Name: _____ ( ) Class:_____

YEAR FOUR INTEGRATED PROGRAMME
END OF YEAR EXAMINATION

MATHEMATICS

Paper 2 **Wednesday 27 September 2023**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, staple all your work securely together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

Q1		**Q4**		**Q7**		**Paper 1**	**/ 90**
Q2		**Q5**		**Q8**		**Paper 2**	**/ 90**
Q3		**Q6**		**Q9**			
						Total	**/180**

This document consists of **21** printed pages and **1** blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount } = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone } = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere } = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length } = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area } = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean } = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation } = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

[Turn over for Question 1]

1 **(a)** Simplify $\frac{6p^2q}{5} \div \frac{9p}{q}$.

Answer …… [1]

(b) Solve $\frac{6}{y-2} - \frac{3}{2y+1} = 0$.

Answer $y =$ …… [2]

(c) **(i)** Express $x^2 - 3x + 12$ in the form $(x+a)^2 + b$, where a and b are constants.

Answer …… [2]

(ii) Hence, solve $x^2 - 3x + 12 = 11$.

Answer $x =$……..… or ……. [2]

(d) Simplify $\dfrac{9m^2 - 4n^2}{6am + 4an - 3bm - 2bn}$.

Answer …… [4]

2 **(a)** Valerie deposited $4000 in a savings account that pays compound interest of 3.8% per annum, compounded monthly.
Calculate the amount of interest Valerie receives at the end of 2 years.

Answer $…… [3]

(b) Ashley invested S$4000 in a savings account in Hong Kong when the exchange rate between Hong Kong dollars and Singapore dollars was HK$ 1 = S$0.15.
The savings account pays 4.2% per annum simple interest.
The investment matured after 3 years when the exchange rate between Hong Kong dollars and Singapore dollars was HK$ 1 = S$0.19

Find the total amount she received after 3 years.
Give your answer in Singapore dollars correct to the nearest cent.

Answer S$…… [4]

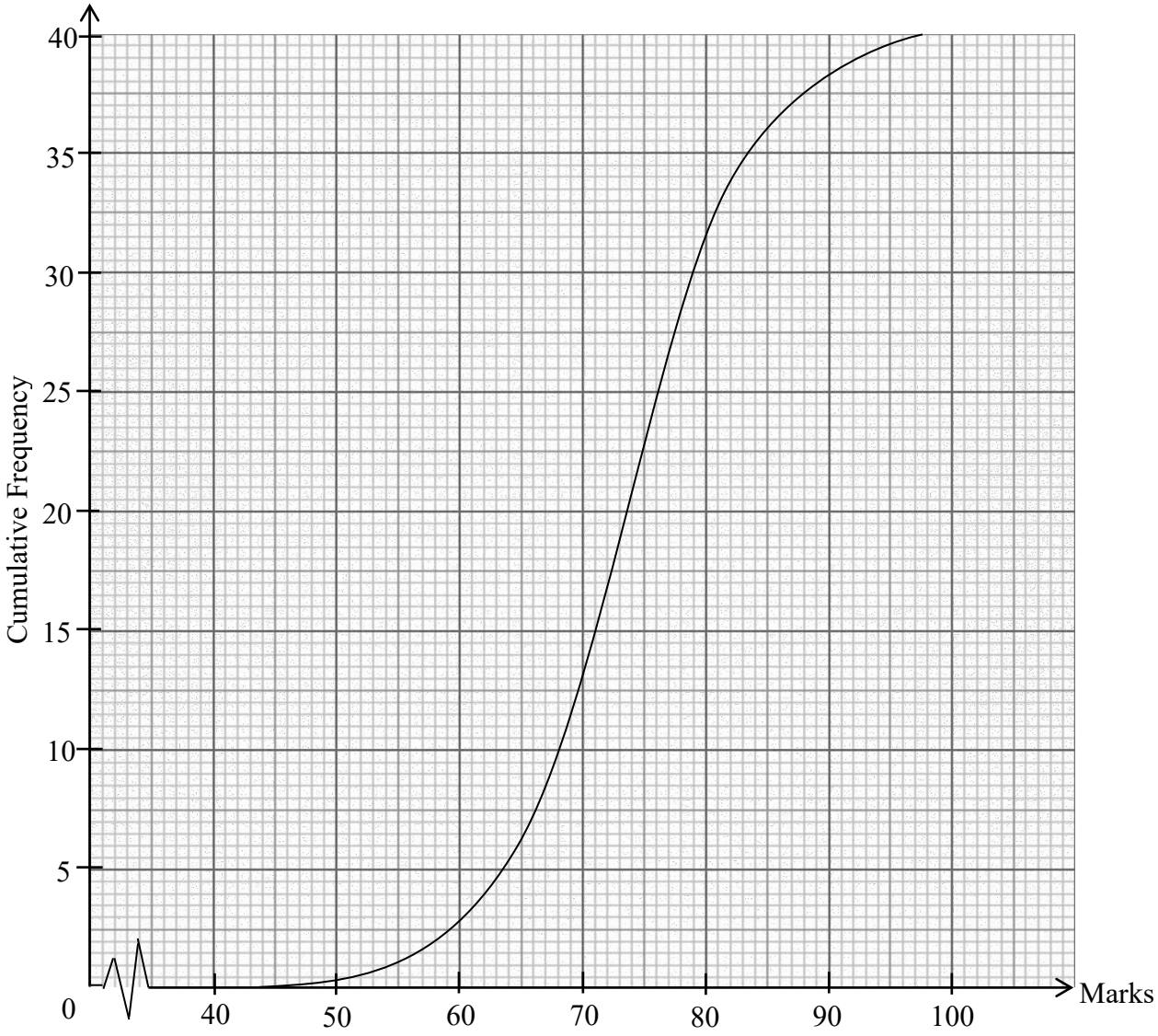
(c) Kenneth purchased a refrigerator on hire purchase.
The cash price of the refrigerator was $6000.
He paid $2000 down payment and subsequently made 24 monthly instalments.
The hire purchase scheme charges a simple interest of 2.5% per annum.
Find the amount of each monthly instalment.

