CHIJ ST. THERESA'S CONVENT
PRELIMINARY EXAMINATION 2023
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS

4049/2

Paper 2

25 Aug 2023
2 hours 15 minutes

Candidates answer on the Question Paper as well as on the graph paper provided.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 Given that f(x) = $6x^3+7x^2-x-2$, show that x + 1 is a factor of f(x) and hence factorise f(x) completely. [3]

2 The equation of a circle is $(x-3)^2+(y+4)^2=26$.

Determine if the origin O lies inside or outside the circle. [3]

3 Solve the equation $3e^{x}+2 = e^{-x}$, giving your answer(s) correct to 3 significant figures. [5]

4 Given that $y = \dfrac{16}{\sqrt[3]{5-3x}}$, find

(i) the value(s) of x for which $\dfrac{dy}{dx} = 1$, [4]

(ii) the value of $\int_0^1 y \, dx$, giving your answer correct to 3 significant figures. [4]

5(a) **(i)** Using the substitution $z = 1+x$, write down all the terms in the expansion of $[1+(1+x)]^6$, leaving each term in the form $k(1+x)^p$ where k and p are integers. [2]

(ii) **Hence** show that the remainder when $(2+x)^6$ is divided by $1+x$ is 1. [1]

(b) Given that the coefficient of x^2 in the expansion of $(1+2x)(1-px)^5$ is 20, find the two possible values of the constant p. [5]

6 At time t seconds, the velocity, v m/s, of a metal ball falling through a very thick liquid is given by $v = 8(1 - e^{-t/3})$.

(i) State the initial velocity of the ball. [1]

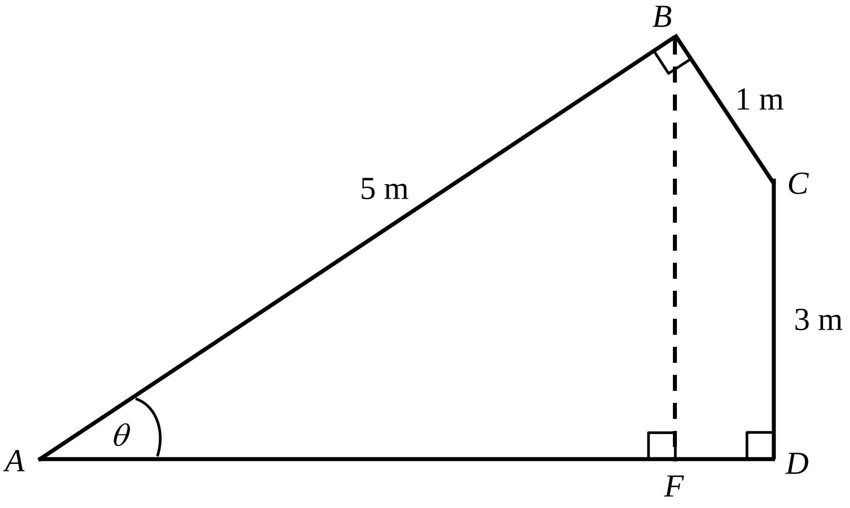
The depth of the ball from the surface of the liquid at time t seconds is s metres. The ball is initially on the surface of the liquid.

(ii) Find an expression for the depth of the ball in terms of t. [4]

(iii) Show that the ball is accelerating throughout its motion. [3]

(iv) Deduce the motion of the ball for large values of t. [1]

7



The diagram shows a quadrilateral *ABCD* with angles *ABC* and *ADC* both equal to $90°$. Angle *BAD* = θ. The lengths of *AB*, *BC* and *CD* are 5 m, 1 m and 3 m respectively. The point *F* on *AD* is such that *BF* is perpendicular to *AD*.

