Question 1

No.	Suggested Solution	Remarks for Student
	$\mathrm{i}z+2w=-1 \quad ...(1)$ $(2-\mathrm{i})z+\mathrm{i}w=6 \quad ...(2)$ $(2)\times 2\mathrm{i}$: $2\mathrm{i}(2-\mathrm{i})z-2w=12\mathrm{i} \quad ...(3)$ (1)+(3): $\mathrm{i}z+4\mathrm{i}z+2z=-1+12\mathrm{i}$ $z=\frac{-1+12\mathrm{i}}{2+5\mathrm{i}}$ $z=\frac{(-1+12\mathrm{i})(2-5\mathrm{i})}{29}=\frac{58+29\mathrm{i}}{29}=2+\mathrm{i}$ Sub back into (1): $w=\frac{-1-\mathrm{i}z}{2}=\frac{-1-\mathrm{i}(2+\mathrm{i})}{2}=-\mathrm{i}$	This is a non-calculator question. Detailed working needs to be shown.

Question 2

No.	Suggested Solution	Remarks for Student
(a)	$\mathrm{f}(x)=\tan^{-1}\left(\sqrt{2}+x\right)$ $\mathrm{f}'(x)=\frac{1}{1+\left(\sqrt{2}+x\right)^2}=\left[1+\left(\sqrt{2}+x\right)^2\right]^{-1}$ $\mathrm{f}''(x)=-\left[1+\left(\sqrt{2}+x\right)^2\right]^{-2}\left(2\left(\sqrt{2}+x\right)\right)=\frac{-2\left(\sqrt{2}+x\right)}{\left[1+\left(\sqrt{2}+x\right)^2\right]^2}$	
(b)	$\mathrm{f}(0)=\tan^{-1}\sqrt{2}=0.95532$ $\mathrm{f}'(0)=\frac{1}{1+\left(\sqrt{2}\right)^2}=\frac{1}{3}=0.33333$ $\mathrm{f}''(0)=\frac{-2\sqrt{2}}{[1+2]^2}=\frac{-2\sqrt{2}}{9}=-0.31427$ $\mathrm{f}(x)=0.955+0.333x-\frac{0.31427}{2}x^2+...$ $=0.955+0.333x-0.157x^2+...$	Note that all calculated values are to be in radians – check that you set calculator to the correct mode. Note the degree of accuracy for this question is 3 s.f.

Question 3

No.	Suggested Solution	Remarks for Student
(a)	$x=\frac{1}{2}\left(\mathrm{e}^{3t}+2\mathrm{e}^{-3t}\right)=\frac{1}{2}\mathrm{e}^{3t}+\mathrm{e}^{-3t}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{3}{2}\mathrm{e}^{3t}-3\mathrm{e}^{-3t}$ $y=\frac{1}{2}\left(\mathrm{e}^{3t}-2\mathrm{e}^{-3t}\right)=\frac{1}{2}\mathrm{e}^{3t}-\mathrm{e}^{-3t}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{3}{2}\mathrm{e}^{3t}+3\mathrm{e}^{-3t}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{3}{2}\mathrm{e}^{3t}+3\mathrm{e}^{-3t}}{\frac{3}{2}\mathrm{e}^{3t}-3\mathrm{e}^{-3t}}=\frac{\mathrm{e}^{3t}+2\mathrm{e}^{-3t}}{\mathrm{e}^{3t}-2\mathrm{e}^{-3t}}$ $t=\frac{1}{3}\ln 2,\ \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2+1}{2-1}=3$ Gradient of normal = $-\frac{1}{3}$	$\mathrm{e}^{\ln x}=x$ for $x>0$
(b)	$x+y=\mathrm{e}^{3t}$ $x-y=2\mathrm{e}^{-3t}$ $(x+y)(x-y)=\mathrm{e}^{3t}\left(2\mathrm{e}^{-3t}\right)$ $x^2-y^2=2\Rightarrow\frac{x^2}{\left(\sqrt{2}\right)^2}-\frac{y^2}{\left(\sqrt{2}\right)^2}=1,\ x\geq\sqrt{2}$	Since question says state the restriction for x, we can use GC to draw $x=\frac{1}{2}\left(\mathrm{e}^{3t}+2\mathrm{e}^{-3t}\right)$ to get the restriction on x. (x as Y_1 and t as X in GC)

