CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 2
JC1 Promotional Examination

CANDIDATE NAME	

CLASS		INDEX NUMBER				

MATHEMATICS — 9758/01

Paper 1

03 Oct 2022
3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approving graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Total**
Marks													
Total	5	6	6	7	8	8	8	9	9	10	12	12	**100**

This document consists of 25 printed pages and 1 blank page.

1 **(i)** Sketch, on the same diagram, the graphs of $y=|2x+3|$ and $y=-2x^2-8x-3$.

(You are not required to label any axial intercepts and stationary points.) [2]

(ii) Solve exactly the inequality $|2x+3|>-2x^2-8x-3$. [3]

2 The second, fifth and tenth terms of an arithmetic series with non-zero common difference are the first three terms of a geometric series respectively.

(i) Find the common ratio. [3]

It is further given that the first terms of the arithmetic and geometric series are 7 and 9 respectively.

(ii) Find the least value of n for which the nth term of the arithmetic series is smaller than the nth term of the geometric series by at least 1000. [3]

3 The curve C defined by $y=\dfrac{ax+1}{bx-2}$ passes through $(3, 13)$ and has vertical asymptote $x=2$.

(i) Find the values of a and b. [2]

(ii) Describe a sequence of three transformations that transform the graph of $y=\dfrac{1}{x}$ onto the graph of C. [4]

4 It is known that the nth term of a sequence is given by

$$u_n = p\left(4^{-n}\right)+qn+r\ ,$$

where p, q and r are constants.

It is given that $u_1=6$, $u_2=0$ and $u_3=-\dfrac{15}{4}$.

(i) Find p, q and r. [3]

(ii) Show $\sum_{r=1}^{n} u_r = A+B\left(4^{-n}\right)+Cn+Dn^2$ where A, B, C and D are constants to be determined. [4]

5 **(a)** Differentiate the following expressions with respect to x.

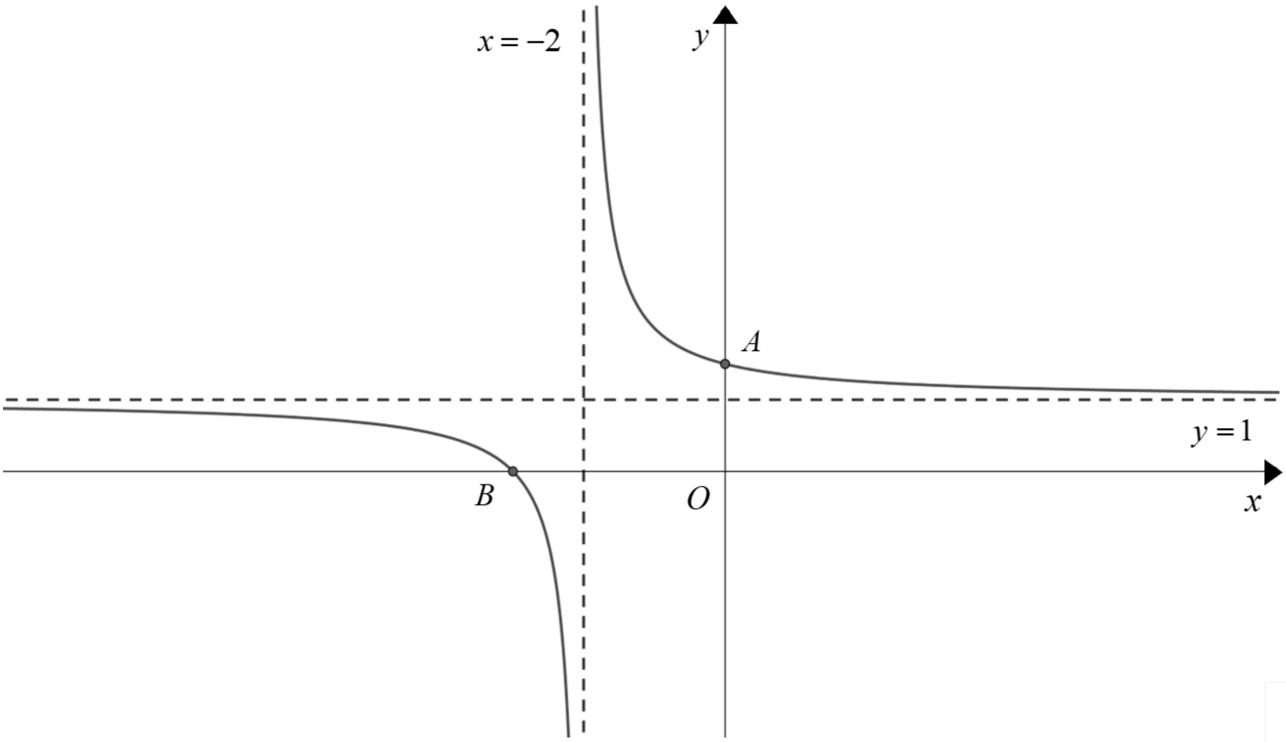
(i) $\dfrac{\ln x}{2+3x}$ [2]

