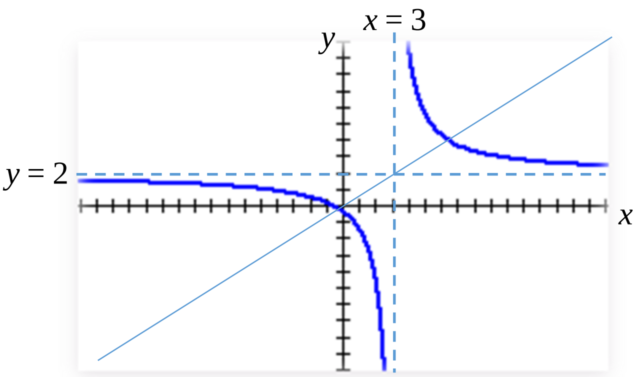
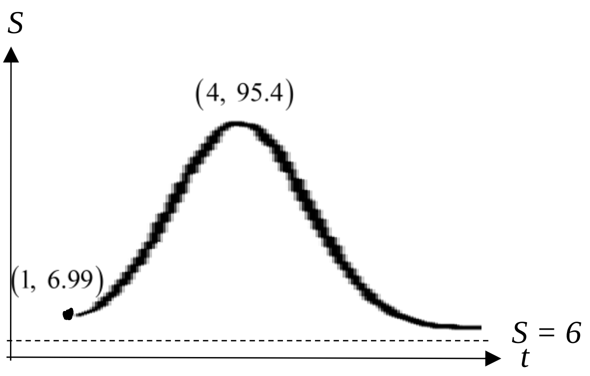
Qn	Suggested Solution
1	$\frac{1}{3}\log_2 8^x+4=y\cdots(1)$ $\sqrt{x^2+y}=y-3\cdots(2)$ From (1), $\frac{1}{3}\log_2 8^x+4=y$ $\frac{1}{3}\log_2 2^{3x}+4=y$ $\frac{1}{3}(3x)\log_2 2+4=y$ $x+4=y\cdots(3)$ Substituting (3) into (2), $\sqrt{x^2+(x+4)}=(x+4)-3$ $x^2+x+4=\left(x+1\right)^2$ $x^2+x+4=x^2+2x+1$ $x=3, y=7$

Qn	Suggested Solution
2	$kx^2+\left(2-k\right)x+k+\frac{7}{4}<0$ $k<0$ and discriminant <0 $\left(2-k\right)^2-4k\left(k+\frac{7}{4}\right)<0$ $-3k^2-11k+4<0$ $k<0$ and $\left(1-3k\right)\left(k+4\right)<0$ $k<-4$ or $k>\frac{1}{3}$ $\Rightarrow k<-4$ $\left\{k\in\mathbb{R}:k<-4\right\}$

Qn	Suggested Solution
3(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(5x^2-3\mathrm{e}^{\pi})^3$ $=3(5x^2-3\mathrm{e}^{\pi})^2(10x)$ $=30x(5x^2-3\mathrm{e}^{\pi})^2$
(b)	$\int \frac{\left(-\sqrt{x}+3\right)^2}{\sqrt{x}}\ \mathrm{dx}$ $= \int \frac{9+x-6\sqrt{x}}{\sqrt{x}}\ \mathrm{dx}$ $=\int \frac{9}{\sqrt{x}}+\frac{x}{\sqrt{x}}-6\ \mathrm{dx}$ $=\int 9x^{-\frac{1}{2}}+x^{\frac{1}{2}}-6\ \mathrm{dx}$ $=18x^{\frac{1}{2}}+\frac{2}{3}x^{\frac{3}{2}}-6x+C$ $=18\sqrt{x}+\frac{2}{3}x\sqrt{x}-6x+C$
(c)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\frac{x}{\sqrt{3-2x^2}}\right)\right)$ $=\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln x-\frac{1}{2}\ln\left(3-2x^2\right)\right)$ $=\frac{1}{x}-\frac{1}{2}\left(\frac{-4x}{3-2x^2}\right)$ $=\frac{1}{x}+\frac{2x}{3-2x^2}$ $=\frac{3}{x\left(3-2x^2\right)}$ $\int_{\frac{1}{2}}^{1}\frac{6}{x\left(3-2x^2\right)}\ \mathrm{d}x=2\int_{\frac{1}{2}}^{1}\frac{3}{x\left(3-2x^2\right)}\ \mathrm{d}x$ $=2\left[\ln\left(\frac{x}{\sqrt{3-2x^2}}\right)\right]_{\frac{1}{2}}^{1}$ $=2\left(\ln\left(\frac{1}{\sqrt{1}}\right)-\ln\left(\frac{\left(\frac{1}{2}\right)}{\sqrt{\frac{5}{2}}}\right)\right)$ $=-2\ln\frac{1}{\sqrt{10}}$ $=-2\ln 10^{-\frac{1}{2}}$ $=\ln 10 \quad \therefore a=1,\ b=10$