Question 4

No.	Suggested Solution	Remarks for Student
(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\cos x}{\sin x}\right)$ $= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$ (using quotient rule) $= \frac{-1}{\sin^2 x}$ (since $\sin^2 x + \cos^2 x = 1$) $= -\operatorname{cosec}^2 x$	
(b)	Note that $\sin 2x = 2\sin x\cos x$ and $\tan x = \frac{\sin x}{\cos x}$, therefore $\sin 2x\tan x = (2\sin x\cos x)\frac{\sin x}{\cos x} = 2\sin^2 x$	This is a "show" question
(c)	$\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \operatorname{cosec} 6x \cot 3x \ \mathrm{d}x$ $= \int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \frac{1}{\sin 6x \tan 3x} \ \mathrm{d}x$ $= \int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \frac{1}{2\sin^2 3x} \ \mathrm{d}x$ (from (b)) $= \frac{1}{2}\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \operatorname{cosec}^2 3x \ \mathrm{d}x$ $= -\frac{1}{6}\left[\cot 3x\right]_{\frac{\pi}{18}}^{\frac{\pi}{9}}$ (from (a)) $= -\frac{1}{6}\left[\cot\frac{\pi}{3} - \cot\frac{\pi}{6}\right]$ $= -\frac{1}{6}\left[\frac{1}{\sqrt{3}} - \sqrt{3}\right]$ $= -\frac{1}{6}\left[\frac{1-3}{\sqrt{3}}\right]$ $= \frac{1}{3\sqrt{3}}$ or $\frac{\sqrt{3}}{9}$	Note that you are expected to simplify the answer. You can also use GC to verify whether your answer is correct.

Question 5

No.	Suggested Solution	Remarks for Student
(a)	$(x+8)^2+(mx-14)^2=52$ $x^2+16x+64+m^2x^2-28mx+196=52$ $(m^2+1)x^2+(16-28m)x+208=0...(1)$ line is tangent to curve means (1) has one repeated root, $(16-28m)^2-4(m^2+1)(208)=0$ $256-896m+784m^2-832m^2-832=0$ $-48m^2-896m-576=0$ $3m^2+56m+36=0$	
(b)	since tangents intersects at (0,0), equations of both tangents will be $y=m_1x$ and $y=m_2x$, where m_1 and m_2 satisfy the equation in (a), solving $3m^2+56m+36=0$ $3m^2+56m+36=0$ $m=\frac{-56\pm\sqrt{56^2-4(3)(36)}}{6}$ $=\frac{-56\pm 52}{6}$ $=-18$ or $-\frac{2}{3}$ Sub $m=-18$ into (1) in (a), $325x^2+520x+208=0$ $x=-0.8$ (from GC or equation can be reduced to $(x+0.8)^2=0$) $y=-18(-0.8)=14.4$ Sub $m=-\frac{2}{3}$ into (1) in (a), $\frac{13}{9}x^2+\frac{104}{3}x+208=0$ $x=-12$ (from GC or equation can be reduced to $(x+12)^2=0$) $y=-\frac{2}{3}(-12)=8$ Coordinates are $(-12,8)$ and $(-0.8,14.4)$.	

Question 6

No.	Suggested Solution		Remarks for Student
(a)	$\mathrm{f}(x)=\dfrac{ax+k}{x-a}=\dfrac{a(x-a)+a^2+k}{x-a}=a+\dfrac{a^2+k}{x-a}$ Sequence of transformation: 1. Translate a units in the positive x direction 2. Scale the graph by a factor (a^2+k) parallel to the y-axis 3. Translate a units in the positive y direction		
(b)	$y=\dfrac{ax+k}{x-a}$ $yx-ya=ax+k$ $x(y-a)=ay+k$ $x=\dfrac{ay+k}{y-a}$ $\mathrm{f}^{-1}(x)=\dfrac{ax+k}{x-a}=\mathrm{f}(x)$		
(c)	$\mathrm{f}^2(x)=\mathrm{ff}(x)=\mathrm{f}^{-1}\mathrm{f}(x)=x$		
(d)	$\mathrm{f}^{2023}(1)=\mathrm{f}^{2022}\mathrm{f}(1)=\mathrm{f}(1)=\dfrac{a+k}{1-a}$	From (c), $\mathrm{f}^{2022}(x)=\mathrm{f}^2\left(\mathrm{f}^2\left(\cdots\mathrm{f}^2(x)\right)\right)=x$	

