TEMASEK JUNIOR COLLEGE, SINGAPORE

Preliminary Examination 2015
Higher 2

MATHEMATICS 9740/01

Paper 1

31 August 2015

Additional Materials: Answer paper
List of Formulae (MF15)

3 hours

READ THESE INSTRUCTIONS FIRST

Write your Civics group and name on all the work that you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of **5** printed pages.

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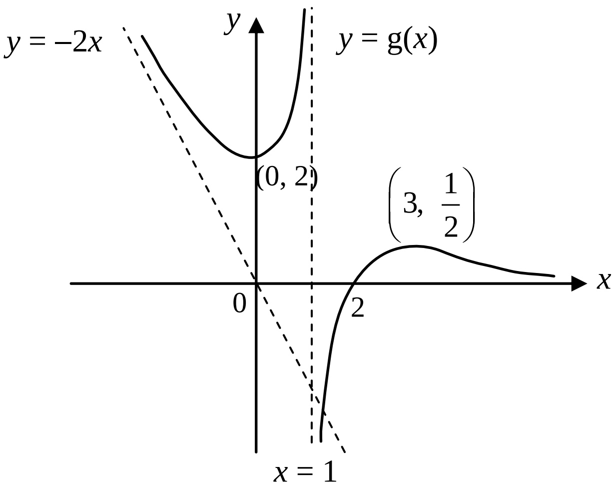
PASSION PURPOSE DRIVE

1 The equation of a circle M is given by $x^2 + y^2 + Ax + By + C = 0$ where A, B and C are real constants. The line $y = -2(x + 1)$ passes through the centre of M and the graph of $y = |\,x\,|$ intersects M at the points where $x = -2$ and $x = -8$. Find the equation of M. [4]

2 The diagram below shows the graph of $y = \mathrm{g}(x)$. The graph has a minimum point at $(0, 2)$ and a maximum point at $\left(3,\ \frac{1}{2}\right)$. The equations of the asymptotes are $x = 1$, $y = 0$ and $y = -2x$.



On separate diagrams, sketch the graphs of

(i) $y = \mathrm{g}'(x)$, [2]

(ii) $y = \dfrac{1}{\mathrm{g}(x)}$, [2]

showing clearly in each case, the equations of the asymptotes and the coordinates of the turning points and axial intercepts, where applicable.

3 Without using a calculator, solve the inequality $\dfrac{3x^2}{1-2x} > 1$. Hence solve $\dfrac{3x^2}{1-2|x|} > 1$. [5]

4 The sequence of real numbers $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{n+2}{n+4}u_n \text{ and } u_1 = a, \text{ where } n \geq 1 \text{ and } a \in \mathbb{R}.$$

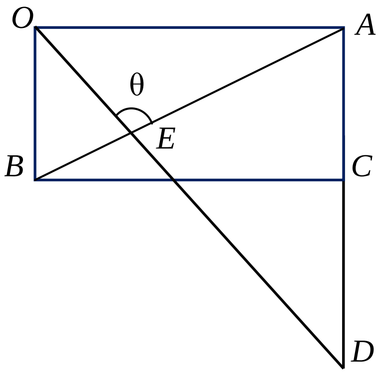
(i) Prove by mathematical induction that $u_n = \dfrac{12a}{(n+2)(n+3)}$ for $n \geq 1$. [4]

(ii) Determine the limit of $n(n+2)\dfrac{u_n}{u_1}$ as $n \to \infty$. [2]

5 The complex number z satisfies the equation $\dfrac{1+z^3}{1-z^3} = \sqrt{3}\,\mathrm{i}$.

Without the use of a graphing calculator, express z^3 in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $r \geq 0$ and $-\pi < \theta \leq \pi$. Hence find the roots of the equation. [6]

6 The figure below shows a rectangle $OACB$ where $OA = 2OB$. Point D is on AC produced such that $AD : AC = \lambda : 1$ where λ is a constant. The lines OD and AB intersect at point E. It is given that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\angle OEA = \theta$.



