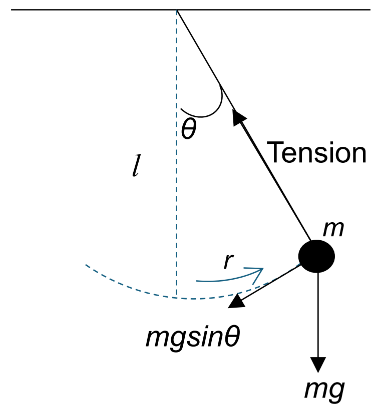
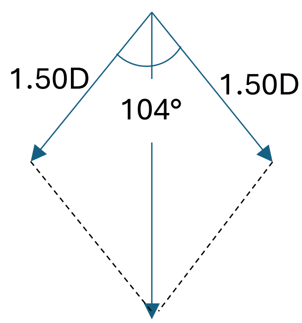
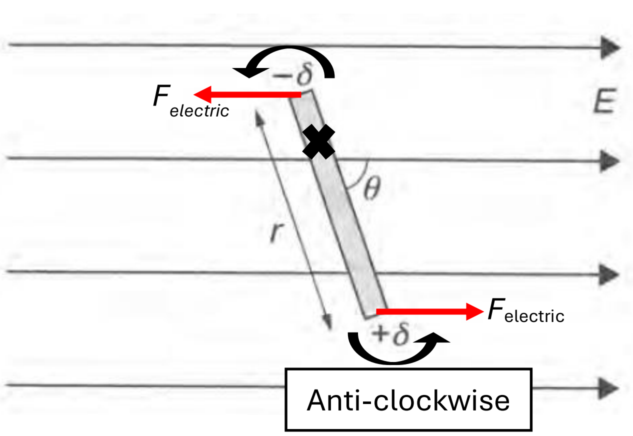
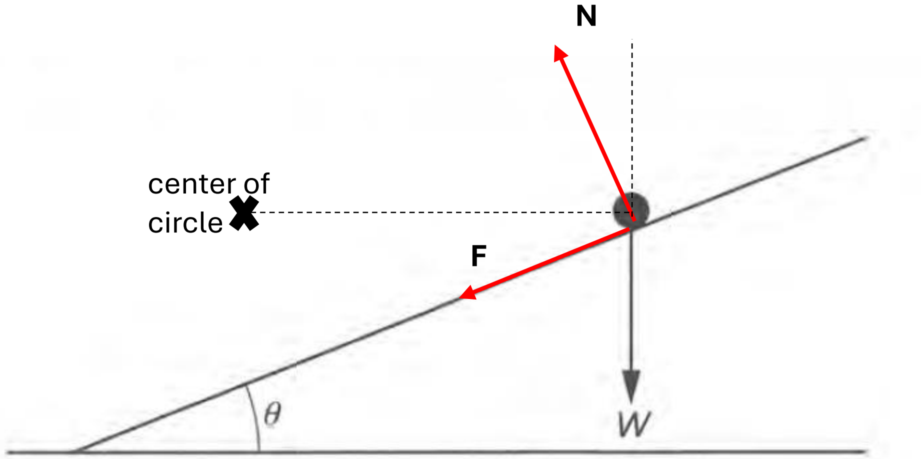
2023 H3 A level Suggested Solutions

1(a)(i)	The restoring force (tangential component of weight) is the net force. By Newton's 2nd law, $-mg\sin\theta = ma$ $\sin\theta = \frac{r}{l}$ where *r* is the amplitude of the oscillation. For small angles, $\sin\theta \approx \theta$, so the amplitude *r* becomes $l\theta$. The motion of pendulum is approximated to be a simple harmonic motion, hence, $a = -\omega^2 r = -\omega^2 l\theta$  Rewriting Newton's 2nd law, $-mg\theta = m\left(-\omega^2 l\theta\right)$ $\omega^2 = \frac{g}{l}$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$
1(a)(ii)	$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ $f = \frac{1}{2\pi}\sqrt{\frac{9.81}{0.300}} = 0.910$ Hz (3 s.f.)
1(b)(i)	$V_L + V_C = 0$ $L\frac{di}{dt} + \frac{q}{C} = 0$ $L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$ $\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$ The differential equation for *q* is analogous to the simple harmonic motion.