Question 7

No.	Suggested Solution	Remarks for Student
(a)	$y = x^{-3}\ln x = \dfrac{\ln x}{x^3}$ $\dfrac{dy}{dx} = \dfrac{x^2 - 3x^2\ln x}{x^6} = \dfrac{1-3\ln x}{x^4}$ $\dfrac{dy}{dx} = 0 \Rightarrow 1-3\ln x = 0 \Rightarrow x = e^{\frac{1}{3}}$ $y = \left(e^{\frac{1}{3}}\right)^{-3}\ln e^{\frac{1}{3}} = \dfrac{1}{3}e^{-1}$ Coordinates are $\left(e^{\frac{1}{3}}, \dfrac{1}{3}e^{-1}\right)$	
(b)	$\int_1^3 x^{-3}\ln x\ dx$ $= \left[-\dfrac{1}{2}x^{-2}\ln x\right]_1^3 + \dfrac{1}{2}\int_1^3 x^{-3}\ dx$ $= -\dfrac{1}{18}\ln 3 - \dfrac{1}{4}\left[x^{-2}\right]_1^3$ $= -\dfrac{1}{18}\ln 3 - \dfrac{1}{36} + \dfrac{1}{4}$ $= \dfrac{2}{9} - \dfrac{1}{18}\ln 3$	

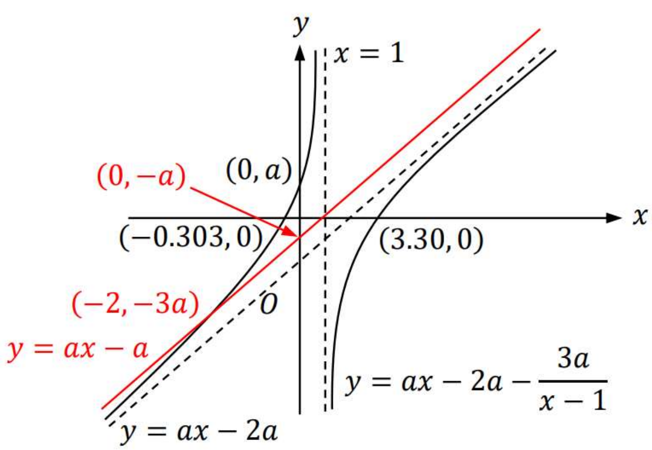
Question 8

No.	Suggested Solution	Remarks for Student
(a)	$\int \frac{2x-1}{x^2+2x+1}\ \mathrm{d}x$ $= \int \frac{2x+2-3}{x^2+2x+1}\ \mathrm{d}x$ $= \int \frac{2x+2}{x^2+2x+1}\ \mathrm{d}x - \int \frac{3}{(x+1)^2}\ \mathrm{d}x,\ x \neq -1$ $= \ln\left\|x^2+2x+1\right\| + \frac{3}{x+1} + c$ $= \ln(x+1)^2 + \frac{3}{x+1} + c$	Idea is to apply $\int \frac{\mathrm{f}'(x)}{\mathrm{f}'(x)}\mathrm{d}x$
(b)	$\int_0^2 \frac{\left\|2x-1\right\|}{x^2+2x+1}\ \mathrm{d}x$ $= -\int_0^{\frac{1}{2}} \frac{2x-1}{x^2+2x+1}\ \mathrm{d}x + \int_{\frac{1}{2}}^2 \frac{2x-1}{x^2+2x+1}\ \mathrm{d}x$ $= -\left[\ln\left(x^2+2x+1\right) + \frac{3}{x+1}\right]_0^{\frac{1}{2}} + \left[\ln\left(x^2+2x+1\right) + \frac{3}{x+1}\right]_{\frac{1}{2}}^2$ $= -\left[\ln\frac{9}{4} + 2 - 3\right] + \left[\ln 9 + 1 - \ln\frac{9}{4} - 2\right]$ $= \ln\frac{16}{9}$ $= 2\ln\frac{4}{3}$	Note: $\left\|2x-1\right\| = \begin{cases} 2x-1, & \frac{1}{2} \leq x \leq 2 \\ -(2x-1), & 0 \leq x < \frac{1}{2} \end{cases}$

