Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

18 August 2023 Friday 2 hours 15 min

2023 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use														
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

Category	Accuracy	Units	Symbols	Others
Question No.				

TOTAL MARKS
90

Setter: Mr Gregory Quek
Vetter: Mr Tan Lip Sing

This question paper consists of **17** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The line $y = 2x + 15$ intersects the curve $y = x^2 + 6x + 3$ at points A and B. Find the value of p for which the distance AB can be expressed as $p\sqrt{5}$. [5]

2 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 12\mathrm{e}^{2x} + \mathrm{e}^{-x}$. The curve intersects the y-axis at $P(0, 5)$ and the tangent to the curve at P is parallel to $y = 4x + 3$. Find the equation of the curve. [6]

3 A function is defined by $\mathrm{f}(x) = x^2 + 2kx + 2k + 3$ for all real values of x, where k is a constant.

(a) Find the discriminant of $\mathrm{f}(x)$ in terms of k. [2]

(b) Show that the discriminant of $\mathrm{f}(x)$ in **part (a)** can be expressed in the form $4(k-a)^2 - b$, where a and b are integers. [2]

(c) Find the range of values of k for which $\mathrm{f}(x) = 0$ has no real roots. [3]

4 It is given that $\mathrm{f}(x) = 2x^3 - 5x^2 - 4x + 12$.

(a) Show that $2x+3$ is a factor of $\mathrm{f}(x)$. [2]

(b) Factorise $\mathrm{f}(x)$ completely. [2]

(c) Hence find the roots of the equation $2\left(2^{3y}\right) - 5\left(2^{2y}\right) - 4\left(2^{y}\right) + 12 = 0$. [3]

5 **(a)** Using long division, show that $\dfrac{x^3 - 2x^2 + 5x - 10}{x^2 + 5} = x - 2.$ [2]

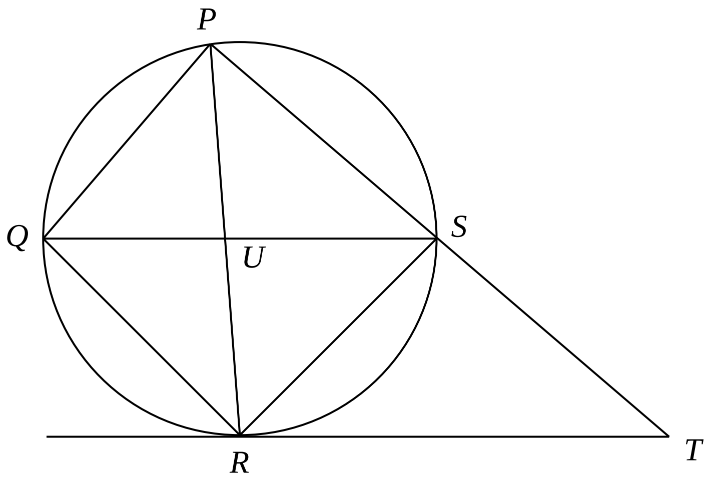
(b) Hence, by first expressing the denominator as a product of two factors,

express $\dfrac{2x^2 + 1}{x^3 - 2x^2 + 5x - 10}$ in partial fractions. [5]

6 **(a)** Find the first 3 terms, in ascending powers of x, of the binomial expansion of $\left(2+\frac{ax}{4}\right)^8$, where a is a non-zero constant. Give each term in its simplest form. [2]

(b) Given that the coefficient of x^2 is –320 in the expansion of $(3-x)^2\left(2+\frac{ax}{4}\right)^8$, find the possible value(s) of a. [4]

7



The diagram shows a quadrilateral $PQRS$ whose vertices lie on the circumference of a circle.
The diagonals PR and QS intersect at U. The tangent at R meets PS produced at T.
If $QR = RS$, prove that

(a) $QS \parallel RT$, [3]

(b) triangle PQR is similar to triangle QUR. [3]

8 **(a)** The equation of a curve is $y = \ln\left(xe^{-3x}\right)$.