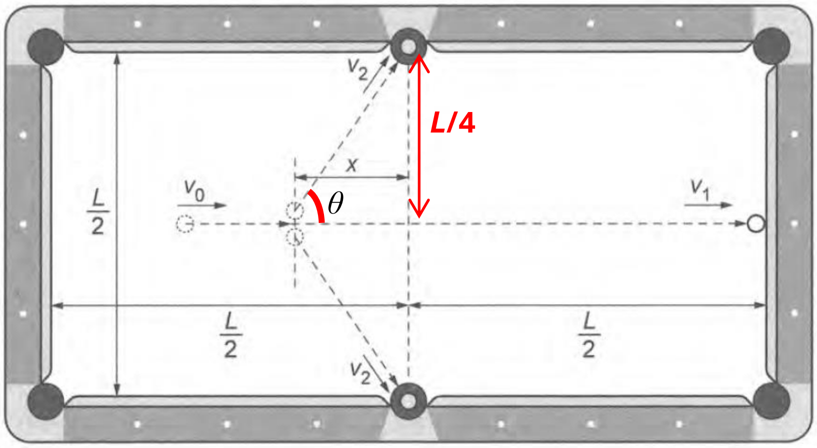
	$\omega^2 = \frac{1}{LC} = (2\pi f)^2$ $f = \frac{1}{2\pi\sqrt{LC}}$
1(b)(ii)	$f_{LC} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{82\times10^{-3}(39000\times10^{-6})}} = 2.81\text{ Hz}$ $\frac{f_{LC}}{f_{pendulum}} = \frac{2.81}{0.910} = 3.09$ $f_{LC} \approx 3\times f_{pendulum}$
1(c)	The natural frequency of LC circuit must be 3 times smaller. So, the effective capacitance of all the capacitors should be 9 times greater. To achieve this, 9 capacitors should be connected in parallel. However, this does not exactly match, $f_{LC} = 0.938$ Hz and $f_{pendulum} = 0.910$ Hz. To **exactly** match the frequencies, connect 10 capacitors in series this gives $f_{LC} = 0.890$ Hz and increase the length of the pendulum to 31.4 cm, which gives $f_{pendulum} = 0.890$ Hz.

2(a)	The magnitude of the electric dipole moment is the product of charge on an atom and distance between the centres of positive and negative charges.
2(b)	$q = \frac{p}{d} = \frac{1.50 \times 3.34 \times 10^{-30}}{(0.784)(95.8 \times 10^{-12})} = 6.67 \times 10^{-20}$ C $\frac{6.67 \times 10^{-20}\ \text{C}}{1.60 \times 10^{-19}\ \text{C}} \times e$ $q = 0.417e$
2(c)	 By applying cosine rule, $p_{net} = (1.50\text{D})^2 + (1.50\text{D})^2 - 2(1.50\text{D})(1.50\text{D})\cos 104°$ $p_{net} = 1.85D$
2(d)(i)	 The centre of mass should be closer to the oxygen atom as it is heavier.
2(d)(ii)	$\tau = F_{electric} r \sin\theta = qEr \sin\theta = pE \sin\theta$
2(d)(iii)	Initially, the dipole is stationary. It has zero rotational kinetic energy. Its potential energy is $pE\cos(63°)$.

	The dipole will have maximum kinetic energy when it has minimum potential energy (the dipole vector is parallel with the electric field, hence the angle is zero). The potential energy at this moment is $pE\cos(0°)$. The maximum rotational kinetic energy would be the change in potential energy between its initial state and when the dipole moment vector is parallel with the electric field. $= pE\cos(0°) - pE\cos(63°)$ $= 1.85 \times 3.34 \times 10^{-34} \times 9500 \times (1 - \cos(63°))$ $= 3.21 \times 10^{-26}$ J

