




Name: _____ Class: _____ TG: _____ Date: _____

Chapter 3: Probability of Combined Events

Success Criteria




	Success Criteria	Indicate confidence level for each success criteria.   
	3.1 Probability of single event	
1.	I can list the sample space of the possible outcomes of a probability experiment.	
2.	I can find the probability of a single event.	
3.	I can understand the probability of an impossible event is 0.	
4.	I can understand the probability of certain event is 1.	
5.	I can apply the complement of probability to solve word problem.	
	3.2 Probability of combined events	
6.	I can draw a possibility diagram to represent the sample space of a 2-stage probability experiment.	
7.	I can calculate the probability of combined events using possibility diagrams.	
8.	I can draw a tree diagram to represent the sample space of a multi-stage probability experiment.	
9.	I can calculate the probability of combined events using tree diagrams.	
10.	I can solve the probability problems using either the possibility diagram or tree diagram.	
	3.3 Addition Law of Probability and mutually exclusive events	
11.	I can identify mutually exclusive events.	
12.	I can state the Addition Law of Probability.	
13.	I can solve probability problems involving mutually exclusive events using the Addition Law of Probability.	
	3.4 Multiplication Law of Probability and independent events	
14.	I can identify independent events.	
15.	I can identify dependent events.	

16.	I can state the Multiplication Law of Probability.	
17.	I can solve probability problems involving independent events using the Multiplication Law of Probability.	

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Chapter 3: Probability of Combined Events

Success Criteria




	Success Criteria	Indicate confidence level for each success criteria.   
	3.1 Probability of single event	
18.	I can list the sample space of the possible outcomes of a probability experiment.	
19.	I can find the probability of a single event.	
20.	I can understand the probability of an impossible event is 0.	
21.	I can understand the probability of certain event is 1.	
22.	I can apply the complement of probability to solve word problem.	
	3.2 Probability of combined events	
23.	I can draw a possibility diagram to represent the sample space of a 2-stage probability experiment.	
24.	I can calculate the probability of combined events using possibility diagrams.	
25.	I can draw a tree diagram to represent the sample space of a multi-stage probability experiment.	
26.	I can calculate the probability of combined events using tree diagrams.	
27.	I can solve the probability problems using either the possibility diagram or tree diagram.	
	3.3 Addition Law of Probability and mutually exclusive events	
28.	I can identify mutually exclusive events.	
29.	I can state the Addition Law of Probability.	
30.	I can solve probability problems involving mutually exclusive events using the Addition Law of Probability.	
	3.4 Multiplication Law of Probability and independent events	
31.	I can identify independent events.	
32.	I can identify dependent events.	

33.	I can state the Multiplication Law of Probability.	
34.	I can solve probability problems involving independent events using the Multiplication Law of Probability.	

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Chapter 3: Probability of Combined Events

Success Criteria




	Success Criteria	Indicate confidence level for each success criteria.   
	3.1 Probability of single event	
35.	I can list the sample space of the possible outcomes of a probability experiment.	
36.	I can find the probability of a single event.	
37.	I can understand the probability of an impossible event is 0.	
38.	I can understand the probability of certain event is 1.	
39.	I can apply the complement of probability to solve word problem.	
	3.2 Probability of combined events	
40.	I can draw a possibility diagram to represent the sample space of a 2-stage probability experiment.	
41.	I can calculate the probability of combined events using possibility diagrams.	
42.	I can draw a tree diagram to represent the sample space of a multi-stage probability experiment.	
43.	I can calculate the probability of combined events using tree diagrams.	
44.	I can solve the probability problems using either the possibility diagram or tree diagram.	
	3.3 Addition Law of Probability and mutually exclusive events	
45.	I can identify mutually exclusive events.	
46.	I can state the Addition Law of Probability.	
47.	I can solve probability problems involving mutually exclusive events using the Addition Law of Probability.	
	3.4 Multiplication Law of Probability and independent events	
48.	I can identify independent events.	
49.	I can identify dependent events.	

50.	I can state the Multiplication Law of Probability.	
51.	I can solve probability problems involving independent events using the Multiplication Law of Probability.	