Answer $…… [3]

(d) Sarah had lunch at a Japanese restaurant which offered a 10% discount.
The marked price of the meal was $75.
Given that there was a service charge of 10% and the GST was 8%, find the amount she had to pay.

Answer $…… [3]

3 A group of 40 students took a Mathematics test.
The cumulative frequency curve below shows the distribution of their marks.



(a) Use the graph to estimate

(i) the median mark,

Answer ……. [1]

(ii) the interquartile range,

Answer ……. [2]

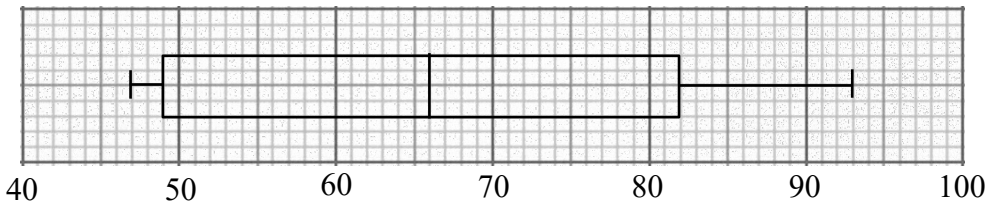
(iii) the passing mark, if 90% of students pass the test.

Answer …… [2]

(b) Two students were selected at random. Find the probability that one of the students obtained less than 55 marks and the other obtained more than 85 marks.

Answer …… [2]

(c) The same group of students took a Science test.
The box-and-whisker plot shows the distribution of their marks.



Make two comparisons between the performance of the students in the two tests.

……...

……...

……...

……...

……... [2]

4

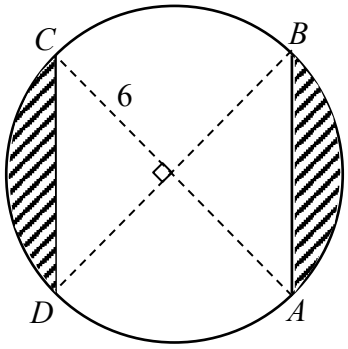


Diagram 1

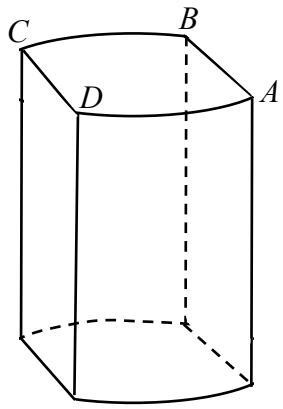


Diagram 2

In Diagram 1, *ABCD* is part of a circle of radius 6 cm, with the shaded parts removed from the circle.
DC = *AB* and *AC* intersects *BD* at right angles at the centre of the circle.