(ii) $\sin^{-1}\left(x^3+2x\right)$ [2]

(b) The curve C has equation $2xy-y^2=(1+y)^2$.

Express $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y and show that there are no tangents to C which are parallel to the x-axis. [4]

6 The diagram below shows the graph of $y=\mathrm{f}(x)$. The curve cuts the axes at $A(0,1.5)$ and $B(-3,0)$. The asymptotes of the curve are $x=-2$ and $y=1$.



Sketch, on separate diagrams, the graphs of

(i) $y=-\mathrm{f}(1+x)$, [3]

(ii) $y=\dfrac{1}{\mathrm{f}(x)}$, [3]

(iii) $y=\mathrm{f}'(x)$, [2]

indicating clearly the asymptotes, axial intercepts and the points corresponding to A and B where possible.

7 **(i)** Sketch the graph of $(x-2)^2+4(y+2)^2=16$, stating the coordinates of all the vertices. [3]

(ii) On the same diagram, sketch the graph of $y=\dfrac{x^2-2x+6}{x+1}$, stating the equations of any asymptotes, the coordinates of turning points and the points of intersection with the axes. [3]

(iii) Find the range of values of m, where $m>0$, such that $(x-2)^2+4m^2\left(\dfrac{x^2-2x+6}{x+1}+2\right)^2=16$ has no real solutions. [2]

8 **(i)** Show that $\dfrac{1}{r!}-\dfrac{1}{(r+1)!}=\dfrac{r}{(r+1)!}$. [1]

(ii) Hence find $\displaystyle\sum_{r=1}^{N}\frac{r}{(r+1)!}$ in terms of N. [3]

(iii) Explain why the series in part **(ii)** is convergent, and hence state the value of $\displaystyle\sum_{r=1}^{\infty}\frac{r}{(r+1)!}$. [2]

(iv) By using the result in part **(ii)**, find $\displaystyle\sum_{r=2}^{N}\frac{r+2}{(r+3)!}$. [3]

9 Relative to the origin *O*, the position vectors of points *A*, *B* and *C* are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, where $\mathbf{b}$ is a unit vector. It is given that $\mathbf{b}$ and $\mathbf{b}-\mathbf{a}$ are perpendicular and *C* lies on *AB* such that $AC:CB=3:1$.

