frac{v}{\sqrt{(v^2 + w^2)}} v$ $P = kv^2 (v^2 + w^2)^{\frac{1}{2}}$
	(iii)	<p>In cross wind v, $P_{cross} = kv^2 (v^2 + v^2)^{\frac{1}{2}} = kv^3 \sqrt{2}$</p> <p>In still air where speed of air is zero, $P_{still} = kv^2 (v^2 + 0)^{\frac{1}{2}} = kv^3$</p> $\frac{P_{cross}}{P_{still}} = \frac{\sqrt{2}kv^3}{kv^3} = \sqrt{2}$

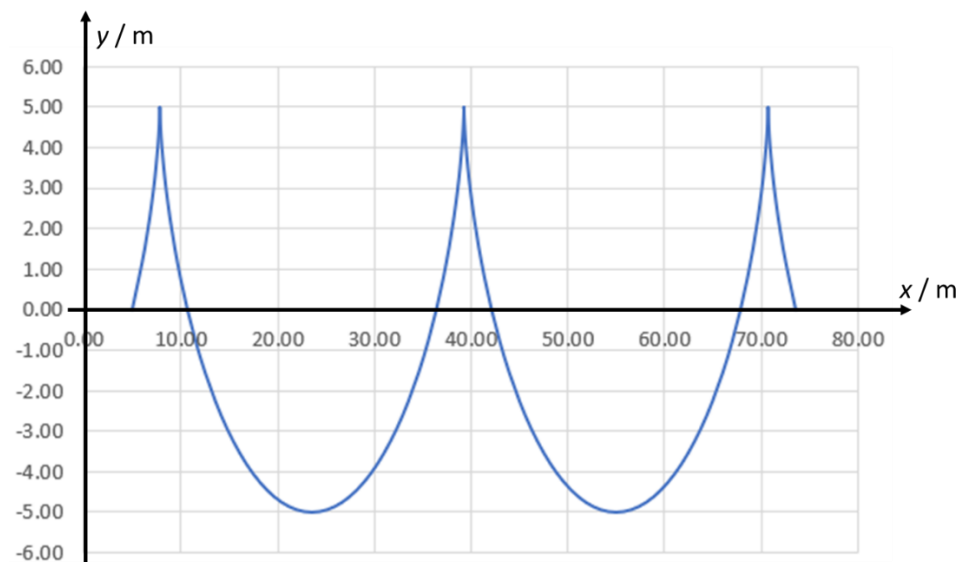
6	(a)	<p>As the body rotates in a circle with a constant angular speed and constant linear speed, its direction keeps changes. Hence, the change in velocity is not zero. The acceleration, which is a vector, is defined as the rate of change of velocity. There must be an acceleration and hence net force acting on the body performing the uniform circular motion. The direction of the acceleration is towards the centre of the circle.</p>
	(b)(i)	$x = vt + R\cos(\omega t)$ $y = R\sin(\omega t)$ $(x, y) = (vt + R\cos(\omega t), R\sin(\omega t))$
	(b)(ii)	$v_x = \frac{dx}{dt} = v - \omega R\sin(\omega t)$ $v_y = \frac{dy}{dt} = \omega R\cos(\omega t)$ $(v_x, v_y) = (v - \omega R\sin(\omega t), \omega R\cos(\omega t))$ <p>The velocity of the light source, v, could be calculated from $v = \sqrt{v_x^2 + v_y^2}$.</p>
	(b)(iii)	$a_x = \frac{dv_x}{dt} = -\omega^2 R\cos(\omega t)$ $a_y = \frac{dv_y}{dt} = -\omega^2 R\sin(\omega t)$ $(a_x, a_y) = (-\omega^2 R\cos(\omega t), -\omega^2 R\sin(\omega t))$ $a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-\omega^2 R\cos(\omega t))^2 + (-\omega^2 R\sin(\omega t))^2} = \omega^2 R$

(b)(iv)

Given that $\omega = \pi$, then the period is $T = \frac{2\pi}{\omega} = 2.0 \text{ s}$.