The normal to the curve at the point P has a gradient of $\frac{1}{2}$. Find the coordinates of P. [4]

(b) The normal to the curve at P meets the x-axis at Q.
Find the area of triangle OQP, where O is the origin. [3]

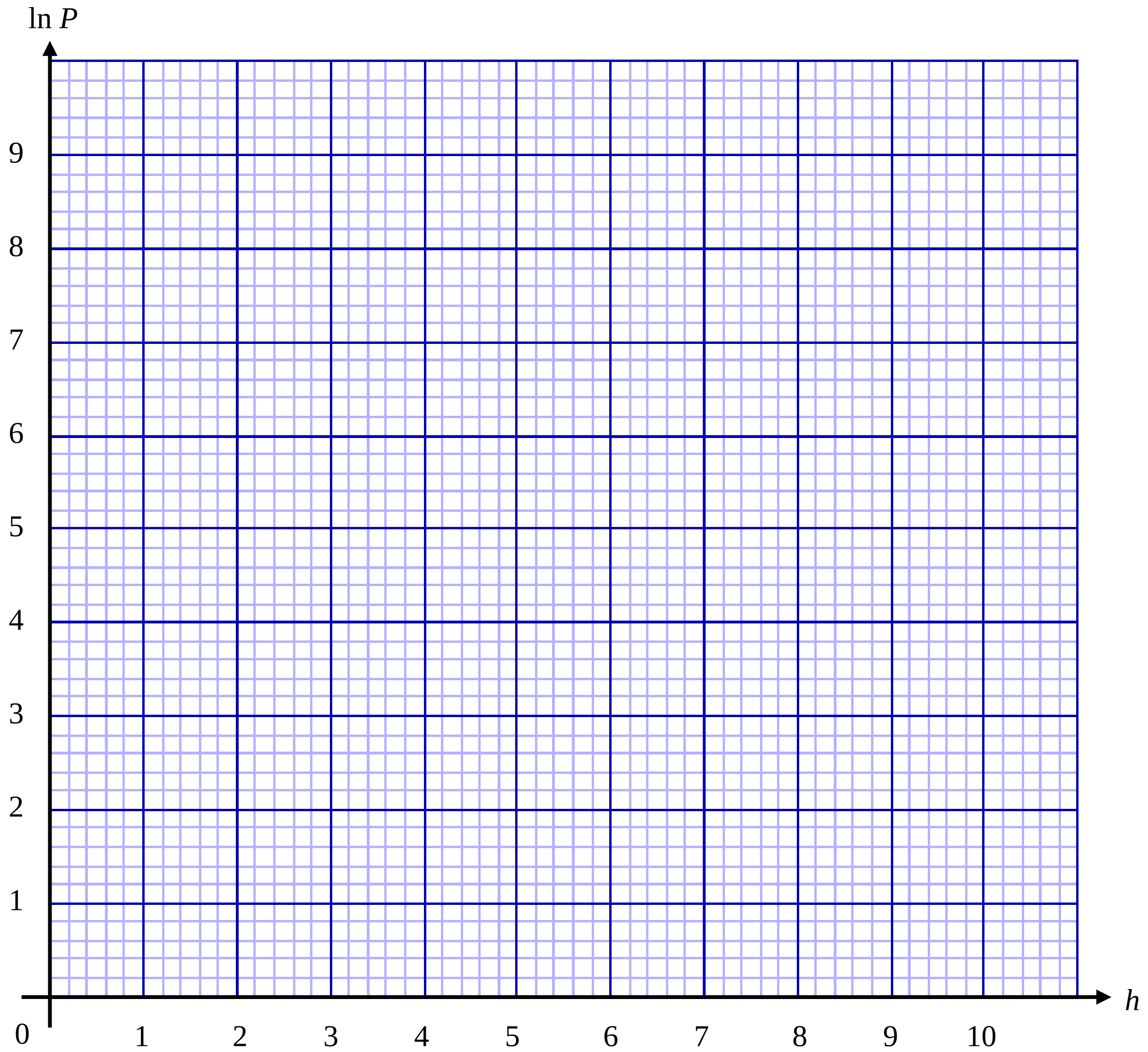
9 Atmospheric pressure is a measure of the force exerted by the mass of air on an object. Altitude is the vertical height above sea level.

The atmospheric pressure, P millibars, exerted at the altitude h kilometres is related by the equation $P = Ae^{bh}$, where A and b are constants.

The following table shows the mean atmospheric pressure at various altitudes.

h (kilometres)	2	4	6	8	10
P (millibars)	810	595	446	340	262

(a) Plot $\ln P$ against h and draw a straight line graph to illustrate the information. [2]



(b) Express the equation $P = Ae^{bh}$ in a form that will yield the straight line graph in **part (a)**.