3(a)	
3(b)	The net force in vertical direction: $\mathbf{N}\cos\theta - mg - \mathbf{F}\sin\theta = 0$ (1) The net force in horizontal direction: $\mathbf{F}\cos\theta + \mathbf{N}\sin\theta = \frac{mv_{max}^2}{r}$ (2) The frictional force is $\mu\mathbf{N}$. Substituting this into (1): $\mathbf{N}\cos\theta = mg + \mu\mathbf{N}\sin\theta$ $\mathbf{N} = \frac{mg}{\cos\theta - \mu\sin\theta}$ The equation (2) then becomes $\mu\mathbf{N}\cos\theta + \mathbf{N}\sin\theta = \frac{mv_{max}^2}{r}$ $\frac{mg}{\cos\theta - \mu\sin\theta}(\mu\cos\theta + \sin\theta) = \frac{mv_{max}^2}{r}$ $\frac{mg\cos\theta(\mu + \tan\theta)}{\cos\theta(1 - \mu\tan\theta)} = \frac{mv_{max}^2}{r}$ $v_{max}^2 = \frac{rg(\mu + \tan\theta)}{(1 - \mu\tan\theta)}$ $v_{max} = \sqrt{\frac{rg(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}$
3(c)(i)	$1 - \mu\tan\theta$ should be the smallest possible so that the speed would be the largest. This can be achieved when the angle of 45°, not 12°. $r = 20.0 + 0.90 \times \cos 45° = 20.637 = 20.6$ m (3 s.f.)

3(c)(ii)	$v_{max} = \sqrt{\dfrac{20.6(9.81)(0.70 + \tan 45°)}{(1 - 0.70 \tan 45°)}} = 33.9 \text{ m s}^{-1}$

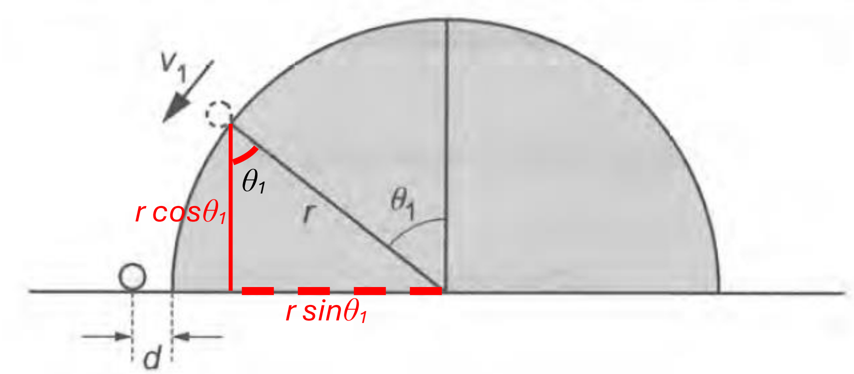
4(a)	By PCOLM, $mv_0 = mv_1 + mv_2\cos\theta + mv_2\cos\theta$ $v_0 = v_1 + 2v_2\cos\theta$ From the diagram,  $\cos\theta = \dfrac{x}{\sqrt{x^2 + \dfrac{L^2}{16}}}$ $v_0 = v_1 + \dfrac{2x}{\sqrt{x^2 + \dfrac{L^2}{16}}} v_2$ Therefore, $f(x) = \dfrac{2x}{\sqrt{x^2 + \dfrac{L^2}{16}}}$
4(b)	$v_1 t = \dfrac{L}{2} + x \rightarrow t = \dfrac{\dfrac{L}{2} + x}{v_1}$ $v_2 t = \sqrt{x^2 + \dfrac{L^2}{16}} \rightarrow v_2 = \dfrac{\sqrt{x^2 + \dfrac{L^2}{16}}}{t} = \dfrac{\sqrt{x^2 + \dfrac{L^2}{16}}}{\dfrac{L}{2} + x} v_1$