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Chapter 3: Probability of Combined Events

Success Criteria

	Success Criteria	Indicate confidence level for each success criteria.   
	3.1 Probability of single event	
52.	I can list the sample space of the possible outcomes of a probability experiment.	
53.	I can find the probability of a single event.	
54.	I can understand the probability of an impossible event is 0.	
55.	I can understand the probability of certain event is 1.	
56.	I can apply the complement of probability to solve word problem.	
	3.2 Probability of combined events	
57.	I can draw a possibility diagram to represent the sample space of a 2-stage probability experiment.	
58.	I can calculate the probability of combined events using possibility diagrams.	
59.	I can draw a tree diagram to represent the sample space of a multi-stage probability experiment.	
60.	I can calculate the probability of combined events using tree diagrams.	
61.	I can solve the probability problems using either the possibility diagram or tree diagram.	
	3.3 Addition Law of Probability and mutually exclusive events	
62.	I can identify mutually exclusive events.	
63.	I can state the Addition Law of Probability.	
64.	I can solve probability problems involving mutually exclusive events using the Addition Law of Probability.	
	3.4 Multiplication Law of Probability and independent events	
65.	I can identify independent events.	
66.	I can identify dependent events.	

67.	I can state the Multiplication Law of Probability.	
68.	I can solve probability problems involving independent events using the Multiplication Law of Probability.	

1. 360 people go on a school trip to one of the four places.
The table shows the number of people at each place.

	adventure park	botanic gardens	wildlife center	red castle	Total
Boys	65	12	b	36	a
Girls	64	9	62	28	163
Staff	15	3	c	7	37
Total	144	24	121	71	360

- (a) Find the value of a , b and c .
(b) One person is picked at random.
Find the probability that the person is a girl who visits the wild life centre.

[2023, Crest Sec, Prelims, P2, Q3a & b]

2. The table shows results from a survey of the shoe sizes of a class of 40 students.

Shoe Size	4	5	6	7	8	9	10
Frequency	2	15	6	p	q	3	1

The probability of a student's shoe size being larger than 7 is $\frac{3}{20}$.

Find the value of q .

[2023, Bukit View Sec, Prelims, P2, Q3a]

3. The stem-and-leaf diagram shows the pH value of 16 different types of detergent.

8	9
9	
10	1 2 2 7
11	0 3 4 4 5 9
12	6 6 6 6 8

Key: 10 | 1 = pH 10.1

In a scientific investigation, pH indicator was added to the different detergent.
The colour change of the detergent when pH indicator was added is shown below.

pH range of detergent	Colour change when pH indicator is added
≤ 11	turns from colourless to blue
> 11	turns from colourless to purple

One of the detergents was selected to conduct further testing.

Find the probability that

- (i) the detergent selected turns from colourless to blue,
(ii) the detergent selected is pH 11.4.

[2023, Greenridge Sec, Prelims, P2, Q3b]

4. A bag contains some red marbles, some yellow marbles and some blue marbles.
The probability of choosing a red marble at random is 0.48.
The probability of choosing a yellow marble at random is 0.36.

- (a) Find the probability of choosing a blue marble at random.
(b) In the bag, there are 9 more red marbles than yellow marbles.
Find the total number of marbles in the bag.

[2023, Hillgrove Sec, Prelims, P1, Q13]

5. A round spinner has five equal sectors.
Each sector has a different number from 1 to 5.
The spinner is spun twice and the product of the resulting numbers is recorded on a possibility diagram.

product	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6		
3	3	6	9		
4	4				
5	5				

- (a) Complete the table to show all the possible outcomes.
(b) Find the probability
(i) the product of the two numbers is a prime number.
(ii) the product of the two numbers is a multiple of 6.

[2023, Bendemeer Sec, Prelims, P1, Q12]

6. A bag contains five counters, numbered 1, 2, 3, 5 and 7.
Two counters are taken from the bag at random, one after another, **without replacement**.
The product of the two numbers is recorded and some of the possible products are shown in the table below.