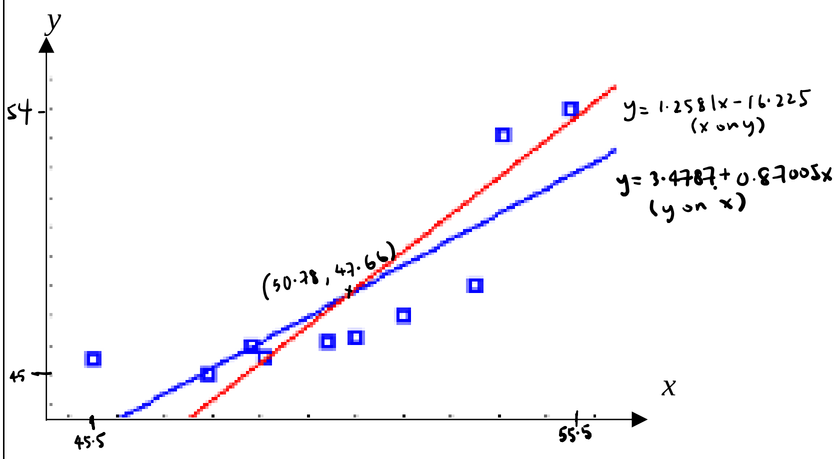
Qn	Suggested Solution
4(a)	$y=2+\frac{7}{x-3}$ HA:$y=2$ VA:$x=3$
(b)	 Axial intercepts: $(0,-\frac{1}{3})$ & $(-\frac{1}{2},0)$
(c)	$2x-6\le\frac{21}{x-3}$ $\frac{2}{3}x-2\le\frac{7}{x-3}$ $\Rightarrow\ \ 2+\frac{7}{x-3}\ge\frac{2}{3}x$ From GC, $x\le -0.240$ or $3<x\le 6.24$
(d)	Using GC, area of bounded region $=\int_4^{6.2404}(2+\frac{7}{x-3})\ -\frac{2}{3}x\,\mathrm{d}x$ $=5.06$ units^2 OR $\int_4^{6.2404}(2+\frac{7}{x-3})\,\mathrm{d}x-\frac{1}{2}(\frac{8}{3}+4.1602)(6.2404-4)$
(e)	$(x-3)^2+\left(\frac{7}{x-3}\right)^2=m^2$ $(x-3)^2+\left(2+\frac{7}{x-3}-2\right)^2=m^2$ For the equation $(x-3)^2+\left(\frac{7}{x-3}\right)^2=m^2$ to have no real roots, there is no intersection between $y=2+\frac{7}{x-3}$ and $(x-3)^2+\left(y-2\right)^2=m^2$ which is a circle centred at (3,2) radius m. Using GC, possible values of m are $m=1,2,3.$