	$v_0 = v_1 + \frac{2x}{\sqrt{x^2+\frac{L^2}{16}}}\left(\frac{\sqrt{x^2+\frac{L^2}{16}}}{\frac{L}{2}+x}\right)v_1$ $= v_1\left(1+\frac{4x}{L+2x}\right)$ $v_0 = \left(\frac{6x+L}{2x+L}\right)v_1$
4(c)	Since the collision is elastic, the kinetic energy would remain the same after the collision. $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + 2\left(\frac{1}{2}mv_2^2\right)$ $v_0^2 = v_1^2 + 2v_2^2$ From earlier part, express this in terms of v_1 only, $\left(\frac{6x+L}{2x+L}v_1\right)^2 = v_1^2 + 2\left(\frac{x^2+L^2/16}{(x+L/2)^2}v_1^2\right)$ $\left(\frac{6x+L}{2x+L}\right)^2 - \left(\frac{2x+L}{2x+L}\right)^2 = 2\left(\frac{x^2+L^2/16}{(2x+L)^2/4}\right)$ $24x^2 + 8xL - L^2/2 = 0$ Solving this quadratic equation for x, $x_1 = 0.0538L$ and $x_2 = -0.378L$ Substituting this into $x = \frac{L}{12}\left(\sqrt{b}-2\right)$ x_1 produces $\sqrt{b} = \sqrt{7} \rightarrow b = 7$ and x_2 produces $\sqrt{b} = -2.644$. Hence, x_1 is accepted and x_2 is rejected. Therefore, the value of b is 7.
4(d)	The friction between the table and the balls produces same deceleration effect on the target balls and cue ball. They will all be moving at slower speed and the trick shot may still be achieved. However, the time for the cue ball hitting the right-hand cushion and the time for target balls being potted would be different.

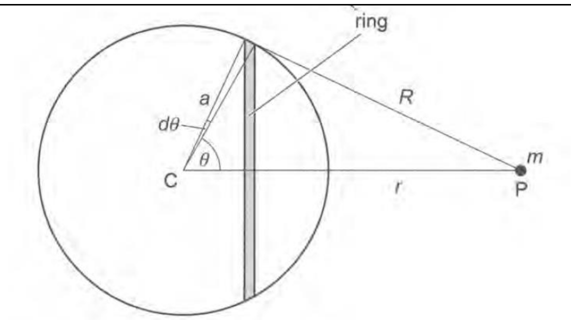
5(a)(i)	By parallel-axis theorem, $I_{\text{single square plate}}$ through the central axle at a distance l away from the centre of square plate is $I_{\text{single square plate}} = \frac{Ma^2}{12} + Ml^2 = M\left(\frac{a^2}{12} + l^2\right)$ $= \left((8360)(0.12)^2(0.003)\right)\left(\frac{0.120^2}{12} + 0.250^2\right)$ $= 0.023 \text{ kg m}^2$
5(a)(ii)	$I_{\text{cylindrical rod}} = \frac{m_{rod} l^2}{3} = \frac{8360(\pi(0.003)^2 0.250(0.250)^2}{3} = 0.00123 \text{ kg m}^2$ $I_{\text{total}} = 8\left(I_{\text{single square plate}} + I_{\text{cylindrical rod}}\right) = 0194 \text{ kg m}^2$ $I_{\text{total}} = 019 \text{ kg m}^2$ (2 s.f.)
5(a)(iii)	By PCOE, $2mgh + 0 + 0 = 0 + \frac{1}{2} I_{\text{total}} \omega^2 + 2\frac{1}{2} mv^2$ $2mgh = \frac{1}{2} I_{\text{total}} \omega^2 + 2\frac{1}{2} m\omega^2 r_{\text{drum}}^2$ $\omega^2 = \frac{4mgh}{I_{\text{total}} + 2mr_{\text{drum}}^2} = \frac{4(1.81)(9.81)(11.0)}{0.194 + 2(1.81)(0.043)^2}$ $\omega = 62.4 \text{ rad s}^{-1}$
5(a)(iv)	$h = \frac{1}{2} at^2 \rightarrow a = \frac{2h}{t^2} \rightarrow$ where a is constant $\omega = \omega_0 + \alpha t = 0 + \frac{a}{r_{drum}} t = \frac{2h}{t^2} \frac{t}{r_{drum}} = \frac{2h}{tr_{drum}}$ $t = \frac{2h}{\omega r_{drum}} = \frac{2(11.0)}{(62.4)(0.086/2)} = 8.2 \text{ s}$
5(a)(v)	$V_{air} =$ (Area of square plate)(distance travelled) $= A(vt)$ $V_{air} = (A)(\omega_{avg} l t)$ $\omega_{avg} \rightarrow$ because α is constant $V_{air} = (0.12)^2(62.4/2)(0.25)(8.2)$ $V_{air} = 0.92 \text{ m}^3$
5(b)(i)	Assume that, the loss in gravitational potential energy is fully converted to increase the temperature of water only.

