2022 PRELIM 4E5N EM P2 Marking Scheme

Q	Solution
1	(a) $\mathbf{A} = \begin{pmatrix} 300 & 300 & 250 \\ 450 & 250 & 200 \end{pmatrix}$ ---B1 (b) $\mathbf{P} = \begin{pmatrix} 300 & 300 & 250 \\ 450 & 250 & 200 \end{pmatrix}\begin{pmatrix} 0.16 \\ 0.48 \\ 0.18 \end{pmatrix}$ $= \begin{pmatrix} 300\times0.16+300\times0.48+250\times0.18 \\ 450\times0.16+250\times0.48+200\times0.18 \end{pmatrix}$ $= \begin{pmatrix} 237 \\ 228 \end{pmatrix}$ ---B1, B1 for each correct element (c) Each element in P represent the <u>total</u> cost of making a sponge cake and butter cake respectively. ---B1 (d) $a = 1$ ---B1 $b = 1$ ---B1
2	(a) No. of students = 148 – 92 = 56 ---M1 $\therefore$ Probability $= \frac{56}{160} = \frac{7}{20}$ o.e ---A1 (b) 50% of 160 = 80 students From the graph, $x =$ 55 ---B1 (c) 90% of the students scored 70 marks and lesser. Or 10% of the students scored more than 70 marks. ---B1 (d) 25% of 160 = 40 students Q_1= 48 marks 75% of 160 = 120 students Q_3= 66 marks –M1 for obtaining Q_1 and Q_3 IQR= 66-48 = 18 marks ---A1 (e) I do not agree. [B1] For the students who have the highest marks, they can be the outliers (extreme values). OR To compare the performance, median marks of each exam should be used instead. The median mark for the Science exams is higher than that of the Maths exams so the students performed better in the Science exams. [B1]
3	(a) $y = \left(\frac{1}{2}x\right)(x+1)(x)+\frac{1}{3}\left(\frac{1}{2}x\right)(x+1)(3)$ ---M1 $= \left(\frac{1}{2}x^2\right)(x+1)+\left(\frac{1}{2}x^2\right)+\frac{1}{2}x$

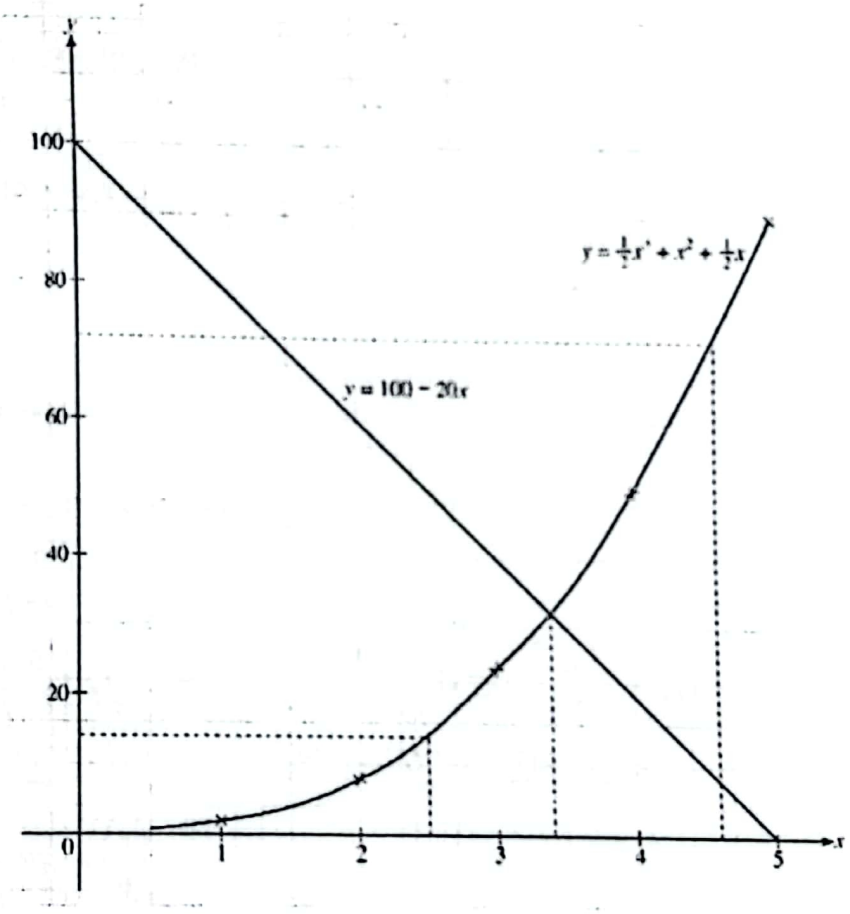
$$= \frac{1}{2}x^3 + \frac{1}{2}x^2 + \frac{1}{2}x^2 + \frac{1}{2}x$$