×	1	2	3	5	7
1			3	5	7
2	2			10	14
3	3	6			
5	5		15		35
7	7	14		35	

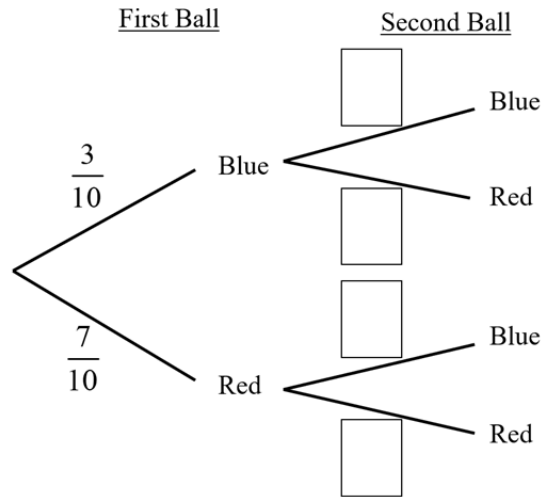
- (a) Complete the table to show all the possible outcomes.
(b) Find, **as a fraction in its lowest term**, the probability that, the product of the two numbers is
(i) an odd number,
(ii) a prime number,
(iii) a number more than 14.

[2023, Regent Sec, Prelims, P1, Q18]

7. Two six-sided dice, one coloured red and the other blue, are thrown together.
(a) Draw a possibility diagram to represent all the possible outcomes from the throw.
(b) Hence or otherwise, find the probability that
(i) the total score on the two dice is 13,
(ii) the total score on the two dice is 6,
(iii) the score on the red die is twice the score on the blue die,
(iv) the numbers on both dice are the same.

[2019, West Spring Sec, Prelims, P2, Q7]

8. A bag contains 3 blue balls and 7 red balls.
Two balls are picked from the bag without replacement.
(i) The probability tree diagram is drawn below.
Fill in the blanks.



- (ii) Find the probability that the balls are of different colours.

[2023, Ahmad Ibrahim Sec, Prelims, P2, Q11b]

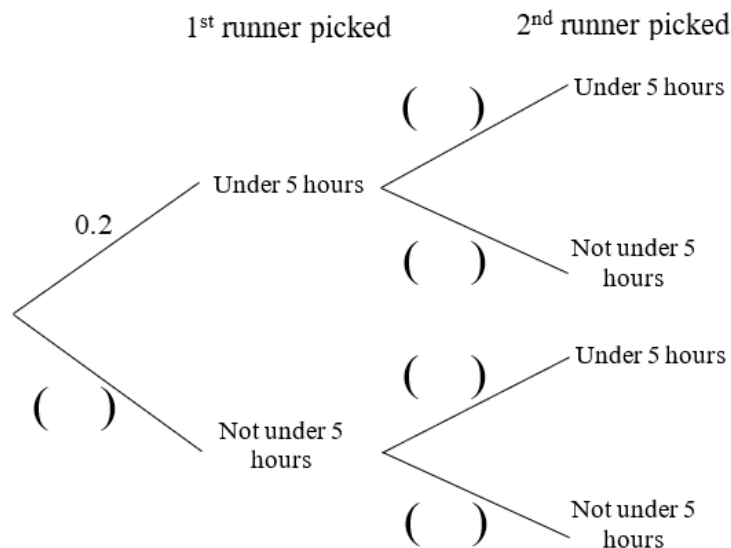
9. Lisa has a bag containing 5 green cards, 4 red cards and 3 blue cards. She takes a card at random from the bag without putting it back. She then takes a second card at random from the bag. Calculate the probability that the two cards are

- (i) the same colour,
(ii) different colours.

[2019, St. Anthony's Canossian Sec, Prelims, P2, Q11a]

10. The probability that any given runner takes under 5 hours to complete the marathon is 0.2.

- (i) Copy and complete the tree diagram showing the probabilities that two runners picked at random completes the marathon under 5 hours or not.