(a) Calculate the area of *ABCD*.
Leave your answer in terms of π.

Answer ……..……..……cm^2 [3]

(b) Diagram 2 shows a solid which has a uniform cross sectional area *ABCD*.
The vertical height of the solid is 25 cm.

(i) Calculate the total surface area of the solid.
Give your answer correct to nearest cm^2.

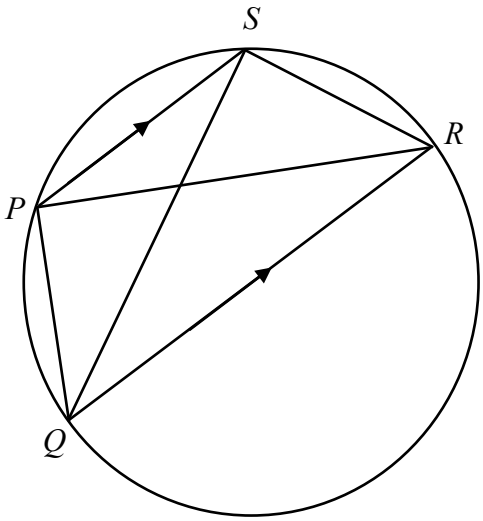
Answer ……..……..……cm^2 [3]

(ii) A glass ornament is modelled with the same dimensions as the solid.
1 cubic centimetre of glass has a mass of 0.0025 kg.

Calculate the mass of the glass ornament in grams.

Answer ……..……...…… g [3]

5



The diagram shows a circle $PQRS$ where QR is the diameter of the circle.
$PQ = SR$ and PS is parallel to QR.

(a) Show that triangles PSQ and SPR are congruent.

Answer [3]

It is given that the radius of the circle is 9 cm, the area of triangle $PSQ = 37.4$ cm^2 and $PS = 10$ cm.

(b) Calculate the area of triangle SQR.

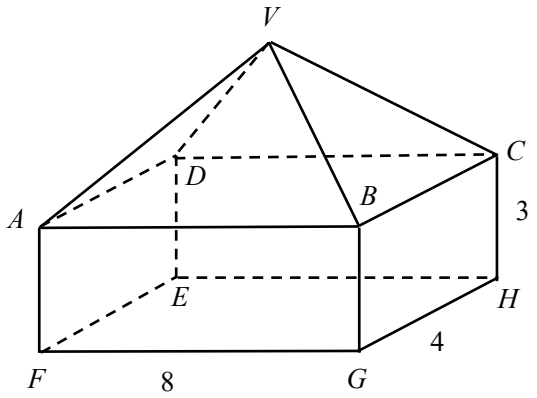
Answer ……cm^2 [3]

(c) T is a point such that $\angle PTS = 40°$.
Determine if point T lies on the circumference of the circle. Justify your answer.

……..……...

……..…… [3]

6 The diagram shows a greenhouse.
It consists of a right pyramid roof *VABCD* attached to a cuboid *ABCDEFGH*.
The dimensions of the cuboid are 8 m × 4 m × 3 m and angle *BVC* is 50°.



Calculate

(a) the length *VA,*

Answer ……..……...…… m [2]

(b) the vertical height of the pyramid *VABCD*,

Answer ……..……...…… m [2]

(c) the area of triangle *FDH*.

Answer ……..……...…… m^2 [4]

7 **(a)** S is a point (3, 0). $\overrightarrow{RS} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$ and $\overrightarrow{ST} = \begin{pmatrix} 9 \\ h \end{pmatrix}$.

(i) Find the gradient of RS.

Answer ……. [1]

(ii) Find the equation of the line RS.

Answer ……. [2]

(b) S, R and T are collinear.

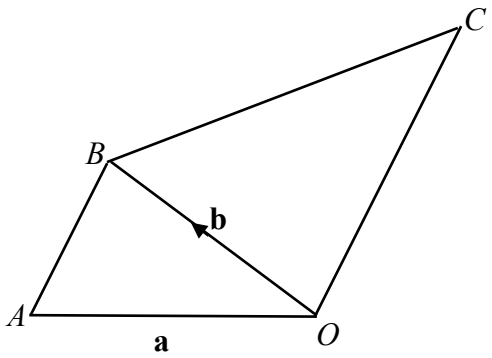
(i) Find the value of h.

Answer ……. [2]

(ii) Find the position vector of T.

Answer ……. [1]

(c)



$OABC$ is a trapezium with AB parallel to OC.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

$AB : OC = 3 : 5$.

