Section A: Pure Mathematics [40 marks]

1 (a) The complex number z satisfies $|z-3-4i|<5$.

(i) On an Argand diagram, sketch and shade the region in which the point representing z can lie. [3]

(ii) Find the range of possible values of $|z+1|$. [2]

(b) Using a single Argand diagram, sketch the loci given by

(i) $|z-2i|=1$,

(ii) $\arg(z-i)=\frac{1}{4}\pi$. [3]

Hence, find the value of z, in cartesian coordinate form $x+iy$, that satisfy both (i) and (ii). [1]

2 The function f is defined by

$$f: x \mapsto e^{2x}-1, \quad x \in \mathbb{R}.$$

(i) Show that f^{-1} exists and state its range. [2]

(ii) Find f^{-1} and write down its domain. [3]

(iii) Sketch the graph of f^{-1}, giving the equations of any asymptotes and the exact coordinates of any points where the curves cross the axes. [3]

(iv) Hence, find the value(s) of x that satisfies the equation

$$e^{2x}=x+1.$$ [3]

3 The curve C has parametric equations

$$x = t^2 + 1 \ , \quad y = \frac{1}{t-2} \ , \ t \neq 2 \ .$$

(i) Find $\frac{dy}{dx}$ in terms of t. [3]

(ii) Find the equation of the normal to curve C at point P (2, -1). [4]

(iii) Find the values of t for the other points of intersection between the curve C and the normal found in part (ii). [3]

4 (a) Prove that $\sum_{r=1}^{n} r(r-4) = \frac{1}{6} n(n+1)(2n-11)$. [5]

(b) (i) Prove by the method of differences that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}$. [3]

(ii) Explain why $\sum_{r=1}^{\infty} \frac{2}{r(r+1)}$ is a convergent series, and find the value of the sum to infinity. [2]

Section B: Statistics [60 marks]

5 (a) An interior designer is designing a tile pattern of 3 blue, 3 red, 3 green, 1 yellow and 1 purple tiles in a line. All the tiles are identical except for the colour. Find the number of different possible ways to arrange the tiles if

(i) the green, yellow and purple tiles must be placed as a single block, [3]

(ii) no blue tiles are placed next to each other, [3]

(iii) a red tile to be placed at the beginning and at the end of the line. [3]

(b) A group of 10 people consists of 9 men and 1 woman. Find the number of ways which the group can be seated at a round table with identical chairs if

(i) there is no restriction, [1]

(iii) 2 particular men, Caleb and James, do not want to sit beside the woman, but will like to sit together. [3]

The chairs at the table are replaced with 10 chairs of different colours.

(iii) Find the number of ways which the group can be seated at a round table if Caleb and James must still sit together, but they need not be separated from the woman. [3]

6 There are 4 white and 5 green balls placed in a bag and the player picks 2 balls from the bag without replacement. The player gets to roll a fair die once if both balls are of the same colour, and twice if both balls are of different colours. The player wins a DVD every time he rolls a 1 on the die. Find the probability that

(i) the player wins exactly one DVD; [2]

(ii) the 2 balls picked are of different colours given that he did not win any DVD. [2]

John picks a white and a green ball from the bag. Find the

(iii) probability that he wins two DVDs; [1]

(iv) expected number of DVDs he could win. [2]

7 An experiment was conducted to measure how the mass of a substance, x (in grams), varies with time t (in minutes) in a particular chemical reaction. The results are summarised in the following table.

t (minutes)	10	15	20	35	50	65	80
x (grams)	1.75	1.58	1.40	1.10	0.97	0.89	0.80

(i) Calculate, correct to 4 decimal places, the value of the product moment correlation coefficient betweeen x and t. [1]

(ii) Give a sketch of the scatter diagram for the data and hence comment on the suitability of a linear model. [2]

(iii) State, with a reason, which of the following models is appropriate for the data collected, where a and b are constants, and $b > 0$.

A: $x = a + \frac{b}{t}$

B: $x = a + b\ln t$ [2]

Using the appropriate model selected in part (iii),

(iv) calculate the values of a and b using the least squares method and hence obtain an estimate of the mass of the chemical at the 45^{th} minute. Comment on the reliability of your estimate. [4]

8 (a) A sales executive sells on average 10 packets of macademia nuts in a week. Sales take place independently at random times. The sales executive will be paid a bonus if he sells more than 11 packets for the week.Taking a year to consist of 52 weeks, find

(i) the least value of k such that the probability that the sales executive is paid a bonus for more than k weeks in the year is less than 0.08. [4]

(ii) the probability that the sales executive is paid a bonus for the 4^{th} time on the 11^{th} week of the year. [2]

(b) A random variable X has a binomial distribution with mean 3 and variance 2.85. Another independent random variable Y has a binomial distribution such that $Y \sim B(80, 0.02)$.

(i) State the approximate distribution of $X + Y$, showing your reason(s). [3]

(ii) Hence find $P(X + Y = 3)$. [1]

9 The time spent by youths at an ice skating rink per month can be assumed to be normally distributed. The number of hours spent at an ice skating rink by youths in Singapore and the United States of America are independent normal distributions with means 10.1 and 9.3 and standard deviations 3.2 and 2.3 respectively.

(i) Find the probability that a randomly chosen Singaporean youth spends less than 5 hours per month at an ice skating rink. [2]

(ii) Find the probability that the total time spent at an ice skating rink by two randomly chosen Singaporean youths exceeds twice the time spent by a randomly chosen American youth in a month. [3]

It is known that an hour at an ice skating rink cost $7. Find the probability that a randomly chosen Singaporean youth will spend more than $120 in a month. [3]

10 In order to conduct a survey on the Earth Week programmes for the Year 3 students, 60 students need to be selected. There are a total of ten Year 3 classes, 3 each from Arts Faculty and Business Faculty and 4 from Science Faculty. The Arts and Business Faculty have 20 students in each class and the Science faculty have 30 students in each class.

(i) 6 students are chosen randomly from each class. State, with a reason, whether this gives a random sample of 60 students from the Year 3 classes. [2]

(ii) Briefly describe how a systematic sample of 60 students could be selected from the Year 3 classes. [2]

11 A drink vending machine is set to dispense 100 ml of hot chocolate into cups. A random sample of sixteen cups were taken, and the cups were found to contain the following volume of hot chocolate, in ml.

102	106	109	98	95	100	101	104
104	100	99	99	103	106	96	104

(i) Carry out an appropriate test, using a 5% level of significance, to test whether the machine is dispensing too much hot chocolate. State any assumptions needed to carry out the test. [4]

(ii) Determine the set of values of α such that it can be concluded that the machine is dispensing too much hot chocolate at α% level of significance. [2]