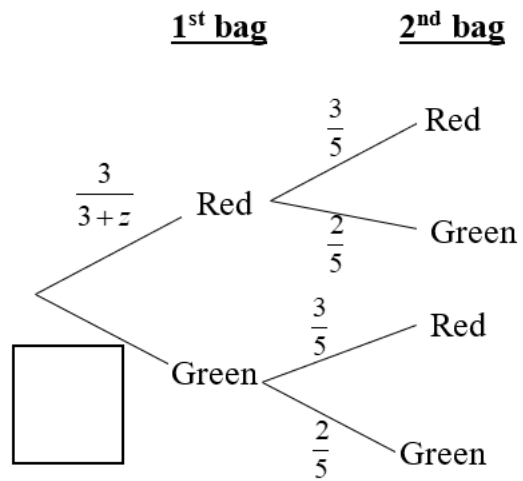


- (ii) Calculate the probability that at least one of the two runners picked complete the marathon under 5 hours.

[2018, CHIJ Katong Convent, Prelims, P2, Q12b]

11. In a bag, there are 3 red socks and z green ones. In another bag containing 10 socks, there are 6 red socks and 4 green socks. The socks are all placed at random and a sock is taken out from **each** bag at random.

- (i) Complete the tree diagram.



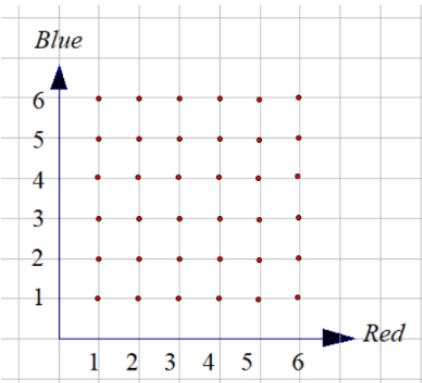
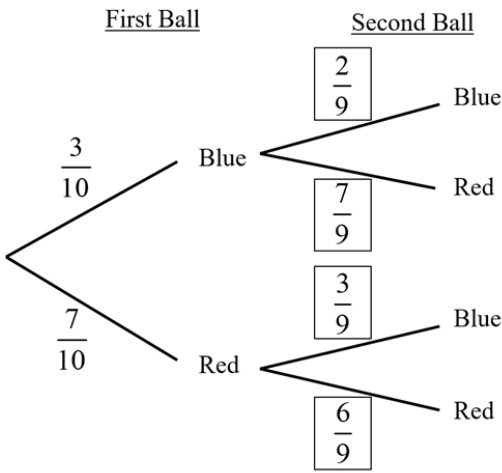
- (ii) The probability that both socks are green is 0.25.

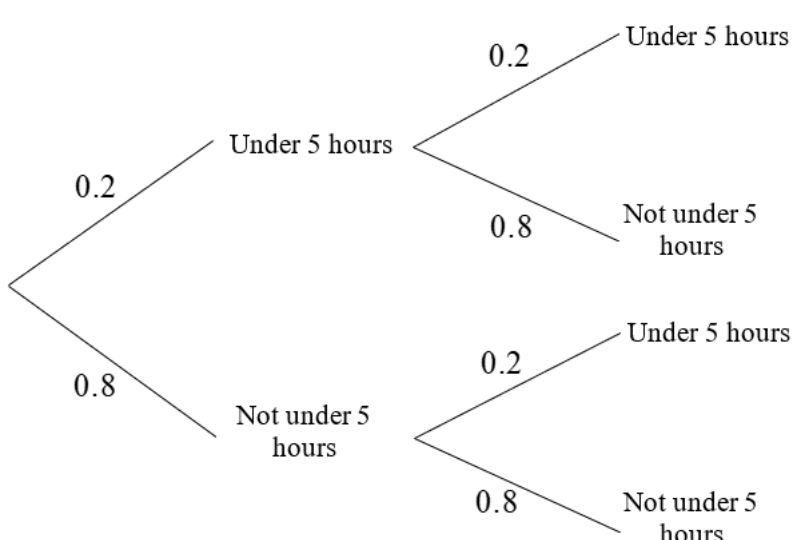
Write down an equation in z and solve it to find the value of z .