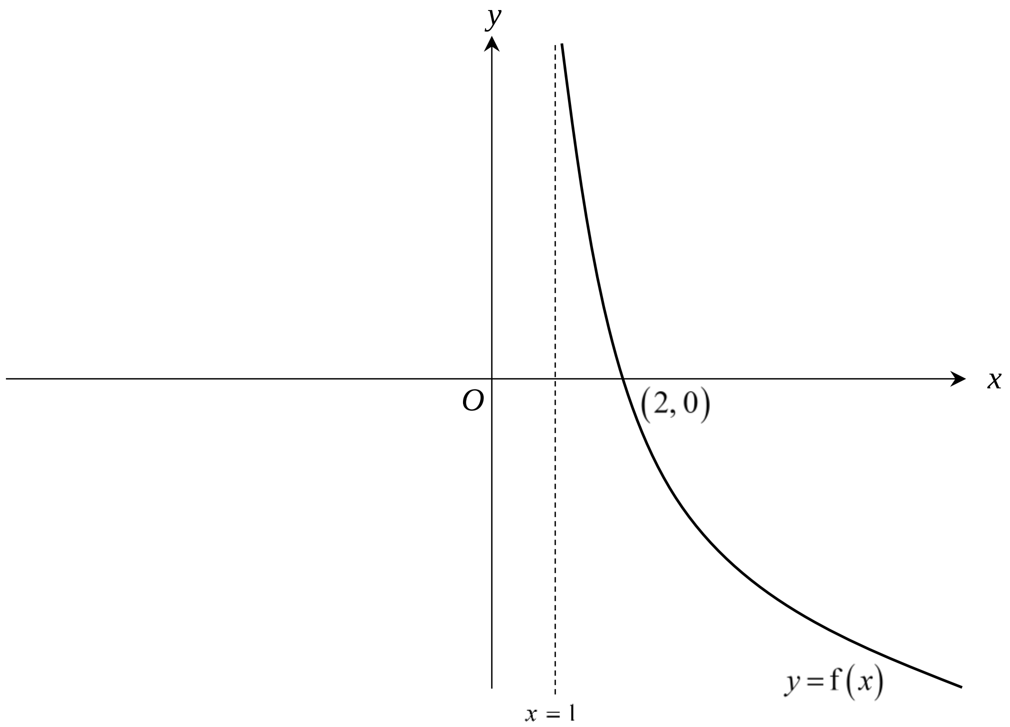
(i) Show that $|\mathbf{a}|=\sec\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$. [3]

By expressing $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$,

(ii) find the value of $|\mathbf{c}\cdot\mathbf{b}|$ and state the geometrical interpretation of $|\mathbf{c}\cdot\mathbf{b}|$, [4]

(iii) find the value of $\dfrac{|\mathbf{b}\times\mathbf{c}|}{|\mathbf{a}\times\mathbf{b}|}$. [2]

10 **(a)** The diagram below shows the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$, $x > 1$. The curve $y = \mathrm{f}(x)$ has an asymptote $x = 1$ and passes through $(2, 0)$. Sketch on this same diagram the graph of $y = \mathrm{f}^{-1}(x)$, showing clearly the geometrical relationship between the two graphs. [2]



(b) Functions g and h are defined by

$$\mathrm{g} : x \mapsto \frac{1}{1 - x^2}, \qquad x \in \mathbb{R}, x > 1,$$
$$\mathrm{h} : x \mapsto 1 - 2x, \qquad x \in \mathbb{R}.$$

(i) Explain why the composite function gh does not exist. [2]

(ii) Find $\mathrm{hg}(x)$. [1]

(iii) Find the range of $\mathrm{hg}(x)$. [2]

(iv) By using the result in part **(ii)**, or otherwise find $(\mathrm{hg})^{-1}(4)$. [3]

11 In a virtual game, a player controls a destroyer with the aim to destroy targets. Points (x, y, z) are defined relative to the origin, and the destroyer is assumed to be moving in a straight line between any 2 points. Initially, the destroyer is placed at point A with coordinates $(2, 2, 3)$. The first target is at point B on the plane p_1 with cartesian equation $x - y + 2z = -6$. It is also known that point B is the closest to point A.

(i) Find the coordinates of point B and hence find the exact distance between the destroyer and the first target. [5]

The destroyer moves towards the first target and after destroying the first target upon reaching it, the destroyer changes its path and moves from point B in the direction parallel to $\mathbf{i} + \mathbf{j} + \mathbf{k}$ towards the second target at point C with coordinates (p, q, r), where p, q and r are positive constants.

(ii) Find the acute angle that the path BC makes with p_1. [2]

It is known that the second target lies on a plane p_2 that is parallel to p_1, where the distance between p_1 and p_2 is $2\sqrt{6}$ units.