$$= \frac{1}{2}x^3 + x^2 + \frac{1}{2}x \text{ ---A1}$$

(b) Answer $= \frac{1}{2}(3)^3 + (3)^2 + \frac{1}{2}(3)$

$= 24$ ---B1

(c) Plot all 5 points correctly ---P2 (Award P1 for plotting 3 or 4 points correctly)
Draw smooth curve through all---P1



(d) When height = 5.5 cm, $x = 5.5 - 3$

$= 2.5$

From the graph, when $x = 2.5$, $y = 14$

$\therefore$ volume of solid = 14 cm^3 (±1)---B1

(e) Volume of another cuboid $= 5(4)(5 - x)$

$= (100 - 20x)$ cm^3---M1

M1- drawing line $y = 100 - 20x$

From the graph, the required value of $x = 3.4$ (±0.1)---A1

4

(a) $\frac{-5}{w+2}=5$

-5 = 5(w + 2) ---M1

-5 = 5w + 10

5w = -15

w = -3 ---A1

(b)

$\frac{3x-5}{4} \leq \frac{2x+5}{6}$

$6(3x-5) \leq 4(2x+5)$ ---M1 for multiplying by 24 on both sides

$18x-30 \leq 8x+20$

$18x-8x \leq 20+30$

$10x \leq 50$

$x \leq 5$ ---A1

(ci)

$a=\frac{7(6)+6(-1)}{10-6}$

$a=\frac{36}{4}$

$a = 9$ ---B1

(cii)

$a=\frac{7b+6c}{10-b}$

$a(10-b)=7b+6c$

$10a-ab=7b+6c$

$-ab-7b=6c-10a$ or $10a-6c=7b+ab$ ---M1 for putting terms with b on one side

$b(-a-7)=6c-10a$ or $10a-6c=b(7+a)$

$b=\frac{6c-10a}{-a-7}$ or $b=\frac{10a-6c}{7+a}$ ---A1

5

(ai) $\overrightarrow{AC} = \overrightarrow{AO}+\overrightarrow{OC}$

$=\begin{pmatrix}-1\\-7\end{pmatrix}+\begin{pmatrix}-2\\9\end{pmatrix}$

$=\begin{pmatrix}-3\\2\end{pmatrix}$ ---B1

(ii) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{OB} = \begin{pmatrix}4\\3\end{pmatrix} - \begin{pmatrix}-1\\-7\end{pmatrix}$

$= \begin{pmatrix}5\\10\end{pmatrix}$

Answer = B (5, 10) ---B1

(iii) $\overrightarrow{DC} = \overrightarrow{AB} = \begin{pmatrix}4\\3\end{pmatrix}$

$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD}$ or $\overrightarrow{DC} = \overrightarrow{DO} + \overrightarrow{OC}$

$\overrightarrow{OD} = \begin{pmatrix}-2\\9\end{pmatrix} - \begin{pmatrix}4\\3\end{pmatrix}$ ---M1 $\begin{pmatrix}4\\3\end{pmatrix} = \overrightarrow{DO} + \begin{pmatrix}-2\\9\end{pmatrix}$ ---M1