[2023, Regent Sec, Prelims, P2, Q12b]

Answer Key:

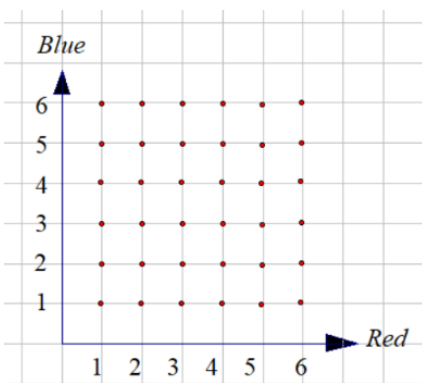
1.	(a)	$a = 160$ $b = 47$ $c = 12$																																								
	(b)	$P(\text{girl visit wild life centre}) = \frac{31}{180}$																																								
2.	$q = 2$																																									
3.	(i)	$P(\text{colourless to blue}) = \frac{3}{8}$																																								
	(ii)	$P(\text{pH } 11.4) = \frac{1}{8}$																																								
4.	(a)	$P(\text{blue}) = 0.16$																																								
	(b)	total no. of marbles = 75																																								
5.	(a)	<table><tr><td>product</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>1</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>2</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>3</td><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td></tr><tr><td>4</td><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td></tr><tr><td>5</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td></tr></table>					product	1	2	3	4	5	1	1	2	3	4	5	2	2	4	6	8	10	3	3	6	9	12	15	4	4	8	12	16	20	5	5	10	15	20	25
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	(b)	(i)	$P(\text{prime number}) = \frac{6}{25}$																																							
		(ii)	$P(\text{product is multiple of 6}) = \frac{4}{25}$																																							
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	(b)	(i)	$P(\text{odd no.}) = \frac{3}{5}$																																							
		(ii)	$P(\text{prime no.}) = \frac{2}{5}$																																							
		(iii)	$P(\text{no. more than 14}) = \frac{3}{10}$																																							

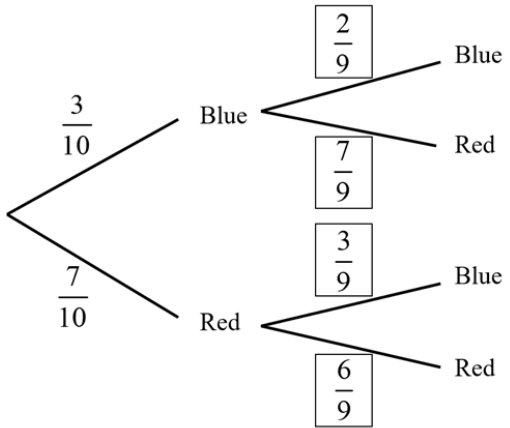
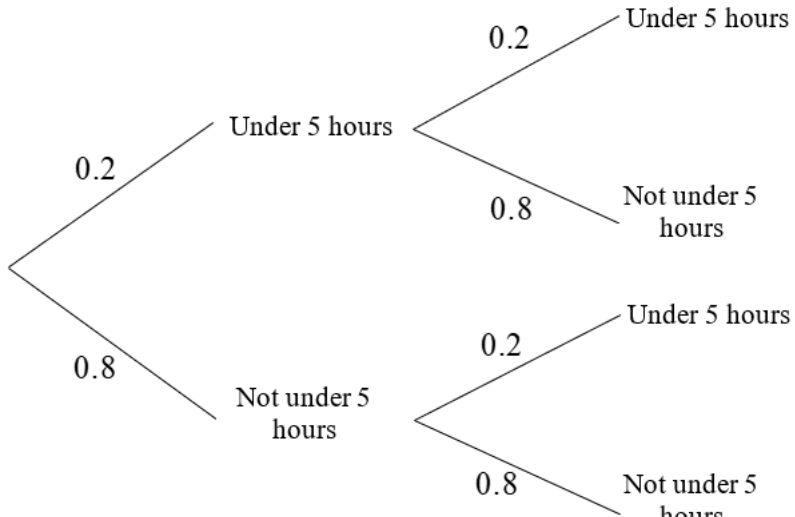
7.	(a)		
	(b)	(i)	$P(\text{total} = 13) = 0$
		(ii)	$P(\text{total} = 6) = \frac{5}{36}$
		(iii)	$P(\text{score on red is twice on blue}) = \frac{1}{12}$
		(iv)	$P(\text{same no. on both dice}) = \frac{1}{6}$
8.	(i)		
	(ii)	$P(\text{different colours}) = \frac{7}{15}$	
9.	(i)	$P(\text{same colour}) = \frac{19}{66}$	
	(ii)	$P(\text{different colour}) = \frac{47}{66}$	

