EUNOIA JUNIOR COLLEGE

JC1 Promotional Examination 2024

General Certificate of Education Advanced Level

Higher 2

CANDIDATE NAME	

CIVICS GROUP		INDEX NO.	

MATHEMATICS

9758/01

Paper 1

07 October 2024

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages and **1** blank page.

1 A small furniture factory produces standardised stools, chairs and tables using both metal and plastic as raw materials. The table below shows the amount of each raw material required to make one unit of each type of furniture.

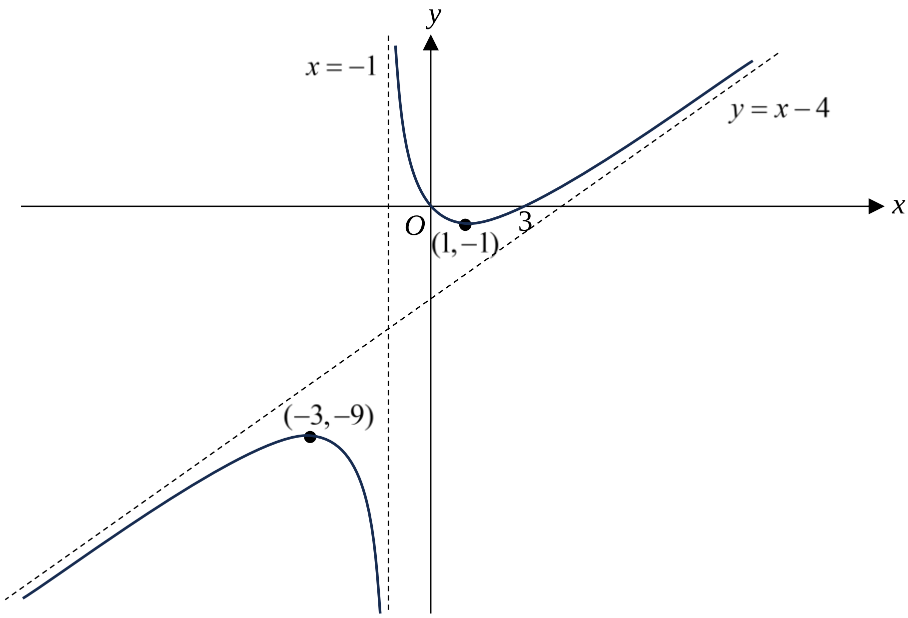
Furniture Item	Amount of raw material needed (in kg)	
	Metal	Plastic
Stool	1	2
Chair	4	2
Table	10	4

Given 68 kg of metal and 32 kg of plastic, determine the number of each type of furniture that could be produced such that all three types of furniture are made, and all the raw materials are fully utilised. [4]

2 Solve the inequality

$$3-\left|1-x^2\right| \geq \frac{x+5}{2} \ .$$ [3]

3 The graph of $y=\mathrm{f}\left(x\right)$ is given below. It has one vertical asymptote $x=-1$ and one oblique asymptote $y=x-4$. It cuts the x-axis at $x=0$ and $x=3$. It has turning points at $\left(-3,-9\right)$ and $\left(1,-1\right)$.



On separate diagrams, sketch the graphs of the following, stating the equations of any asymptotes, and indicating any axial intercepts and turning points.

(a) $y=\dfrac{1}{\mathrm{f}(x)}$, [3]

(b) $y=\mathrm{f}'(x)$. [3]

4 In a triangle ABC, $AB = 5$ cm and $AC = 4$ cm. If angle BAC is decreasing at a constant rate of 0.2 radians per second, find the rate of change of the length BC at the instant where $\angle BAC = \frac{\pi}{3}$ radians, giving your answer in centimetres per second to 3 significant figures. [5]

5 **a** and **b** are vectors such that $|\mathbf{a}| = \sqrt{2}$ and $|\mathbf{b}| = \sqrt{6}$. The angle between **a** and **b** is $\frac{5\pi}{6}$.