$= \begin{pmatrix}-6\\6\end{pmatrix}$

Answer= D (–6, 6) ---A1

(bi) $\overrightarrow{OM} = \mathbf{a} + \mathbf{b}$ ---B1

(ii)

$\overrightarrow{RS} = 4\mathbf{b} - 3\mathbf{a}$

$\overrightarrow{MR} = 2\mathbf{a} - \mathbf{b}$

$\overrightarrow{RN} = \frac{3}{7}(-3\mathbf{a} + 4\mathbf{b})$ ---M1

$\overrightarrow{MN} = \overrightarrow{MR} + \overrightarrow{RN}$

$= 2\mathbf{a} - \mathbf{b} + \frac{3}{7}(-3\mathbf{a} + 4\mathbf{b})$ ---M1

$= \frac{5}{7}(\mathbf{a} + \mathbf{b})$ ---A1

(iii) $\overrightarrow{MN} = \frac{5}{7}(\mathbf{a} + \mathbf{b})$

$\overrightarrow{OM} = \mathbf{a} + \mathbf{b}$

$\overrightarrow{MN} = \frac{5}{7}\overrightarrow{OM}$ ---M1 (can be seen or implied)

$OM : ON = 7 : 12$---A1

	(iv) $\frac{\text{area of } \Delta PRM}{\text{area of } \Delta ORM} = \frac{1}{3}$ ---B1
6	(a) $\angle EFG + \angle EGF + \angle FEG = 180°$ ($\angle$ sum of Δ) $\angle EFG + 81° + 17° = 180°$ $\angle EFG = 82°$ ---B1 (b) $\angle CAE = \angle EGC$ ($\angle$s in the same segment) $= 63°$ ---B1 (c) $\angle CEF + \angle CGF = 180°$ (opp $\angle$s of cyclic quad) $\angle CEF + \angle EGC + \angle EGF = 180°$ $\angle CEF + 63° + 81° = 180°$ $\angle CEF = 36°$ $\angle CEG = \angle CEF - \angle FEG$ $= 36° - 17°$ $= 19°$ ---B1 (d) $\angle COE = 2 \times \angle EGC$ ($\angle$ at centre = 2$\angle$ at circumference) $= 2 \times 63°$ $= 126°$ ---B1 (e) $OC = OE$ (radii of the same circle) $\therefore \angle OCE = \angle OEC$ (base $\angle$s of isos. Δ) $\angle OCE + \angle OEC + \angle COE = 180°$ ($\angle$ sum of Δ) $\angle OCE + \angle OCE + 126° = 180°$ $2\angle OCE = 54°$ $\angle OCE = 27°$ ---M1 $\angle OCB = 90°$ (tangent $\perp$ radius) $\angle BCE = \angle OCB - \angle OCE$ $= 90° - 27°$ $= 63°$ ---A1 OR $\angle AEC = 90°$ (right $\angle$ in semicircle) $\angle CAE = 63°$ (from b) $\angle ACE = 90° - 63°$ $= 27°$ ---M1 $\angle BCE = 90° - 27°$ (tangent $\perp$ radius) $= 63°$ ---A1
7	(ai) $AB = (x+6)$ m or $(6+x)$---B1 (aii) $BC = (\frac{180}{x} + 6)$ m or $(\frac{180+6x}{x})$m ---B1

(b) $(x+6)(\frac{180}{x}+6)-180=198$ ---M1 for forming eqn

$(x+6)(\frac{180}{x}+6)=378$

$180+6x+\frac{1080}{x}+36=378$ ---M1 for expansion of terms in the brackets

$6x+\frac{1080}{x}-162=0$

$6x^2-162x+1080=0$

$x^2-27x+180=0$ (Shown) ---A1

(c) $x^2-27x+180=0$

$x=\frac{-(-27)\pm\sqrt{(-27)^2-4(1)(180)}}{2(1)}$ or $(x-15)(x-12)=0$ ---M1

$x=\frac{27\pm\sqrt{9}}{2}$

$\therefore\ x=\frac{27+\sqrt{9}}{2}$ or $x=\frac{27-\sqrt{9}}{2}$

$=15$ ---A1 $=12$ ---A1

(d) Since PQ is the length of the rectangle, the value of x must be longer than its breadth. Hence, 12 must be rejected. [OR] If 12 is taken to be the value of x, it will result in the breadth of the lawn being longer than its length, hence 12 must be rejected. [Accept any other reasonable answers]