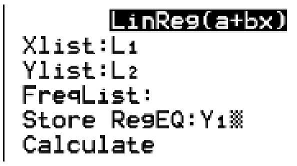
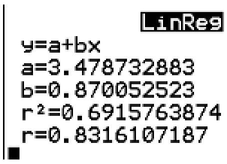
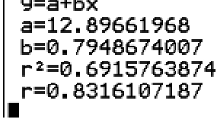
Qn	**Suggested Solution**
5(a)	$y=p+q^2x^2+2^{r+x}+s\sqrt{x}$ $(0,9):\quad p+2^r=9$ $(9,1787):\quad p+81q^2+512\left(2^r\right)+3s=1787$ $(4,201):\quad p+16q^2+16\left(2^r\right)+2s=201$ $(1,29):\quad p+\ q^2+2\left(2^r\right)+s=29$ From GC, $p=7$ $q^2=9\Rightarrow q=3,\ 2^r=2\Rightarrow r=1, s=9$ The key is $(7,3,1,9)$
(b)	From G.C, maximum *S* occur at *t* = 4 Maximum *S* = 95.4 million (3 s.f.)
(c)	$S=0.03\mathrm{e}^{-0.5t^2+4t}+6$ $\frac{\mathrm{d}S}{\mathrm{d}t}=0.03(-t+4)e^{-0.5t^2+4t}$ when $t=5$, $\frac{\mathrm{d}S}{\mathrm{d}t}=0.03(-5+4)\mathrm{e}^{-0.5(5)^2+4(5)}=-0.03\mathrm{e}^{\frac{15}{2}}$ $S=0.03e^{-0.5(5)^2+4(5)}+6=0.03\mathrm{e}^{\frac{15}{2}}+6$ Equation of tangent: $S-\left(0.03\mathrm{e}^{\frac{15}{2}}+6\right)=-0.03\mathrm{e}^{\frac{15}{2}}(t-5)$ $S=-0.03\mathrm{e}^{\frac{15}{2}}t+0.18\mathrm{e}^{\frac{15}{2}}+6$
(d)	
(e)	For $t\geq 7$, as *t* increases from 7 as $t\to\infty$, $\frac{\mathrm{d}S}{\mathrm{d}t}$ increases from -2.9804 (gradient less negative) until it approaches 0. Hence the manager expects the sales to **decrease at a slower/decreasing rate** until it **stabilises at 6 million.**

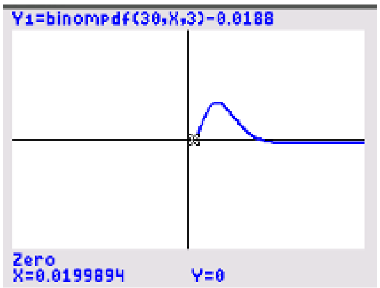
Qn	Suggested Solution
6(a)	Number of ways $=4!\times 3!\times {}^5C_2\times 2!=2880$
(b)	Required probability $=\dfrac{\text{P(DAY grouped and start end vowel)}}{\text{P(start and end with vowel)}}$ $=\dfrac{\text{no. of ways (\_ DAY \_)}}{\text{no. of ways (start and end with vowel)}}$ $=\dfrac{2}{{}^3C_2\times 2!\times {}^6C_3\times 3!}$ $=\dfrac{1}{360}$

Qn	Suggested Solution
7(a)(i)	$\text{P}(A\cap B)$ $=\text{P(fall, rise, rise)}+\text{P(fall, fall, rise)}$ $=(0.4\times 0.15\times 0.6)+(0.4\times 0.85\times 0.15)$ $=0.087$
(ii)	$\text{P}(B)$ $=\text{P}(A\cap B)+\text{P}(A'\cap B)$ $=0.087+\text{P(rise, rise, rise)}+\text{P(rise, fall, rise)}$ $=0.087+(0.6\times 0.6\times 0.6)+(0.6\times 0.4\times 0.15)$ $=0.339$
(iii)	$\text{P}(B\,\|\,A)$ $=\dfrac{\text{P}(B\cap A)}{\text{P}(A)}$ $=\dfrac{0.087}{0.4}$ $=0.2175$
(b)	Since $\text{P}(B\,\|\,A)=0.2175\neq 0.339=\text{P}(B)$, A and B are not independent.
(c)	Let W be the number of Tuesdays in which the unit price of X rises, out of 12 Tuesdays. $W\sim \text{B}(12, 0.6)$ $\text{P}(W=5)=0.101\ (3\text{ s.f.})$

Qn	Suggested Solution
8(a)	• Set B will have a smaller r.