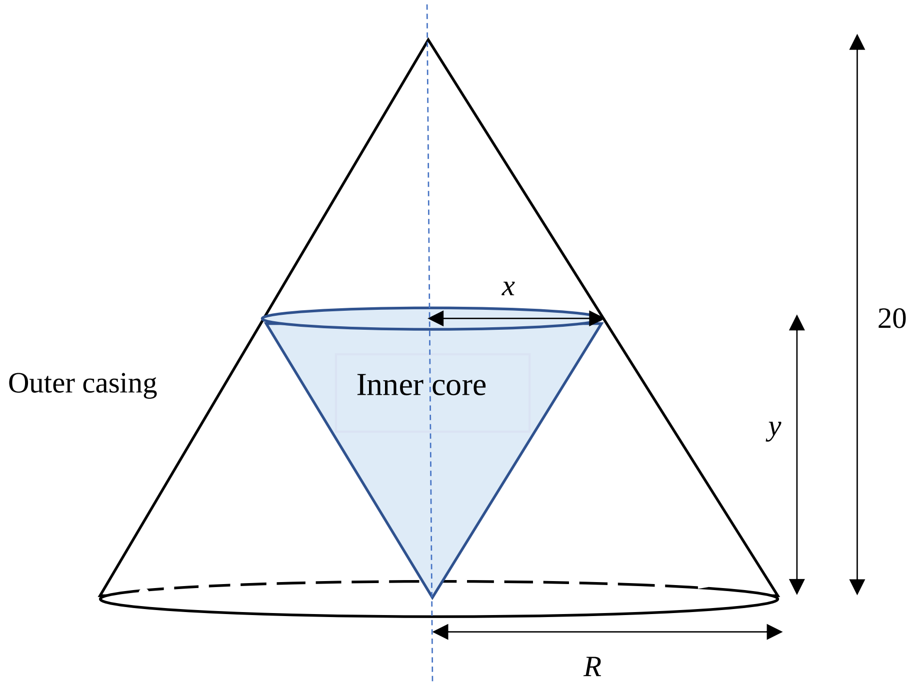
(iii) Find the coordinates of point C. [3]

The destroyer moves towards the second target and destroy the target upon reaching it. The final target is at point D with coordinates $(18, 22, 17)$.

(iv) Determine if the destroyer needs to change its path to reach the final target. Justify your answer. [2]

12 In a popular fantasy online game Ginseng Impact, there is a powerful in-game item known as the “Ring of Knowledge” which the players can buy or “craft” for their game characters. “Crafting” is a game mechanic where the player gets to make a particular item by gathering the required materials and deciding on some of its visual details.

Nathan, an avid player, decides to craft the “Ring of Knowledge”, which is an enchanted jewel set on ring that is worn on the finger. The enchanted jewel itself consists of two components: a conical outer casing, and a conical inner core, as shown in the diagram.



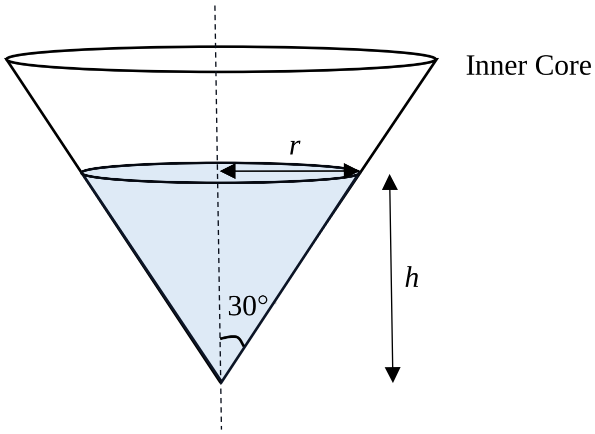
The outer casing has fixed height of 20 units, and fixed base radius of R units. Nathan is able to decide the base radius x, and the height y, of the inner core during his crafting. The inner core acts as a store of destructive power that can be unleashed on enemies, hence Nathan wants the volume of the inner core, V to be as large as possible.

[The volume of a cone is $\frac{1}{3} \times \text{base area} \times \text{height}$.]

(a) **(i)** Show that $V = \frac{20\pi}{3}x^2 - \frac{20\pi}{3R}x^3$. [2]

(ii) Find the largest possible volume of the inner core that Nathan can craft, giving your answer in terms of R. [6]

Focusing on the inner core, Nathan decides to make the values of x and y such that the slant edge of the core makes an angle of $30°$ with the centre axis, as shown in the diagram.



When the Ring is in use, the core continually charges up and the accumulated energy slowly fills up the space within the core, starting from the bottom. The height and base radius of this accumulated energy at time t are given as h and r respectively.

If the rate of energy charge is 8 unit^3 per second,

(b) (i) show that $r = \dfrac{h}{\sqrt{3}}$, [1]

(ii) find the rate of change of h, when it reaches the height of 5 units. [3]

- END -