Question 9

No.	Suggested Solution	Remarks for Student
(a)	$a+2d=ar$ $a+14d=ar^2$ $\dfrac{a+2d}{a}=\dfrac{a+14d}{a+2d}$ $a^2+4ad+4d^2=a^2+14ad$ $4d^2-10ad=0$ $d=0$ (rejected since $d\neq 0$) or $d=\dfrac{5a}{2}$	
(b)	$S_\infty=\dfrac{\sin\theta}{1-(-\cos\theta)}$ $=\dfrac{\sin\theta}{1+\cos\theta}$ $=\dfrac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$ $=\tan\dfrac{\theta}{2},\ k=\dfrac{1}{2}$	
(c)	$\theta=\dfrac{\pi}{3},\ a=\sin\theta=\dfrac{\sqrt{3}}{2}$ $r=-\cos\theta=-\dfrac{1}{2}$ $S_7=\dfrac{a(1-r^7)}{1-r}$ $=\dfrac{\frac{\sqrt{3}}{2}\left(1-\left(-\frac{1}{2}\right)^7\right)}{1-\left(-\frac{1}{2}\right)}$ $=\dfrac{\sqrt{3}}{2}\left(1+\dfrac{1}{2^7}\right)\times\dfrac{2}{3}$ $=\dfrac{\sqrt{3}}{3}\left(\dfrac{129}{128}\right)$ $=\dfrac{43\sqrt{3}}{128}$	

Question 10

No.	Suggested Solution	Remarks for Student
(a)	$y=ax+b+\dfrac{a+2b}{x-1}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=a-\dfrac{a+2b}{(x-1)^2}$ C has no stationary points, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\ a(x-1)^2-(a+2b)=0$ has no solution $ax^2-2ax-2b=0$ has no solution $\therefore 4a^2-4a(-2b)<0$ $a(a+2b)<0$ Since $a>0$, $a+2b<0\Rightarrow a<-2b$ or $b<-\dfrac{1}{2}a$.	
(b)		Given $b=-2a$, $-2b=4a$. Since $a>0$, $a<-2b$ From (a), C has no stationary point.
(c)	As shown in diagram for (b)	
(d)	$a=1$ From graphs, $x\le -2$ or $x>1$	

Question 11

No.	Suggested Solution	Remarks for Student
(a)	$\lvert\overrightarrow{QP}\rvert = 939 \Rightarrow \left\lvert \begin{pmatrix} 936 \\ 72 \\ p+15 \end{pmatrix} \right\rvert = 939$ $936^2 + 72^2 + (p+15)^2 = 939^2$ $(p+15)^2 = 441$ $p = 6$ (rejected since $p < -15$) or $p = -36$	
(b)	$\begin{pmatrix} -100 \\ 400 \\ 50 \end{pmatrix} \times \begin{pmatrix} -200 \\ 940 \\ 30 \end{pmatrix} = 50\begin{pmatrix} -2 \\ 8 \\ 1 \end{pmatrix} \times 10\begin{pmatrix} -20 \\ 94 \\ 3 \end{pmatrix} = 500\begin{pmatrix} -70 \\ -14 \\ -28 \end{pmatrix} = -7000\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$ $\underset{\sim}{r} \cdot \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 400 \\ 600 \\ -20 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} = 2560$ Cartesian equation of the plane is $5x + y + 2z = 2560$	Question asked for cartesian equation of the plane
(c)	$l_{PQ} : r = \begin{pmatrix} 200 \\ 20 \\ -15 \end{pmatrix} + \lambda \begin{pmatrix} 312 \\ 24 \\ -7 \end{pmatrix}$ Sub into $5x + y + 2z = 2560$ $5(200 + 312\lambda) + (20 + 24\lambda) + 2(-15 - 7\lambda) = 2560$ $\lambda = 1$ Coordinates are $(512, 44, -22)$	Question asked for coordinates
(d)	Let α be angle of PQ made with vertical $\cos\alpha = \dfrac{\begin{pmatrix} 312 \\ 24 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{\left\lvert \begin{pmatrix} 312 \\ 24 \\ -7 \end{pmatrix} \right\rvert \left\lvert \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rvert} = \dfrac{7}{313}$ $\alpha = 88.719^\circ$ Thus, the angle of PQ made with horizontal is $90^\circ - \alpha = 1.3^\circ$ (1 d.p.)	The horizontal refers to a plane perpendicular to the z-axes. 178.7^o is also acceptable in this case as question did not specify acute angle.