	The data points for **Set *B* lie relatively closer to a straight line** with negative gradient whereas **Set *A*'s r value will be closer to 0** since the data points are more scattered with weak linear correlation between *x* and *y*.
(b) (i)	
(ii)	$r = 0.83161 = 0.832$ (3s.f.) Since r value is close to 1, it indicates a **strong positive linear** correlation between x and y which is seen in the scatter diagram where the data points lie close to a **straight line with positive gradient**.

(iii)	equation of regression line of y on x: $y = 3.4787 + 0.87005x$ $y = 3.48 + 0.870x$  LinReg(a+bx) Xlist:L_1 Ylist:L_2 FreqList: Store RegEQ:Y_1 Calculate  LinReg y=a+bx a=3.478732883 b=0.870052523 r^2=0.6915763874 r=0.8316107187 equation of regression line of x on y: $x = 12.897 + 0.79487y$ $y = \frac{x - 12.897}{0.79487}$ $y = 1.2581x - 16.225$ $y = 1.26x - 16.2$ $(\bar{x}, \bar{y}) = (50.78, 47.66)$  LinReg(a+bx) Xlist:L_2 Ylist:L_1 FreqList: Store RegEQ: Calculate  LinReg y=a+bx a=12.89661968 b=0.7948674007 r^2=0.6915763874 r=0.8316107187
(iv)	Using equation of regression line of y on x: For $x = 50$, $y = 3.4787 + 0.87005(50) = 46.9812$ The mean household expenditure is estimated to be **$46 981**. The estimate is reliable since it is an **interpolation** where x = 50 ($45.5 \le x \le 55.5$) and ***r* is close to 1.**
(v)	It is not valid because a strong positive linear correlation between income and expenditure does not imply causation.

Qn	Suggested Solution
9(a)	$X \sim \text{B}(30, p)$ $\text{P}(X = 3) = 0.0188$ Using GC graph, $p = 0.0199894$ or $p = 0.2644435$ (rej since $0 < p < 0.2$) $p = 0.01999$ (5 d.p.)  Y_1=binompdf(30,X,3)-0.0188 Zero X=0.0199894 Y=0

(b)	$X \sim \mathrm{B}(30, 0.1)$ $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1)$ $= 1 - 0.183695$ $= 0.81630$ $= 0.816$ (3 s.f.)
(c)	Let Y be the number of boxes with at least 2 defective phones, out of 10 boxes. $Y \sim \mathrm{B}(10, 0.81630)$ $\mathrm{P}(X < 6) = \mathrm{P}(X \leq 5)$ $= 0.023192$ $= 0.0232$ (3 s.f.)
(d)	E(X) = 30(0.1) = 3 Var(X) = 30(0.1)(0.9) = 2.7 Since n is large, by Central limit theorem $\overline{X} \sim \mathrm{N}(3, \frac{2.7}{n})$ approximately $\mathrm{P}(\overline{X} \leq 3.5) \leq 0.998$ Using GC table, $n = 88, \mathrm{P}(\overline{X} \leq 3.5) = 0.9978 < 0.998$ $n = 89, \mathrm{P}(\overline{X} \leq 3.5) = 0.998 = 0.998$ $n = 90, \mathrm{P}(\overline{X} \leq 3.5) = 0.9981 > 0.998$ Hence, greatest n is **89.**
(e)	The boxes are picked without replacement, hence the trials are not independent.