	$m_{\text{water}}c\Delta T = 2mgh$ $\Delta T = \dfrac{2mgh}{m_{\text{water}}c} = \dfrac{2(1.81)(9.81)(11.0)}{(999\times 93.2\times 10^{-3})(4190)}$ $\Delta T = 1.0\times 10^{-3}\text{ K} = 1.0\text{ mK}$
5(b)(ii)	Total supply of energy is the lost in GPE of system, $2mgh = 2(1.81)(9.81)(11.0) = 390.6\text{ J}$ 1. Translational kinetic energy of moving masses, $2\left(\frac{1}{2}mv^2\right) = 1.81(0.305)^2 = 0.1684\text{ J}$ 2. Rotational kinetic energy of rotating system, $\frac{1}{2}I\omega^2 = 0.194\left(\frac{0.305}{0.043}\right)^2 = 4.880\text{ J}$ 3. Ignoring other forms of energies conversion (due to friction, radiation etc), the thermal energy converted, $390.6 - 0.1684 - 4.880 = 385.6\text{ J}$ Hence, the energy associated with paddles & insulated copper cylinder would then be $385.6/13 = 29.66\text{ J}$ Since 29.66494 > (0.1684 + 4.880), item (2) has a higher impact on temperature rise.
5(b)(iii)	The resolution of the thermometer is the smallest change in the marking of the thermometer. In the older days where instruments are mainly analogue, so from one marking (say a line) to the adjacent marking is a temperature difference of 2.8 mK. Using a magnifier etc, it is possible that Joule could have read the interval in between the two successive markings to tell a temperature difference of 1.0 mK or even less as it is in the same order of magnitude. This can even be enhanced if Joules allowed the masses to fall through an even longer distance of 11 m (there is no change in apparatus used) and a higher temperature change will result.

6(a)	By PCOE, the total energy remains constant, hence, the loss in gravitational potential energy is equal to the gain in kinetic energy. $mg(r - r\cos\theta) = \frac{1}{2}mv^2$ $v^2 = 2gr(1-\cos\theta)$ The object is performing a circular motion. Normal contact force and the component of weight provides the net force towards the centre (centripetal) force. $-N + mg\cos\theta = \frac{mv^2}{r}$ $N = mg\cos\theta - \frac{m2gr(1-\cos\theta)}{r}$ $N = mg\cos\theta - 2mg + 2mg\cos\theta$ $N = mg(3\cos\theta - 2)$
6(b)(i)	When the object loses contact with the hemisphere, the normal contact force acting on it goes to zero. $N = mg(3\cos\theta - 2)$, $N \to 0$ $\cos\theta_1 = \frac{2}{3}$ $\theta_1 = 48.2°$
6(b)(ii)	By PCOE, the total energy remains constant. $mgr + 0 = mgr\cos\theta_1 + \frac{1}{2}mv_1^2$ $v_1^2 = 2gr\left(1-\cos\theta_1\right) = 2gr\left(1-\frac{2}{3}\right)$ $v_1 = \sqrt{\frac{2}{3}gr}$
6(b)(iii)	$v_{1,\text{ hor}} = v_1\cos\theta_1 = \sqrt{\frac{2}{3}gr}\cos(48.2)$ $v_{1,\text{ hor}} = \sqrt{\frac{8}{27}gr}$ towards left

	$v_{1,\text{ ver}} = v_1 \sin\theta_1 = \sqrt{\frac{2}{3}gr}\sin(48.2)$ $v_{1,\text{ ver}} = \sqrt{\frac{2}{3}gr}\,\frac{\sqrt{5}}{3} = \sqrt{\frac{10}{27}gr}$ (downwards)
6(b)(iv)	 Vertical displacemet: $r\cos\theta_1 = v_{1,\text{ ver}}t + \frac{1}{2}gt^2$ $r\cos\theta_1 = \left(\sqrt{\frac{10}{27}gr}\right)t + \frac{1}{2}gt^2$ consider $r = 1$ m $\frac{2}{3} = \left(\sqrt{\frac{10}{27}g}\right)t + \frac{1}{2}gt^2$ $\frac{1}{2}gt^2 + \left(\sqrt{\frac{10}{27}g}\right)t - \frac{2}{3} = 0$ $t = 0.222$ s Horizontal displacement: $d + (r - r\sin\theta_1) = v_{1,\text{ hor}}t$ $d = v_{1,\text{ hor}}t - (r - r\sin\theta_1)$, consider $r = 1$ m $d = \left(\sqrt{\frac{8}{27}(9.81)}\right)0.222 - (1-(1)\sin 48.2°)$ $d = 0.124$ m $\frac{r}{8} = 0.125 \rightarrow d \approx \frac{r}{8}$