Hence explain how the graph may be used to determine the value of A and of b. [3]

(c) Use your graph to estimate the atmospheric pressure, to the nearest millibar, when an object is at sea level. [1]

(d) The atmospheric pressure at the summit of Mount Everest is 300 millibars. Use your graph to estimate the altitude of Mount Everest. [1]

10 A patient's blood pressure, $\mathrm{P}(t)$ in mmHg, can be modelled by the function

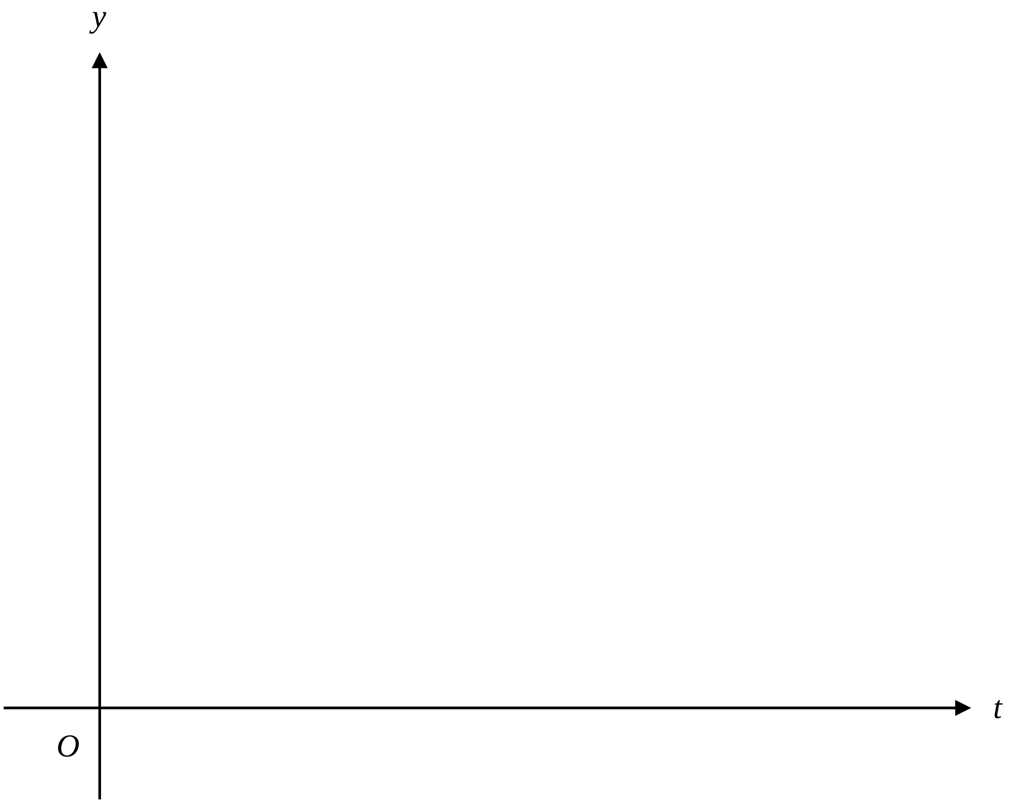
$$\mathrm{P}(t) = 22\cos\left(2.5\pi t\right) + 116,$$

where t is the time in seconds.

The systolic pressure (highest pressure) occurs when the heart beats, and the diastolic pressure (lowest pressure) occurs when the heart is at rest between beats.

(a) State the amplitude and period of $\mathrm{P}(t) = 22\cos\left(2.5\pi t\right) + 116$. [2]

(b) Sketch the graph of $y = \mathrm{P}(t)$ for $0 \le t \le 2$. [2]



(c) The pulse rate is the number of times a heart beats per minute.
A normal resting pulse rate should be between 60 to 100 beats per minute.
Show that the patient's pulse rate is normal. [2]

(d) According to health guidelines, someone with systolic pressure above 140 mmHg or diastolic pressure above 90 mmHg has high blood pressure and should see a doctor.
Determine whether the patient needs to see a doctor. Justify your answer. [1]

11 A particle moves in a straight line so that t seconds after passing through a fixed point O, its velocity v m/s is given by $v = 5\cos\left(\frac{t}{2}\right)$. Find

(a) the initial velocity of the particle, [1]

(b) the value of t, in terms of π, when the particle first comes to instantaneous rest, [3]

(c) the distance travelled by the particle in the first 5 seconds, after passing through O. [4]

12 A curve has the equation $y = 3 + \left(\frac{x}{2} - 1\right)^4$. The point (p, q) is the stationary point on the curve.