BC is produced to a point T such that $BC = \frac{1}{4}BT$.

Express $\overrightarrow{TC}$, as simply as possible, in terms of **a** and **b**.

Answer $\overrightarrow{TC} =$…..……. [4]

8 **(a)** $\xi = \{\text{integers } x : 1 \le x \le 12\}$
$A = \{ 2, 3, 5, 6, 10 \}$
$A \cap B = \{ 3, 5, 6 \}$
$A \cup B = \{ 1, 2, 3, 4, 5, 6, 10, 12 \}$

(i) Illustrate the above information in a Venn diagram.

[2]

(ii) List the elements in $(A \cup B)'$.

Answer ……. [1]

(iii) A number, k, is chosen at random from the set $A \cap B$.
Find the probability that $k \in A$.

Answer ……. [1]

(b) At a carnival, one of the games requires contestants to throw a ball towards a target on the wall. The target has three concentric circles with diameters as shown in **Diagram I**.

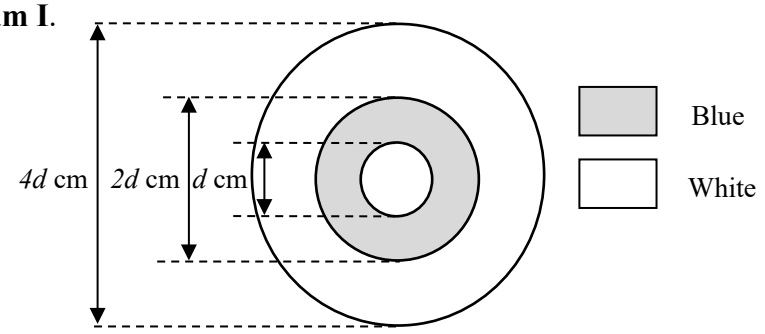


Diagram I

(i) Find the ratio of the area of the smallest circle to area of the largest circle.

Answer …… [2]

If all the balls thrown land within the target, calculate the probability that a ball will hit

(ii) the blue region,

Answer …… [2]

(iii) the white region.

Answer …… [2]

9 Due to the recent Russia-Ukraine war, petrol prices have been on the rise.
The table below shows the difference in prices of two common types of petrol, namely 95-Octane and 98-Octane.
Jack's car has a fuel tank capacity of 55 litres and he uses 98-Octane for his vehicle.

Type of Petrol	**95-Octane**	**98-Octane**
Price before war	\$2.91 per litre	\$3.48 per litre
Current price	\$3.23 per litre	\$3.72 per litre

(a) Calculate the percentage increase in price for 98-Octane.

Answer ……..……...…..……% [2]

(b) Calculate the amount of money Jack can save for each full tank if he switches to using 95-Octane.

Answer \$……...…….…….……. [2]

Jack wants to purchase a new car and he is deciding between a petrol car or an electric car. A petrol car relies on an internal combustion engine that is powered by petrol while an electric car is powered by electricity stored in rechargeable batteries.
Jack is choosing between the Audi Q5 (petrol car) and the Audi E-tron (electric car).
Both cars belong to the same model but are powered differently.
Table I shows some of the specifications of the two cars while **Table II** show the cost of petrol and electric charge

	Audi Q5		**Audi E-tron**
Price	\$309,813	**Price**	\$380,247
Rebates	None	**Rebates**	\$45,000
Fuel Consumption	6.5 litres per 100 km	**Electric Power Consumption**	21.6 kilowatt hour (kWh) per 100 km
Maintenance & Servicing	Estimate \$1558 per year	**Maintenance & Servicing**	Estimate \$600 per year

Table I: Specifications of Audi Q5 and Audi E-tron

	Petrol
95-Octane	\$3.23 per litre
98-Octane	\$3.72 per litre

	Electric Charge
Slow Charge	\$0.4153 per kWh
Fast Charge	\$0.4746 per kWh

Table II: Cost of Petrol and Electric Charge

Jack is planning to drive his new vehicle for **ten years** before he changes his car again.
It is given that there are 52 weeks in a year and the occurrence of leap years can be ignored.
Table III shows the average distance he travels on weekdays and weekends.

	Average distance travelled
Weekdays	46 kilometres per day
Weekends	55 kilometres per day

Table III: Distance travelled

Jack is at a dilemma over which model to choose.
Suggest which car Jack should purchase.
Justify the decision you make and show your calculations clearly.

[7]

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