Xinmin Secondary School
Additional Mathematics Practice
Topic: Solving Trigo Equations

Name: _____ ( ) Date: _____

1 Given that $0° < x < 360°$, solve the equation
(a) $\sin x = -\frac{1}{3}$,
(b) $\text{cosec } x = \tan 60°$.

2 Given that $0° \le x \le 360°$, solve the equation
(a) $2\cos^2 x - 5\cos x - 3 = 0$.
(b) $6\sin^2 x = 2 - 2\cos x$.

3 Given that $0° \le x \le 360°$, solve the equation
(a) $\tan x = 4\sin x$,
(b) $2\cos x + \cot x = 0$.

4 Solve the equation $\cos(2y + 30°) = -\frac{1}{2}$ for all the angles between $0°$ to $360°$ inclusive.

5 Solve the equation $5\sin 2x + 2 = 0$ for $0° \le x \le 360°$.
Correct your answer to 1 decimal place.

6 Find all the angles between 0 and 2π which satisfy $\tan\left(2x - \frac{\pi}{6}\right) = \frac{1}{2}$.
Correct your answer to 3 significant figures.

7 Solve the equation $2\sin\left(2x + \frac{5\pi}{8}\right) = -\frac{7}{4}$ for $1 \le x \le 4.5$.
Correct your answer to 3 significant figures.

8 Find all the angles between 0 and 2π which satisfy the equation $\cos^2 x = 3\sin x \cos x$.
If the answer is not exact, correct the answer to 3 significant figures.

9 Given that $0 \le x \le 2\pi$, solve the equation
(a) $2\sin x - 5\,\text{cosec } x = 9$,
(b) $14\sin x + 54\cos x = 18\sin x + 27\cos x$.

10 Solve the equation $3\cos 2y = \cos y - 3$ for $0° < y < 360°$.
If the answer is not exact, leave the answer to 1 decimal place.

11 Solve the equation $4\cos 2x + 2\sin x = -2$ for $0 \le x \le 2\pi$.

12 Find all the angles between $0°$ and $360°$ which satisfy $5 \tan^2 y + 7 = 11 \sec y$.

13 Solve the equation $\sin^2 y + 2\cos 2y = 2\cos y$ for $0° \le y \le 360°$.

Work Solution

1

(a) $\sin x = -\frac{1}{3}$

Reference angle = $\sin^{-1}\frac{1}{3} \approx 19.471°$

Since $\sin x < 0$, x is in the 3rd or 4th quadrant.

$x \approx 180° + 19.471°, 360° - 19.471°$

$\approx 199.5°, 340.5°$

(b) $\operatorname{cosec} x = \tan 60°$

$\frac{1}{\sin x} = \sqrt{3}$

$\sin x = \frac{1}{\sqrt{3}}$

Reference angle = $\sin^{-1}\frac{1}{\sqrt{3}} \approx 35.264°$

Since $\sin x > 0$, x is in the 1st or 2nd quadrant.

$x \approx 35.264°, 180° - 35.264°$

$\approx 35.3°, 144.7°$

2 **(a)** $2\cos^2 x - 5\cos x - 3 = 0$

$(2\cos x + 1)(\cos x - 3) = 0$

$\cos x = -\frac{1}{2}$ or $\cos x = 3$ (no solution)

$\cos x = -\frac{1}{2}$

Reference angle = $\cos^{-1}\frac{1}{2} = 60°$

Since $\cos x < 0$, x is in the 2nd or 3rd quadrant.

$x = 180° - 60°, 180° + 60°$

$= 120°, 240°$

(b) $6\sin^2 x = 2 - 2\cos x$

$6(1 - \cos^2 x) = 2 - 2\cos x$

$6 - 6\cos^2 x = 2 - 2\cos x$

$3\cos^2 x - \cos x - 2 = 0$

$(3\cos x + 2)(\cos x - 1) = 0$

$\cos x = -\frac{2}{3}$ or $\cos x = 1$

$\cos x = 1$

$x = 0°, 360°$

$\cos x = -\frac{2}{3}$

Reference angle =

$\cos^{-1}\frac{2}{3} \approx 48.190°$

Since $\cos x < 0$, x is in the 2nd or 3rd quadrant.

$x \approx 180° - 48.190°,$

$180° + 48.190°$

$\approx 131.8°, 228.2°$

Hence $x = 0°, 131.8°, 228.2°, 360°$.

3 (a) $\tan x = 4\sin x$

$\frac{\sin x}{\cos x} = 4\sin x$

$\sin x - 4\sin x\cos x = 0$

$\sin x(1 - 4\cos x) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{4}$

$\sin x = 0$

$x = 0°, 180°, 360°$

$\cos x = \frac{1}{4}$

Reference angle = $\cos^{-1}\frac{1}{4} \approx 75.522°$

Since $\cos x > 0$, x is in the 1st or 4th quadrant.

$x \approx 75.522°, 360° - 75.522°$

$\approx 75.5°, 284.5°$

Hence $x = 0°, 75.5°, 180°, 284.5°, 360°$.