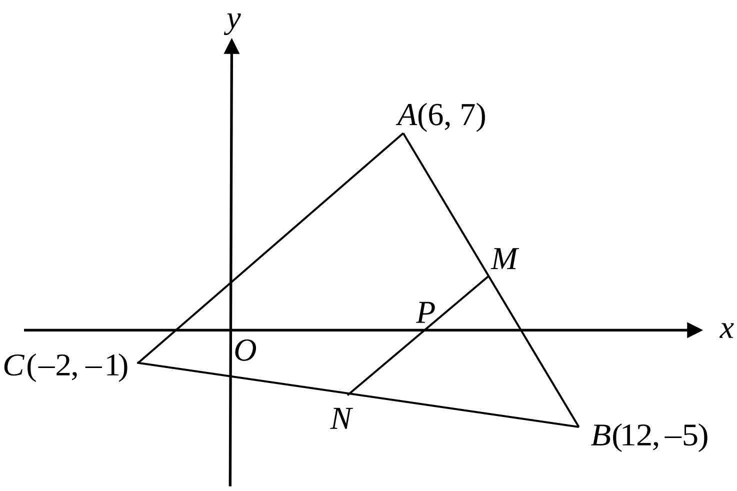
(a) Determine the coordinates of the stationary point (p, q). [4]

(b) **(i)** Justify whether y is increasing or decreasing for values of x less than p. [2]

(ii) Hence infer whether y is increasing or decreasing for values of x greater than p. [1]

(c) What do the results of **part (b)** imply about the stationary point? [1]

13 **Solutions to this question by accurate drawing will not be accepted.**



The diagram above shows a triangle ABC with vertices at $A(6, 7)$, $B(12, -5)$ and $C(-2, -1)$.

M and N are the mid-points of AB and BC respectively. The line MN cuts the x-axis at P.

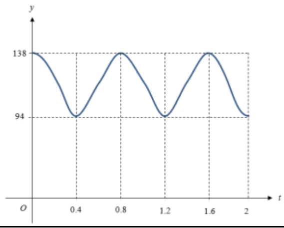
(a) Find the coordinates of P. [4]

(b) Find the ratio $AC : MN$. [1]

(c) Find the area of the quadrilateral *ACNM*. [2]

(d) Explain why quadrilateral *ACNM* is a trapezium. [2]

END OF PAPER

Qn	Answers
1	$p = 8$
2	$y = 3e^{2x} + e^{-x} - x + 1$
3a	$4k^2 - 8k - 12$
3b	$4(k-1)^2 - 16$
3c	$-1 < k < 3$
4b	$f(x) = (2x+3)(x-2)^2$
4c	$y = 1$
5b	$\dfrac{2x^2+1}{x^3-2x^2+5x-10} = \dfrac{1}{x-2} + \dfrac{x+2}{x^2+5}$
6a	$\left(2 + \dfrac{ax}{4}\right)^8 = 256 + 256ax + 112a^2x^2 + \ldots$
6b	$a = \dfrac{2}{3}$ *or* $a = \dfrac{6}{7}$
8a	$P = (1, -3)$
8b	10.5 units^2
9b	$\ln P = bh + \ln A$ The value of A can be determined by finding the **<u>vertical intercept</u>** of the graph. The value of b can be determined by finding the **<u>gradient</u>** of the graph.
9c	$P \approx 1097$ millibars (nearest whole)
9d	$h = 8.8$ km
10a	Amplitude = 22 Period = 0.8
10b	
10c	Patient's pulse rate = 75 beats per minute Hence the patient's pulse rate is normal.
10d	Since the **<u>diastolic pressure</u>** (94 mmHg) is **<u>above 90 mmHg</u>**, the patient has high blood pressure and **<u>should see the doctor</u>**.
11a	5 m/s
11b	$t = \pi$ s
11c	Distance ≈ 14.0 m
12a	Stationary point = $(2, 3)$
12bi	y is **<u>decreasing</u>** when $x < 2$.
12bii	y is **<u>increasing</u>** when $x > 2$.
12c	The stationary point is a **<u>minimum point</u>**.
13a	$P = (8, 0)$
13b	$AC : MN = 2 : 1$
13c	Area of trapezium $ACNM$ $= 54$ units^2