(a) Find the exact length of projection of **a** onto **b**. [2]

(b) By considering $(3\mathbf{a}+2\mathbf{b}).(3\mathbf{a}+2\mathbf{b})$, find the exact value of $|3\mathbf{a}+2\mathbf{b}|$. [4]

6 **(a)** Write down the derivative of $\sin^n \theta$ with respect to θ, where n is a constant and $n \geq 1$. [1]

(b) Using the substitution $x = 2\cos^2\theta$ for $0 \leq \theta \leq \frac{\pi}{2}$, show that

$$\int x\sqrt{1-\frac{x}{2}}\,\mathrm{d}x = 8\int\left(\cos\theta\sin^4\theta - \cos\theta\sin^2\theta\right)\mathrm{d}\theta.$$ [4]

(c) Hence find $\int x\sqrt{1-\frac{x}{2}}\,\mathrm{d}x$. [3]

7 **(a)** The terms $u_1, u_2, u_3, \ldots$ form a geometric sequence with first term a and common ratio r, where $a \neq 0$. It is given that the sum of the first 10 terms is 33 times the sum of the first 5 terms.

(i) Explain why r cannot be equal to 1. [1]

(ii) Show that $r = 2$. [2]

(iii) If u_7 is the first term in the sequence greater than 11, find exactly the range of possible values of a. [2]

(b) Given that $\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)} = \frac{1}{2} - \frac{1}{4n+2}$, find $\sum_{r=4}^{n}\frac{1}{(2r+3)(2r+5)}$ in terms of n. [3]

8 It is given that $y = \tan\left[\ln(1+2x)\right]$.

(a) Show that $(1+2x)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\left(1+y^2\right)$. [2]

(b) By differentiating the result in part **(a)**, find the Maclaurin expansion of $\tan\left[\ln(1+2x)\right]$ up to and including the term in x^2. [3]

(c) Hence, find the Maclaurin expansion of $\mathrm{e}^{\frac{1}{2}y}$, up to and including the term in x^2. [2]

(d) Using your answer in part **(c)**, evaluate $\lim\limits_{x \to 0} \dfrac{\mathrm{e}^{\frac{1}{2}y} - 1}{3x}$. [2]

9 A curve C has parametric equations

$$x = t^2 - 1, \qquad y = \frac{2}{3}t^3 - 2at \qquad \text{for } t \in \mathbb{R}$$

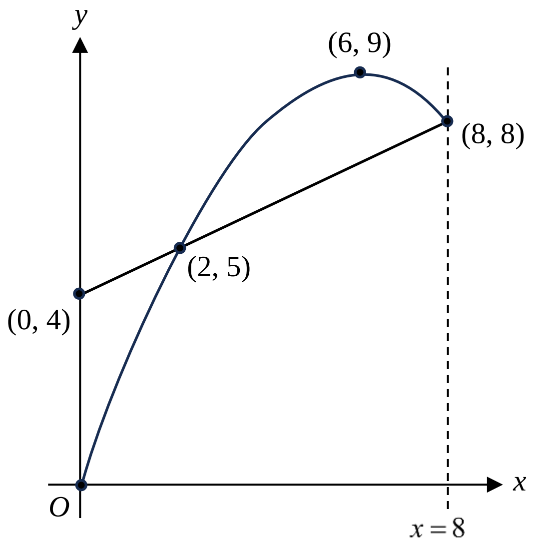
where a is a positive constant. P is a point on C where $t = a$.

(a) Find the equation of the tangent to the curve at P. [3]

(b) Q is another point on C. The tangent to the curve at Q is parallel to the tangent to the curve at P. Find, in terms of a, the coordinates of Q. [4]

(c) Find the range of values of a for which C intersects the positive x-axis. [2]

10 The figure shows a sketch of the graphs $y = \frac{1}{4}x(12 - x)$ and $y = \frac{1}{2}x + 4$, for $0 \le x \le 8$.



The function f is defined as

$$\mathrm{f} : x \mapsto \begin{cases} \frac{1}{4}x(12 - x) & \text{for } 0 \le x \le c, \\ \frac{1}{2}x + 4 & \text{for } c < x \le 8, \end{cases}$$

where the value of c satisfies $0 < c < 8$.