(e) $AB=CD=15+6$

$=21$ m

$AD=BC=12+6$

$=18$ m ---M1 for finding the dimensions

Using Pythagoras Theorem,

$BD^2=21^2+18^2$ ---M1

$BD=\sqrt{765}$ or 27.6586

Radius $=\frac{\sqrt{765}}{2}$

$=13.8$ m(3.s.f.)---A1

or $QC=\sqrt{3^2+3^2}=\sqrt{18}=4.2426$ ---M1

$QR=\sqrt{6^2+7.5^2}=\sqrt{92.25}=9.6046$ ---M1

Radius $=\sqrt{18}+\sqrt{92.25}=13.847=13.8$m(3s.f.)---A1

8 (ai) length of arc $AB=16\times1.2$

$=19.2$ cm ---B1

(ii) area of the sector $ACB=\frac{1}{2}\times16\times16\times1.2$

$=153.6$ cm^2---B1

(iii) area of triangle $ABC=\frac{1}{2}\times16\times16\times\sin(1.2\text{ rad})$

$=119.30$ cm^2 ---M1

area of the shaded region = 153.6 – 119.30

= 34.3 cm^2 (3 s.f.) ---A1

(bi) angle DFE

$= \pi - (\pi - 1.2) - [2(0.234)]$

= 1.2 – 2 (0.234)

= 0.732 rad(shown)---B1(must show working on how the answer is obtained)

(ii) perimeter of the sector FDE

= 8 + 8 + (8 × 0.732) ---M1 for calculating arc length

= 21.856 cm or 21.9 cm(3 s.f.) ---A1

(iii)

$$\left.\begin{aligned} \angle ACF &= (\pi - 0.366 - 0.234)\text{rad} \\ &= (\pi - 0.6)\text{rad} \\ &= 2.5416 \text{ rad} \end{aligned}\right\} \text{M1}$$

By sine rule,

$$\frac{AF}{\sin\angle ACF} = \frac{CF}{\sin\angle DAC}$$

$$\frac{AF}{\sin(2.5416 \text{ rad})} = \frac{8}{\sin(0.234 \text{ rad})} \text{---M1}$$

AF = 19.481 cm

AD = 19.481 – 8

= 11.5 cm (3 s.f.) ---A1

Or

$AB = \sqrt{16^2 + 16^2 - 2(16)(16)\cos 1.2} = 18.0685$---M1

$$\frac{AF}{\sin(\frac{\pi - 0.732}{2})} = \frac{CF}{\sin(0.732)} \text{---M1}$$

AF=25.242

AD=25.242-8 =17.2435

=17.2m(3 s.f.)---A1

Or

Let the mid point of AB be M

	$CM = 16\cos 0.6 = 13.205$ $FM = 8 + 16\cos 0.6 = 21.205$---M1 AM= 16 sin 0.6=9.0342 $AF = \sqrt{21.205^2 + 9.0342^2} = 23.049$---M1 AD=23.049-8 =15.049 =15.0m(3 s.f.)---A1
9	(ai) area of the field $= \frac{1}{2} \times 4 \times 8.35 \times \sin 73.3°$ ---M1 $= 15.99563$ $= 16.0\,m^2$(3 s.f.) ---A1 (ii) $QR^2 = 4^2 + 8.35^2 - 2 \times 4 \times 8.35 \times \cos 73.3°$---M1 $QR = \sqrt{66.52681}$ Total length of rope used $= \sqrt{66.52681} + 4 + 8.35$---M1 $= 20.50639688$ $= 20.5$ m (3 s.f.) ----A1 (bi) Area of triangle $= \frac{1}{2} \times PQ \times RM$ $\frac{1}{2} \times 4 \times RM = 15.996$----M1 $RM = 15.996 \div \frac{1}{2} \div 4$ $= 7.997815$ $= 8.00$ m (3 s.f.) ---A1 <u>Alternative method</u> $\sin 73.3° = \frac{RM}{8.35}$----M1 $RM = \sin 73.3° \times 8.35$ $= 7.997815$ $= 8.00$ m (3 s.f.)----A1 (ii) $PM = \frac{RM}{\tan 73.3°}$ $PM = \frac{7.997815}{\tan 73.3°}$---M1 $= 2.3994$ Angle of elevation from P to Q