(i) Show that the length of *BF* is $\cos\theta + 3$. [2]

(ii) Hence show that $5\sin\theta - \cos\theta = 3$. [2]

(iii) By expressing $5\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, find the value of θ. [4]

8 The equation of a curve C_1 is $y = 6x - 3x^2$.

(i) Show that the coordinates of the maximum point of the curve C_1 are (1, 3). [2]

(ii) Using your answer in part **(i)**, state the range of values of k for which the curve C_1 lies entirely below the line $y = k$. [1]

The curve C_2 is the reflection of the curve C_1 in the line $y = k$, where k takes one of the values found in part **(ii)**.

(iii) Deduce the equation of the curve C_2, giving your answer in terms of k. [3]

Another line $y = x + a$ is a tangent to the curve C_1 at the point P.

(iv) Find the value of the constant a. [3]

(v) Find the coordinates of P. [2]

9 The equation of a circle is $x^2 + y^2 - 4x - 12y + 36 = 0$. The equation of the line L is $y = kx$, where k is a constant.

(i) Find the radius of the circle and the coordinates of its centre. [4]

(ii) State the equations of the horizontal tangents to the circle. [2]

(iii) Find the range of values of k for which L does not intersect the circle. [4]

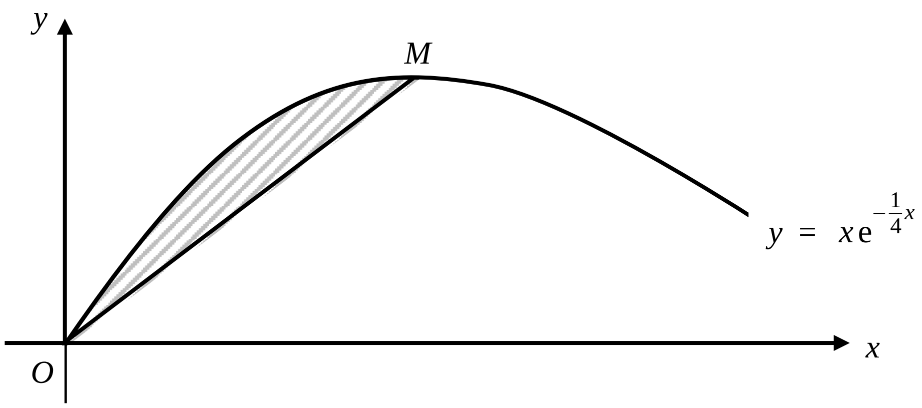
It is given that $k = 1$. The point on the line L that is closest to the circle is P.

(iv) Find the coordinates of P. [3]

10 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left[x\mathrm{e}^{-\frac{1}{4}x}\right] = \left(1-\frac{1}{4}x\right)\mathrm{e}^{-\frac{1}{4}x}$. [2]

(ii) Hence show that $\int x\mathrm{e}^{-\frac{1}{4}x}\,\mathrm{d}x = -16\mathrm{e}^{-\frac{1}{4}x} - 4x\mathrm{e}^{-\frac{1}{4}x} + C$ where C is a constant. [3]

(iii)



The diagram shows part of the curve $y = xe^{-\frac{1}{4}x}$. The point M is the maximum point of the curve and OM is a straight line.

Show that the area of the shaded region is $\left(16 - \frac{40}{e}\right)$ units^2. [7]