Find $\overrightarrow{OD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, and show that $\overrightarrow{OD} \cdot \overrightarrow{AB} = (\lambda - 4)|\mathbf{b}|^2$. [4]

In the case when E is the foot of perpendicular from A to OD, deduce the value of λ. [2]

Using this value of λ and given that $\mathbf{a} = \begin{pmatrix} 4 \\ 4 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, find $|\overrightarrow{OE}|$. [2]

7 The function f is defined by

$$\mathrm{f} : x \mapsto \left|\frac{1-4x}{x+1}\right|, \ x \in \mathbb{R}, \ x \geq k.$$

(i) With the aid of a graph, find the least value of k such that f has an inverse. [2]

(ii) Using the least value of k found in **(i)**,

(a) find $\mathrm{f}^{-1}(x)$ and state its domain, [3]

(b) find the exact solution(s) of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [2]

Describe a sequence of two transformations which would transform the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \left|\dfrac{2+4x}{2-x}\right|$. [2]

8 **(i)** Use the substitution $x = \sin^2\theta$, where $0 < \theta < \dfrac{\pi}{2}$ and $0 < x < 1$, to show that

$$\int \sqrt{\frac{x}{1-x}}\, \mathrm{d}x = \sin^{-1}\sqrt{x} - \sqrt{x(1-x)} + c$$ where c is an arbitrary constant. [5]

(ii) The region R is bounded by the curve $y = \left(\dfrac{x}{1-x}\right)^{\frac{1}{4}}$ and the lines $y = 4x - 1$ and $x = \dfrac{1}{4}$. Find the volume of revolution formed when R is rotated completely about the x-axis, giving your answer in exact form. [5]

[Turn over

9 The planes p_1 and p_2 have equations $2x+z=2$ and $\mathbf{r}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=0$ respectively.

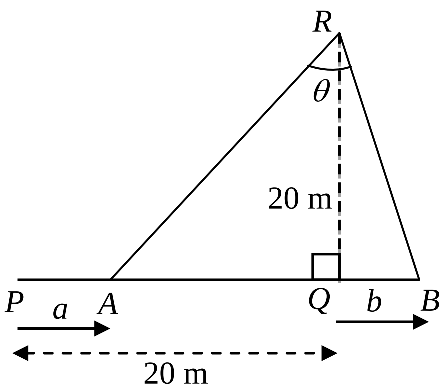
(i) Obtain a vector equation of the line of intersection, l, between p_1 and p_2. [2]

(ii) A third plane p_3 contains l and is perpendicular to p_1. Find a vector equation of p_3, in scalar product form. [3]

(iii) The point S lies on p_1 and the point T lies on p_3 such that the line ST is perpendicular to p_2. If the coordinates of S are $(2,-3,-2)$, find the coordinates of T. [4]

(iv) Find the acute angle between ST and p_1. [2]

10 P and Q are two points lying 20 m apart on a horizontal straight line. Two particles A and B are initially located at P and Q respectively. A begins to move towards Q and B begins to move away from Q. At time t s, the distance travelled by A and B are a m and b m respectively where $0 \le a < 20$. The fixed point R is located 20 m vertically above point Q such that angle $ARB = \theta$.



By considering θ as the sum of two acute angles, show that

$$\tan\theta = \frac{20(20-a+b)}{400-20b+ab}.$$ [3]

(a) On day 1, A and B move in such a way that the distance of B from Q is always twice the distance of A from P, that is, $b = 2a$. Find, using differentiation, the value of a when θ is maximum. [4]
[You do not need to show that θ is maximum.]

(b) On day 2, A and B resume their starting positions at P and Q, and move such that θ remains a constant.

(i) Show that $b = \dfrac{20a}{40-a}$. [2]

(ii) If A moves at a constant speed of 0.5 ms^{-1}, find the speed of B at $t = 30$. [3]

11 **(a)** By using small angle approximations, where x is small enough for x^3 and higher powers of x to be neglected, show that $\dfrac{\sin\left(2x-\dfrac{\pi}{4}\right)}{2-\sin x} \approx \sqrt{2}\left(-\dfrac{1}{4}+px+qx^2\right)$, where p and q are constants to be determined. [5]

(b) A curve has equation $y^2 - xy = 4 - \sin x$.

(i) Show that there is no tangent to the curve that is parallel to the y-axis. [4]

(ii) Given that $y = 2$ when $x = 0$, find the Maclaurin's series for y up to and including the term in x^2. [3]

12 A curve C has parametric equations

$$x = 7 - 4\sin^2 t\,,\ \ y = 4 + 3\sin^3 t$$

where $-\dfrac{\pi}{2} < t \le \dfrac{\pi}{2}$.

(i) Show that the equation of the tangent to the curve at the point with parameter t is

$$8y + 9x\sin t - 63\sin t + 12\sin^3 t - 32 = 0\,.$$

This tangent passes through a fixed point (X, Y). Give a brief argument to explain why there cannot be more than 3 tangents passing through (X, Y). [5]

(ii) Sketch the curve C. [2]

(iii) Show that the coordinates of the points of intersection between C and the line $8y + 9x - 83 = 0$ are $(3, 7)$ and $\left(6,\ \dfrac{29}{8}\right)$. [3]

(iv) Find the area of the region bounded by C and the line $8y + 9x - 83 = 0$. [3]

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