<table>
<tr><th>Qn</th><th>Suggested Solution</th></tr>
<tr><td>10</td><td>Let X be the mass of a randomly chosen mooncake.
$H_0 : \mu = 150$
$H_1 : \mu < 150$

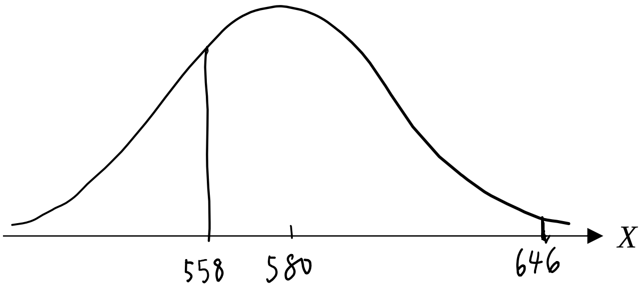
where μ is the population mean mass of mooncakes.
Since sample size of 9 is small, assume X follows a normal distribution.
Under H_0, $\bar{X} \sim N\left(150, \frac{6.73^2}{9}\right)$
Using GC, the test statistics $\bar{x} = 148$ gives $z_{calc} = -0.89153$ and
p-value = 0.186322 ≈ 0.186 (3 sf)

Since the p-value = 0.186 > 0.1, we do not reject H_0 and conclude that there is insufficient evidence at the 10% significance level that the mean mass of the mooncake is less than 150 g, i.e. insufficient evidence to reject owner's claim.</td></tr>
<tr><td>(b) (i)</td><td>Assign each teacher in the country a number in consecutive order. Among these numbers assigned, use a calculator to generate n different numbers randomly and choose the corresponding numbered teacher.</td></tr>
<tr><td>(ii)</td><td>Let Y be the working hours of a randomly chosen teacher in the school.
$H_0 : \mu = 60$
$H_1 : \mu \neq 60$

Under H_0, $\bar{Y} \sim N\left(60, \frac{6.5^2}{n}\right)$

In order to reject H_0, p-value = $2\,P(\bar{Y} \geq 62) \leq 0.05$
Using G.C
<table>
<tr><th>n</th><th>$2P(\bar{Y} \geq 62) - 0.05$</th></tr>
<tr><td>40</td><td>0.00165</td></tr>
<tr><td>41</td><td>$-0.0012 \quad \leq 0$</td></tr>
<tr><td>42</td><td>$-0.0039 \quad \leq 0$</td></tr>
</table>

Least n is 41.</td></tr>
<tr><td>(iii)</td><td>There is a probability of 0.05 that we reject the null hypothesis that the mean working hours of teachers in the school is 60 hours when it is actually true.</td></tr>
<tr><td></td><td></td></tr>
</table>

11(a)	
(b)	$X \sim \mathrm{N}(580,22^2)$ Expected number $=300\times\mathrm{P}(X>600)$ $=300\times 0.18165$ $=54.495$ $=54.5$ (3 s.f.)
(c)	No. By combining the masses, it would give a distribution with 2 peaks instead of a single peak.
(d)	Let K and L be the selling price of a randomly chosen rock melon and watermelon respectively. $K=0.003X, \quad L=0.0028Y$ $K \sim \mathrm{N}(0.003\times 580,\ 0.003^2\times 22^2)$ $K \sim \mathrm{N}(1.74, 0.004356)$ $K_1+K_2 \sim \mathrm{N}(3.48, 0.008712)$ $L \sim \mathrm{N}(0.0028\times 870,\ 0.0028^2\times 30^2)$ $L \sim \mathrm{N}(2.436, 0.007056)$ $K_1+K_2+L \sim \mathrm{N}(5.916, 0.015768)$ $\mathrm{P}(K_1+K_2+L\le 5.9)$ $=0.44930$ $=0.449$ (3s.f.)
(e)	$\mathrm{P}(K\le 1.70)^2\times\mathrm{P}(L\le 2.50)$ $=0.27224^2\times 0.77694$ $=0.057583$ $=0.0576$ (3s.f.)
(f)	Although the probabilities for both events are for at most \$5.90 payment, part (e) is a subset of (d) as part (e) is a special case where the cost of each type of melon is limited to at most \$1.70 and \$2.50, while in part (d) there is no restrictions as long as the total cost is at most \$5.90.
(g)	The masses/price of melons are independent of each other.