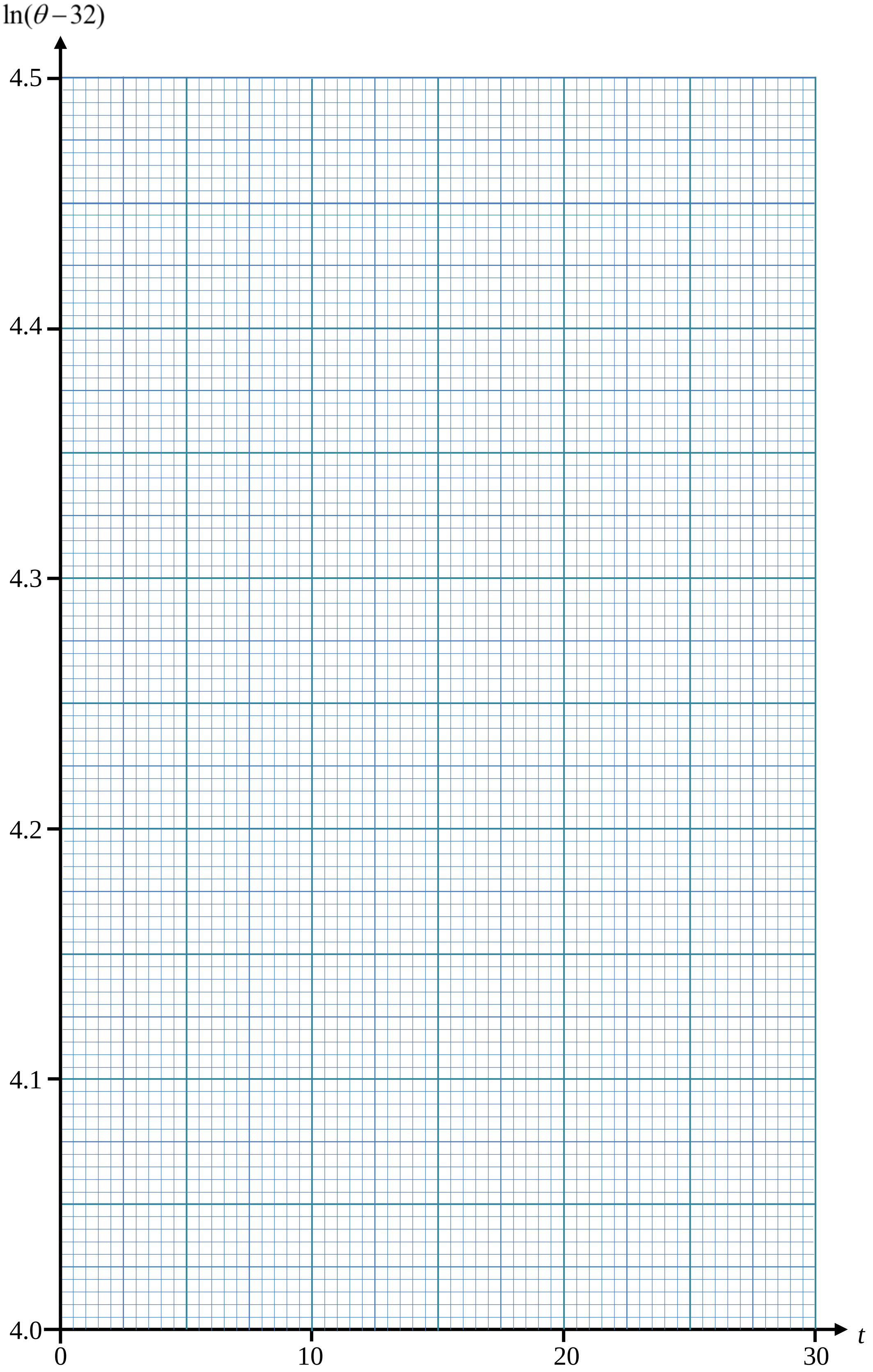
11 The temperature, $\theta °C$, of a liquid placed in a container can be modelled by an equation of the form $\theta = 32 + ae^{-bt}$, where *a* and *b* are constants and *t* is the time in minutes that the liquid has been left in the container. The table below records the value of θ for various values of *t*.

t minutes	0	10	20	30
$\theta °C$	120.0	107.6	97.7	88.0

(i) On the grid given, plot $\ln(\theta - 32)$ against *t* and draw a straight line graph. [3]

(ii) Use the graph to estimate the value of each of the constants *a* and *b*. [5]

(v) Use the graph to estimate when the temperature of the liquid drops to $100 °C$. [2]



~ ~ ~ End of Paper ~ ~ ~