10.	(i)	<div style="display: flex; justify-content: space-around; margin-bottom: 10px;"> 1st runner picked 2nd runner picked </div> 
	(ii)	$P(\text{at least one under 5 hrs}) = 0.36$
11.	(i)	$\frac{z}{z+3}$
	(ii)	$z = 5$

Worked Solutions:

1.	(a)	$a = 360 - 37 - 163$ $= 160$ $b = 160 - 65 - 12 - 36$ $= 47$ $c = 121 - 47 - 62$ $= 12$																																							
	(b)	$P(\text{girl visit wild life centre}) = \frac{62}{360}$ $= \frac{31}{180}$																																							
2.		$\frac{q+3+1}{40} = \frac{3}{20}$ $q+4 = 6$ $q = 2$																																							
3.	(i)	$P(\text{colourless to blue}) = \frac{6}{16}$ $= \frac{3}{8}$																																							
	(ii)	$P(\text{pH } 11.4) = \frac{2}{16}$ $= \frac{1}{8}$																																							
4.	(a)	$P(\text{blue}) = 1 - 0.48 - 0.36$ $= 0.16$																																							
	(b)	$P(\text{difference bet. red and yellow}) = 0.48 - 0.36$ $= 0.12$ $0.12 \text{ units} = 9$ $1 \text{ unit} = \frac{9}{0.12}$ $\text{total no. of marbles} = 75$																																							
5.	(a)	<table border="1"> <thead> <tr> <th>product</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr> </thead> <tbody> <tr> <td>1</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr> <td>2</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr> <tr> <td>3</td><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td></tr> <tr> <td>4</td><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td></tr> <tr> <td>5</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td></tr> </tbody> </table>				product	1	2	3	4	5	1	1	2	3	4	5	2	2	4	6	8	10	3	3	6	9	12	15	4	4	8	12	16	20	5	5	10	15	20	25
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7.	(a)																																									
	(b)	(i)	$P(\text{total} = 13) = 0$																																							
		(ii)	$(1,5), (2,4), (3,3), (4,2), (5,1)$ $P(\text{total} = 6) = \frac{5}{36}$																																							
		(iii)	$(2,1), (4,2), (6,3)$ $P(\text{score on red is twice on blue}) = \frac{3}{36}$ $= \frac{1}{12}$																																							
		(iv)	$P(\text{same no. on both dice}) = \frac{6}{36}$ $= \frac{1}{6}$																																							

8.	(i)	<div style="text-align: center;"> <p><u>First Ball</u> <u>Second Ball</u></p>  </div>
	(ii)	$P(\text{different colours}) = P(\text{blue,red}) + P(\text{red,blue})$ $= \left(\frac{3}{10} \times \frac{7}{9}\right) + \left(\frac{7}{10} \times \frac{3}{9}\right)$ $= \frac{7}{15}$
9.	(i)	$P(\text{same colour}) = P(\text{green}) + P(\text{red}) + P(\text{blue})$ $= \left(\frac{5}{12} \times \frac{4}{11}\right) + \left(\frac{5}{12} \times \frac{3}{11}\right) + \left(\frac{3}{12} \times \frac{2}{11}\right)$ $= \frac{19}{66}$
	(ii)	$P(\text{different colour}) = 1 - P(\text{same})$ $= 1 - \frac{19}{66}$ $= \frac{47}{66}$
10.	(i)	<div style="text-align: center;"> <p>1st runner picked 2nd runner picked</p>  </div>
	(ii)	$P(\text{at least one under 5 hrs}) = 1 - P(\text{none under 5 hrs})$ $= 1 - (0.8 \times 0.8)$ $= 0.36$

11.	(i)	$\frac{z}{z+3}$
	(ii)	$P(\text{both green}) = \left(\frac{z}{z+3}\right)\left(\frac{2}{5}\right)$ $0.25 = \frac{2z}{5z+15}$ $8z = 5z+15$ $z = 5$