Question 12

No.	Suggested Solution	Remarks for Student
(i)	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = kP$ $\displaystyle\int \frac{1}{P}\,\mathrm{d}P = \int k\,\mathrm{d}t$ $\ln\lvert P\rvert = kt + C$ $\ln P = kt + C, \quad \text{since } P > 0$ $P = \mathrm{e}^{kt+C} = A\mathrm{e}^{kt}$, where $A = \mathrm{e}^{c}$ Given that $t = 0,\ P = 50 \Rightarrow A = 50$ $\therefore P = 50\mathrm{e}^{kt}$ $t = 10,\ P = 100 \Rightarrow 100 = 50\mathrm{e}^{10k}$ $\mathrm{e}^{10k} = 2 \Rightarrow \mathrm{e}^{k} = 2^{\frac{1}{10}}$ $k = \dfrac{1}{10}\ln 2$ $\therefore P = 50\left(2^{\frac{t}{10}}\right)$	Need to write P in terms of t as stated in the question
(ii)	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = \lambda P(500 - P)$ $\displaystyle\int \frac{1}{P(500-P)}\,\mathrm{d}P = \int \lambda\,\mathrm{d}t$ $\displaystyle\int \frac{1}{500P} + \frac{1}{500(500-P)}\,\mathrm{d}P = \lambda t + C$ $\dfrac{1}{500}\ln\left\lvert\dfrac{P}{500-P}\right\rvert = \lambda t + C$ $\dfrac{P}{500-P} = A\mathrm{e}^{500\lambda t},\ A = \pm\mathrm{e}^{500C}$ $P = \dfrac{500A\mathrm{e}^{500\lambda t}}{1 + A\mathrm{e}^{500\lambda t}}$ $t = 0, P = 50 \Rightarrow A = \dfrac{1}{9}$ $P = \dfrac{\frac{500}{9}\mathrm{e}^{500\lambda t}}{1 + \frac{1}{9}\mathrm{e}^{500\lambda t}} = \dfrac{500\mathrm{e}^{500\lambda t}}{9 + \mathrm{e}^{500\lambda t}}$ $t = 10, P = 100$ $\dfrac{1}{4} = \dfrac{1}{9}\mathrm{e}^{5000\lambda} \Rightarrow \mathrm{e}^{\lambda} = \left(\dfrac{9}{4}\right)^{\frac{1}{5000}}$	

	$P=\dfrac{500\left(\frac{9}{4}\right)^{\frac{t}{10}}}{9+\left(\frac{9}{4}\right)^{\frac{t}{10}}}$ or $P=\dfrac{500}{1+9\left(\frac{9}{4}\right)^{-\frac{t}{10}}}$	
(c)	Refined model, $P=\dfrac{500\left(\frac{9}{4}\right)^{\frac{t}{10}}}{9+\left(\frac{9}{4}\right)^{\frac{t}{10}}}=\dfrac{500}{9\left(\frac{9}{4}\right)^{-\frac{t}{10}}+1}\to 500$ as $t\to\infty$ Population approaches 500 in the long run. First model, $P=50\left(2^{\frac{t}{10}}\right)\to\infty$ as $t\to\infty$ which is not realistic in real world as no population could indefinitely approach infinity as resources are limited.	Need to compare the 2 models in the long run