	$= \tan^{-1}\left(\frac{6.8}{2.3994}\right)$ —M1 $= 70.6°$(1 d.p.) —A1 (iii) She should stand at Point *P* [B1] because distance of *PM* is longer as compared to *MQ*, hence the angle of elevation is smaller. This means lesser effort to view the kite. [B1] [Can either use distance or angle to explain]
10	(a) Volume of cone $= \frac{1}{3}\pi(4)^2(10)$ ---M1 $= 167.552$ $= 168\ cm^3$ (to 3 sf)---A1 (b) slant height $= \sqrt{10^2+4^2} = \sqrt{116} = 10.77$cm ---M1 Area of paper $= \pi(4)(10.77)$ $= 135\ cm^2$---A1 (c) Volume of water $= \left(\frac{3}{5}\right)^3 \times 167.552$ ----M1 or $\frac{r}{4} = \frac{6}{10}$ $= 36.191\ cm^3$ $r = 2.4$ Volume of water $= \frac{1}{3}\pi(2.4)^2(6) = 36.191\ cm^3$ Volume of empty space $= 167.552 - 36.191$ $=131.361$ $= 131.4\ cm^3$ (3 s.f.) ---A1 (d) No. of drops of oil $= \frac{131.361}{\frac{4}{3}\pi(1)^3}$ ----M1 $= 31.36$ He can add 31 drops of oil. ---A1
11	(a) Chargeable income = \$102 000- \$30 400 = \$71 600---M1 Income tax $= \$550 + \frac{7}{100} \times (\$71600 - \$40000)$ = \$2762 ---A1 (b) \$4155 – \$3350 = \$805 11.5% → \$805 100% →\$ 7000 ---M1 Total chargeable income = \$80 000 +\$7 000 = \$87 000---A1

(c)

	Amy	John
Annual income	$105 000	$90 000
Balance after NS relief and Earned Income relief	$103 250	$86 000
GCR	$3000	-
WMCR	$\frac{15}{100}\times 105000$ =$15 750	
Chargeable income before Parent Relief	$84 500 [M1 for finding Amy's chargeable income before parent relief]	$86 000[M1 for finding John's chargeable income before parent relief John before parent relief]
Case 1		
Chargeable income if John claim 2 parent relief ($11 000)	$84 500	$75 000
Tax payable (John claim full relief)	$3350+\frac{11.5}{100}\times 4500$ = $3867.50	$550+\frac{7}{100}\times 35000$ =$3000
Total	=$6867.50	
Case 2		
Chargeable income if Amy claim 2 parent relief ($11 000)	$73 500	$86 000
Tax payable (Amy claim full relief)	$550+\frac{7}{100}\times 33500$ = $2895	$3350+\frac{11.5}{100}\times 6000$ =$4040
Total	=$6935	
Case 3		
Chargeable income for **shared amount. (Amy $4500, John $6500)**	$80 000	$79 500
Tax payable	$3350	$550+\frac{7}{100}\times 39500$ =$3315
Total	=$6665	

M1, M1 (correct tax payable for Amy & John for either case 1 or 2)

M1 (Calculation of tax payable using shared amount Amy $4500. John $6500 for parent relief (Case 3))

**Note: For those who calculated shared amount of Amy $5500, John $5500 will obtain 5 marks out of 6 marks for qn 6(c).*

	Disagree. The lowest amount of tax payable happens if Amy and John share the amount of parent relief such that both of them will get an annual chargeable income of $80000 and below. ---A1