(a) State the domain of f. [1]

(b) **(i)** Explain why the inverse function f^{-1} does not exist when $c = 6$. [2]

(ii) Find the set of values of c for which the function f^{-1} exists. [1]

(c) **(i)** State the range of f when $c = 6$. [1]

(ii) Find the range of f when $c = 1$. [2]

(iii) Find the set of values of c for which the composite function f^2 exists. [2]

11 A sequence $u_1, u_2, u_3, ...$ is such that $u_1 = 2$, $u_{n+1} = ku_n - 3$ for all $n \geq 1$, where k is a constant.

(a) Let $k = \frac{1}{5}$.

(i) Write down the values of u_2, u_3 and u_4. [2]

(ii) Given that the sequence converges to l, find the value of l. [1]

(b) Let $k = 1$.

(i) Given that the sequence $u_1, u_2, u_3, ...$ is an arithmetic progression, find the value of $\sum_{r=1}^{n} u_{r+4}$. [3]

(ii) Show that the sequence $u_7, u_{10}, u_{13}, u_{16}, ..., u_{3n+4}, ...$ is also an arithmetic progression. [2]

(iii) Hence, find the value of $\sum_{r=1}^{100} u_{3r+4}$. [2]

12 The curve C has equation

$$y = x - 2 + \frac{3k(x-2)}{x^2 - 2k}, \text{ where } k > 0,\ k \neq 2.$$

(a) Without the use of a calculator, find the coordinates of any points of intersection of C with the axes. [3]

(b) Write down the equations of the asymptotes of C in terms of k. [2]

It is now given that $k = 1$.

(c) Show that the x-coordinates of the turning points of C satisfy the equation

$$x^4 - 7x^2 + 12x - 2 = 0.$$

Hence find these x-coordinates. [3]

(d) Sketch C, labelling all essential features. [3]

13 **v** and **n** are unit vectors in 3-dimensional space, with $\mathbf{v.n} \neq 0$.

Let P be a plane with unit normal **n**. The reflection of **v** in plane P, **w**, is given by the formula

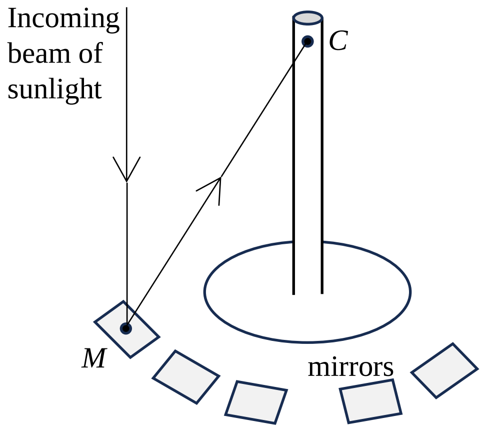
$$\mathbf{w} = \mathbf{v} - 2(\mathbf{v.n})\mathbf{n}. \qquad (*)$$

(a) Calculate the vector **w** when $\mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{n} = \frac{1}{5}\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$. [2]

When the unit vectors **v** and **n** vary, the formula (*) will always result in a unit vector for **w**.

(b) Verify that the vector **w** calculated in part **(a)** is a unit vector. [1]

Before photovoltaic technology allowed for cost-effective direct conversion of sunlight to electrical energy in solar panels, solar power plants relied on arrays of mirrors to concentrate a large area of sunlight into a receiver. In one such plant (see diagram), the receiver is modelled as a single point C with coordinates $(0, 0, 10)$, with mirrors surrounding it. One particular mirror has centre M with coordinates $(-10, -5, 0)$, and its surface is modelled as part of a plane P passing through M with unit normal **n**.



(c) Write down the unit vector **w** in the direction of $\overrightarrow{MC}$. [1]

At midday, incoming beams of sunlight can be modelled as travelling along the direction $\mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$. The mirror is angled so that a beam of sunlight shining on M is reflected to pass through the receiver C.

(d) Using the formula (*) or otherwise, derive that a unit normal for plane P is $\mathbf{n} = \frac{1}{\sqrt{30}}\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$. [3]

(e) Hence find a cartesian equation of plane P. [2]

(f) Find the acute angle that plane P makes with the xy-plane. [2]

(g) State, in the context of the question, an interpretation of the condition $\mathbf{v.n} \neq 0$. [1]