6(c)(i)	**Diameter :** When the diameter of the ball increases, whilst keeping the other properties constant, assume the angle it leaves the hemisphere is the same. This means that the distance between the centre of mass of the ball from the centre of hemisphere is larger when it loses contact, thus, distance d will be larger too . **Mass :** By PCOE, the loss in GPE = gain in translational K.E. + gain in rotational K.E. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ $mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mr^2\omega^2$ $gh = \frac{1}{2}v^2 + \frac{1}{2}\frac{2}{5}v^2$ $h = \frac{7}{10}\frac{v^2}{g}$ For the same drop in height, the translational velocity of centre of mass will be the same, independent of the mass of the sphere. The translational speed is in dependent of the mass of the ball, whilst keeping the other properties constant. Thus, the distance *d* will be the same. **Composition :** When the ball is hollow , its moment of inertia is larger, $\frac{2}{3}mr^2$. The translational velocity of centre of mass will be the smaller for same drop in height and the ball will leave the hemisphere at lower height. Whilst keeping the other properties constant, thus, the distance *d* will be the smaller.
6(c)(ii)	The surface texture of the ball will affect the distance *d*. If the surface of the ball is smooth, the ball will not rotate as it rolls down the hemisphere. The translational velocity of centre of mass will be the larger for same drop in height and the ball will leave the hemisphere at a higher height. Whilst keeping the other properties constant, thus, the distance *d* will be the larger.
6(d)(i)	With the crossed lines, using the many frames from the high-speed camera, one can see the motion of the centre of mass of ball (which is aligned with the cross), is able to calculate the angular velocity and linear velocity of the ball.
6(d)(ii)	The camera is in a fixed position facing the hemisphere. The ball has to roll in a certain path so that plane of the path is parallel to the screen of camera and the cross lines have to face directly towards the camera. This can be done with help of the blue-tac, which helps to fix the intial starting orientation of the ball before rolling.

7(a)(i)	$dM = \sigma dA = \frac{M}{4\pi a^2}(2\pi a \sin\theta) a d\theta$ $dM = \frac{M}{2}(\sin\theta) d\theta$
7(a)(ii)	$d\phi = -\frac{GdM}{R}$ $\phi = \int_0^\pi -\frac{G}{R}\frac{M}{2}(\sin\theta) d\theta$ $\phi = -GM\int_0^\pi \frac{(\sin\theta)}{2R} d\theta$
7(a)(iii)	 Using cosine rule, $R^2 = r^2 + a^2 - 2ar\cos\theta$ R and θ are varying but r and a are constants. Differentiate R with respect to θ, $2RdR = -2ar(-\sin\theta)d\theta = 2ar\sin\theta d\theta$ $\frac{\sin\theta d\theta}{R} = \frac{1}{ar}dR$
7(a)(iv)	$\phi = -GM\int_0^\pi \frac{(\sin\theta)}{2R} d\theta = -\frac{GM}{2}\int_0^\pi \frac{(\sin\theta)}{R} d\theta$ Change the integration limits appropriately, $0 \rightarrow r-a$ and $\pi \rightarrow r+a$ $\phi = -\frac{GM}{2}\int_{r-a}^{r+a}\frac{1}{ar}dR = -\frac{GM}{2ar}\int_{r-a}^{r+a} dR$ Therefore, $k = \frac{GM}{2ar}$, $Y = r-a$ and $Z = r+a$