	(b) $2\cos x + \cot x = 0$ $2\cos x + \frac{\cos x}{\sin x} = 0$ $\cos x \left(2 + \frac{1}{\sin x}\right) = 0$ $\cos x = 0$ or $\sin x = -\frac{1}{2}$ $\cos x = 0$ $x = 90°, 270°$ $\sin x = -\frac{1}{2}$ Reference angle $= \sin^{-1}\frac{1}{2} = 30°$ Since $\sin x < 0$, x is in the 3rd or 4th quadrant. $x = 180° + 30°, 360° - 30°$ $= 210°, 330°$ Hence $x = 90°, 210°, 270°, 330°$.
4	$\cos(2y + 30°) = -\frac{1}{2}$ Reference angle $= 60°$ Take note on range: Since $0° \le y \le 360°$, $0° \le 2y \le 720°$, $30° \le 2y + 30° \le 750°$ $2y + 30° = 180° - 60°,\ 180° + 60°,\ 540° - 60°,\ 540° + 60°$ $2y + 30° = 120°,\ 240°,\ 480°,\ 600°$ $y = 75°,\ 105°,\ 225°,\ 285°$
5	$5\sin 2x + 2 = 0$ $\sin 2x = -\frac{2}{5}$ $\alpha = 23.578°$ (correct to 3 decimal places) $2x = 180° + 23.578°, 360° - 23.578°, 540° + 23.578°, 720° - 23.578°$ $2x = 203.578°, 336.422°, 563.578°, 696.422°$ $x = 101.8°, 168.2°, 281.8°, 348.2°$ (correct to 1 decimal place)

6

$$\tan 2y = -\frac{6}{5}$$

$\alpha = 50.194°$ (correct to 3 decimal places)

$2y = 180° - 50.194°, 360° - 50.194°, 540° - 50.194°, 720° - 50.194°$

$2y = 129.806°, 309.806°, 489.806°, 669.806°$

$y = 64.9°, 154.9°, 224.9°, 334.9°$ (correct to 1 decimal place)

7

$$2\sin\left(2x + \frac{5\pi}{8}\right) = -\frac{7}{4}$$

$$\sin\left(2x + \frac{5\pi}{8}\right) = -\frac{7}{8}$$

$\alpha = 1.0654$ (correct to 5 sig. fig.)

$$2x + \frac{5\pi}{8} = \pi + 1.0654, 2\pi - 1.0654, 3\pi + 1.0654, 4\pi - 1.0654$$

$x = 1.12, 1.63, 4.26, 4.77$ (correct to 3 sig. fig.)

Since $1 \le x \le 4.5$, $x = 1.12, 1.63, 4.26$.

8

$$\cos^2 x = 3\sin x \cos x$$

$$\cos^2 x - 3\sin x \cos x = 0$$

$$\cos x(\cos x - 3\sin x) = 0$$

$\cos x = 0$ or $\cos x - 3\sin x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x = 3\sin x$

$\tan x = \frac{1}{3}$

$\alpha = 0.32175$ (correct to 3 sig. fig.)

$x = 0.32175, \pi + 0.32175$

$x = 0.322, 3.46$ (correct to 3 sig. fig.)

Hence, $x = 0.322, \frac{\pi}{2}, 3.46, \frac{3\pi}{2}$.

9

(a) $2 \sin x - 5 \operatorname{cosec} x = 9$

$$2 \sin x - \frac{5}{\sin x} = 9$$

$2 \sin^2 x - 9 \sin x - 5 = 0$

$(2 \sin x + 1)(\sin x - 5) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 5$ (no solution)

Reference angle $= \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

Since $\sin x < 0$, x is in the 3rd or 4th quadrant.

$x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

	$= \frac{7\rho}{6}, \frac{11\rho}{6}$ (b) $14 \sin x + 54 \cos x = 18 \sin x + 27 \cos x$ $4\sin x = 27\cos x$ $\frac{\sin x}{\cos x} = \frac{27}{4}$ $\tan x = \frac{27}{4}$ Reference angle = $\tan^{-1} \frac{27}{4} \approx 1.424$ Since $\tan x > 0$, x is in the 1st or 3rd quadrant. $x \approx 1.424, \pi + 1.424$ $\approx 1.42, 4.57$
10	$3\cos 2y = \cos y - 3$ $3(-1 + 2\cos^2 y) = \cos y - 3$ $-3 + 6\cos^2 y = \cos y - 3$ $6\cos^2 y - \cos y = 0$ $\cos y(6\cos y - 1) = 0$ Either $\cos y = 0$ $y = 90°, 270°$ Or $6\cos y - 1 = 0$ $6\cos y = 1$ $\cos y = \frac{1}{6}$ $\alpha = 80.406°$ $y = 80.406°, 360° - 80.406°$ $y = 80.4°, 279.6°$ (correct to 1 decimal place) Hence, $y = 80.4°, 90°, 270°, 279.6°$.

11	$4\cos 2x + 2\sin x = -2$ $4(1-2\sin^2 x) + 2\sin x + 2 = 0$ $4\sin^2 x - \sin x - 3 = 0$ $(4\sin x + 3)(\sin x - 1) = 0$ $\sin x = -\frac{3}{4}$ or $\sin x = 1$ Basic $\angle = \sin^{-1}\left(\frac{3}{4}\right)$ $x = \frac{\pi}{2}$ $= 0.84806$ $= 1.57$ $x = \pi + 0.84806, 2\pi - 0.84806$ $= 3.99, 5.44$
12	$5\tan^2 y + 7 = 11\sec y$ $5(\sec^2 y - 1) + 7 = 11\sec y$ $5\sec^2 y - 11\sec y + 2 = 0$ $(5\sec y - 1)(\sec y - 2) = 0$ $\sec y = \frac{1}{5}$ or $\sec y = 2$ $\cos y = 5$ $\cos y = \frac{1}{2}$ (Rej.)
13	$1 - \cos^2 y + 4\cos^2 y - 2 - 2\cos y = 0$ $3\cos^2 y - 2\cos y - 1 = 0$ $(3\cos y + 1)(\cos y - 1) = 0$ $\cos y = -\frac{1}{3}$ or $\cos y = 1$ Basic $\angle = 70.53°$ $y = 0°, 360°$ $y = 109.5°, 250.5°$