RAFFLES INSTITUTION
2023 YEAR 6 TIMED PRACTICE

CANDIDATE NAME

CLASS 23

MATHEMATICS 9758

2 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (extracted from MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper. **You may use the blank page on page 18 if necessary and you are reminded to indicate the question number(s) clearly.**
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 65.

For examiner's use only						
Q1	Q2	Q3	Q4	Q5	Q6	Total
/10	/10	/10	/12	/12	/11	/65

This document consists of **4** printed pages.

1 **(a)** **(i)** Find $\int \sin^2 x \, dx$. [2]

(ii) Hence or otherwise, show that

$$\int x\sin^2 x \, dx = \frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + c.$$ [3]

(b) **(i)** Sketch the graph of $y = x\sin^2 x$ for $0 \le x \le \frac{1}{2}\pi$, labelling the end points clearly. [2]

(ii) Hence, find the exact area of the region bounded by the graph of $y = x\sin^2 x$, the x-axis and the line $x = \frac{1}{2}\pi$. [3]

2 **(a)** A point R has position vector $\mathbf{r} = \begin{pmatrix} a \\ 2 \\ 3 \end{pmatrix} + a\begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$, where a is a real number.

Describe geometrically the set of all possible positions of R as a varies. [2]

(b) The angle between the vectors $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{3}$ radians.

(i) Given that the angle between the vectors $\mathbf{b}$ and $(\mathbf{a} - 2\mathbf{b})$ is a right angle, show that $|\mathbf{a}| = 4|\mathbf{b}|$. [3]

(ii) Given also that $|\mathbf{b}| = 2$, find the exact length of projection of $(2\mathbf{a} + \mathbf{b})$ onto $(\mathbf{a} + 2\mathbf{b})$. [5]

3 **Do not use a calculator in answering this question.**

Two complex numbers p and q are given by $p = \frac{3}{\sqrt{2}} + \mathrm{i}\frac{3}{\sqrt{2}}$ and $q = 1 + \mathrm{i}\sqrt{3}$ respectively.

(a) Find p^*q^2 in the form $a + \mathrm{i}b$, where a and b are exact real values, and p^* denotes the conjugate of p. [3]

(b) **(i)** Find the modulus and argument of p and q. [2]

(ii) Write down the value of p^*q^2 in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \le \pi$. [2]

(c) **(i)** Represent p^*q^2 on an Argand diagram, labelling clearly the information found in parts **(a)** and **(bii)**. [1]

(ii) Hence, find the exact value of $\sin\frac{1}{12}\pi$. [2]

4 Two planes p_1 and p_2 are perpendicular. Plane p_1 has equation

$$\mathbf{r}=\begin{pmatrix}2\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\a\\2\end{pmatrix}+\mu\begin{pmatrix}2\\0\\1\end{pmatrix},$$

where a is a constant, and λ and μ are parameters.

Plane p_2 contains the line l with equation $x-3=z-4,\ y=1$.

(a) Given that the equation of p_1 may also be expressed as

$$\mathbf{r}\cdot\begin{pmatrix}1\\3\\-2\end{pmatrix}=d,$$

where d is a constant, find the values of a and d. [2]

(b) Find a cartesian equation of p_2. [3]

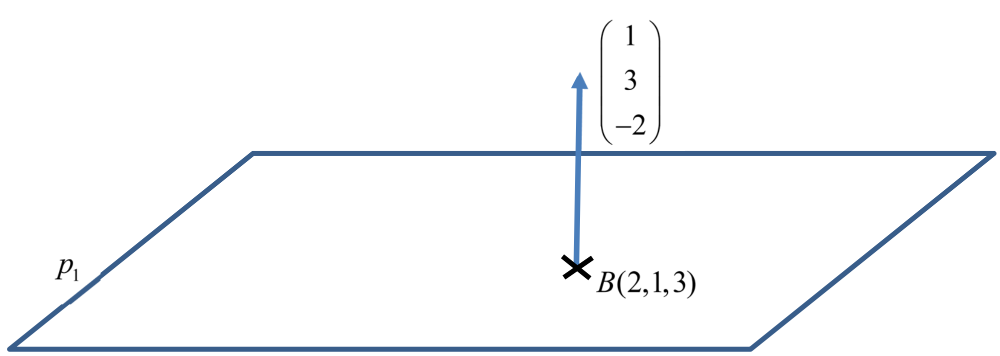
(c) Show that a vector equation of m, the line of intersection of p_1 and p_2, can be expressed as $\mathbf{r}=\begin{pmatrix}2\\1\\3\end{pmatrix}+s\begin{pmatrix}5\\1\\4\end{pmatrix}$, where s is a parameter. [3]

(d) Find the acute angle between l and m. [2]

(e) The points O, A and B have coordinates $(0,0,0)$, $(3,1,4)$ and $(2,1,3)$ respectively.

By considering the dot products of vectors $\overrightarrow{BO}$ and $\overrightarrow{BA}$ with the normal of p_1, determine whether O and A are on the same side or opposite sides of p_1.

You should indicate clearly the vectors $\overrightarrow{BO}$ and $\overrightarrow{BA}$ on the diagram below. [2]



5 John models the nitrate level, N mg/L, present in the aquarium at a time t days after setting up his fish tank. The model assumes that, at any time t, the rate of increase of nitrate level due to ammonia and bacteria in the tank is kN mg/L per day, for some positive constant k. The initial nitrate level is 10 mg/L and it is found to be 20 mg/L after 1 day.

(a) **(i)** Write down a differential equation involving N, t and k. [1]

(ii) Solve this differential equation to find an expression for N in terms of t and show that $k = \ln 2$. [3]

(iii) Find the nitrate level after 3 days. [1]

Hornwort is an aquatic plant that can help to reduce nitrate levels in aquariums. John adds s stalks of hornwort to his fish tank after 3 days. He then models the rate of decrease of nitrate level by the hornwort, x days after it is added, by h mg/L per stalk per day. The rate of increase of nitrate level due to ammonia and bacteria in the tank remains at $N \ln 2$ mg/L per day.

(b) **(i)** Explain why the differential equation in this case is now

$$\frac{\mathrm{d}N}{\mathrm{d}x} = N \ln 2 - sh.$$

[1]

(ii) Solve this differential equation, giving N in terms of x, s and h. [3]

(c) In the case where h = 18.5, find the least number of stalks of hornwort needed in order for the nitrate level to decrease to 0 mg/L at $x = 7$. [3]

6 **(a)** Find the number of arrangements of all ten letters of the word ENGAGEMENT with no restrictions. [1]

(b) Find the number of arrangements of all ten letters of the word ENGAGEMENT in which the first and last letters are vowels. [3]

(c) Find the number of arrangements of all ten letters of the word ENGAGEMENT in which no two vowels are next to each other. [3]

(d) It is now given that 4 letters are randomly selected from the ten letters in the word ENGAGEMENT.

Find the probability that the 4 letters selected contains exactly 2 distinct letters. [4]