t / s	x / m	y / m
0.0	5.00	0.00
0.1	6.33	1.55
0.2	7.19	2.94
0.3	7.65	4.05
0.4	7.83	4.76
0.5	7.85	5.00
0.6	7.88	4.76
0.7	8.06	4.05
0.8	8.52	2.94
0.9	9.38	1.55
1.0	10.71	0.00
1.1	12.52	-1.55
1.2	14.80	-2.94
1.3	17.48	-4.05
1.4	20.45	-4.76
1.5	23.56	-5.00
1.6	26.68	-4.76
1.7	29.64	-4.05
1.8	32.32	-2.94
1.9	34.60	-1.55
2.0	36.42	0.00

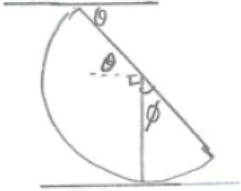
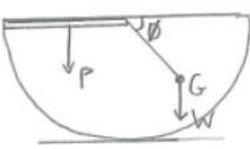
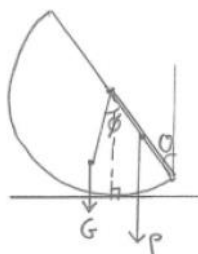
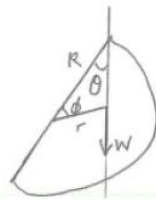
Plotting y against x graph for 5.0 s gives



(b)(v)

No loop indicates that the maximum backward velocity of the rotor blade $\omega R = 5\pi$ is equal to the forward velocity of the helicopter, $v = 5\pi$.

ω_1 must be greater than $\omega = \pi$ so that the maximum backward velocity of the rotor blade is greater than the forward velocity of the helicopter, which results in a backward loop.

7	(a)(i)	<p>$\Sigma(\text{force}) = 0$, net force acting on the system is zero. Translational equilibrium.</p> <p>$\Sigma(\text{torque}) = 0$, net torque acting on the system is zero. Rotational equilibrium.</p>
	(a)(ii)	<p>Correct: The particle is not rotating about its centre of mass, so no torque is acting.</p> <p>Incorrect: The particle is moving with changing linear velocity, so it is not in translational equilibrium.</p> <p>The particle, therefore, is not in equilibrium.</p>
	(b)(i)	<p>1.  $\theta + \phi = 90^\circ$</p>
		<p>2.  moments about O. $P \times \frac{R}{2} = W \times R \cos \phi$</p>
		<p>3.  moments about O. $P \times \frac{R}{2} \sin \theta = W \times r \sin (\phi - \theta)$</p>
		<p>4.  $\frac{R}{\sin (180^\circ - \phi - \theta)} = \frac{r}{\sin \theta} \Rightarrow r = R \sin \theta$ actually, since $\theta + \phi = 90^\circ$, it's actually a \triangle.</p>

	(b)(ii)	<p>1.</p> $\frac{PR}{2} = Wr \cos \phi$ $\frac{PR}{2} \sin \theta = Wr \sin(\phi - \theta) \rightarrow \sin \theta = \frac{\sin(\phi - \theta)}{\cos \phi}$ <p>We know that $\theta + \phi = 90^\circ \rightarrow \cos \phi = \sin \theta$</p> <p>Also, $\phi - \theta = 90^\circ - \theta - \theta = 90^\circ - 2\theta$</p> $\sin \theta = \frac{\sin(90^\circ - 2\theta)}{\sin \theta} \rightarrow \sin^2 \theta = \cos 2\theta$ $\sin^2 \theta = 1 - 2\sin^2 \theta$ $3\sin^2 \theta = 1 \rightarrow \sin^2 \theta = \frac{1}{3} \rightarrow \theta = 35.3^\circ$
		<p>2.</p> $\theta + \phi = 90^\circ \rightarrow \phi = 90^\circ - 35.3^\circ = 54.7^\circ$ $\frac{PR}{2} = Wr \cos \phi$ $P = \frac{2Wr \cos \phi}{R} = \frac{2WR \sin \theta \cos \phi}{R} = 2W \sin \theta \cos \phi$ $P = 2W \sin(35.3^\circ) \cos(54.7^\circ) = 0.668W$
		<p>3.</p> $r = R \sin \theta = R \sin(35.3^\circ) = 0.578R$