7(a)(v)	$\phi = -\frac{GM}{2ar}\int_{r-a}^{r+a} dR = -\frac{GM}{2ar}(r+a-r+a) = -\frac{GM}{r}$ $F = mg = m\left(-\frac{d\phi}{dr}\right) = m\left(-\left(-\frac{GM}{r^2}\right)\right) = \frac{GMm}{r^2}$
7(a)(vi)	This is the gravitational force acting on the point mass *m* along the horizontal direction (CP in figure) and its direction is towards the thin spherical shell mass *M*. The formula in **(a)(v)** is similar to that due to a point mass on a test mass. In other words, the hollow sphere can be treated like a point mass stationed at its centre of mass.
7(a)(vii)	The point mass *m* is now inside the shell, the integration limits will change $0 \rightarrow r-a$ (outside the shell) $\rightarrow a-r$ (inside the shell) $\pi \rightarrow r+a$ (outside the shell) $\rightarrow a+r$ (inside the shell) $\phi = -\frac{GM}{2ar}\int_{a-r}^{a+r} dR = -\frac{GM}{2ar}(a+r-a+r) = -\frac{GM}{a}$ $F = mg = m\left(-\frac{d\phi}{dr}\right) = m(-0) = 0$
7(b)(i)	Mass distribution at r: $dM = \rho dV = \rho\frac{4}{3}\pi r^3 = \frac{M_E}{V}\frac{4}{3}\pi r^3 = \frac{M_E}{\frac{4}{3}\pi R_E^3}\frac{4}{3}\pi r^3 = M_E\frac{r^3}{R_E^3}$ $F = -G\frac{mdM}{r^2} = -G\frac{m}{r^2}\left(\frac{M_E r^3}{R_E^3}\right)$ (the negative sign indicates that the direction of gravitational force is opposite to the displacement of the person from the centre of the Earth.) $F = -G\frac{M_E mr}{R_E^3}$

7(b)(ii)	The gravitational force is the restoring force acting on the person. It is proportional to its displacement of the person from the centre of Earth and is in opposite direction to its displacement of the person, thus, the motion can be considered as a simple harmonic motion. $F_{net} = ma_{SHM} = -m\omega^2 r = -\frac{GM_E mr}{R_E^3}$ $\omega^2 = \frac{GM_E}{R_E^3} \rightarrow \frac{4\pi^2}{T^2} = \frac{GM_E}{R_E^3}$ $T = \sqrt{\frac{4\pi^2 R_E^3}{GM_E}} = \sqrt{\frac{4\pi^2(6370\times10^3)^3}{(6.67\times10^{-11})(5.97\times10^{24})}} = 5060\text{ s}$ $T = 84.3\text{ min}$ Time taken to fly from Singapore to Pantoja is *T*/2, hence, it will be 42 min.
7(b)(iii)	Astronauts in ISS in low Earth orbit, which is 400 km above the surface of Earth. The gravitational force by the Earth provides the centripetal force for the ISS. $\frac{GM_E m}{R_{ISS}^2} = m\omega^2 R_{ISS} = m\left(\frac{2\pi}{T_{ISS}}\right)^2 R_{ISS}$ $T_{ISS} = \sqrt{\frac{4\pi^2 R_{ISS}^3}{GM_E}} = 5560\text{ s}$ $T_{ISS}/2 = 46\text{ min}$ Time taken to move between antipolar for ISS, is slightly larger than 42 min since the radius of orbit is slightly larger than radius of Earth. Thus, the time taken to fly from Singapore to Pantoja in **(b)(ii)** is approximately equal for ISS to complete half of its orbital period 400 km above the Earth's surface.

8(a)	When a motor is used to drive the belt, there is a transfer of electrons from the dissimilar materials of the belt and the two rollers. When the rubber belt rubs against the two plastic rollers, **the rubber belt will become negatively charged while the plastic roller will become positively charged.** Next, the strong electric field surrounding the positive upper roller induces a very high electric field near the points of the nearby upper comb. At the points of the comb, the electric field becomes strong enough to ionize air molecules. The electrons from the air molecules are attracted to the outside of the belt, while the positive ions go to the comb. At the comb, they are neutralized by electrons from the metal, thus leaving the comb and the attached outer shell. Both will have fewer net electrons and has a net positive charge. When the negatively charged belt moves down to the lower roller, the high electric field at the lower comb ionizes the air molecules, which will neutralize the belt. Thus, this process produces a continuous flow of electrons from dome to ground. Since dome is a conductor, the excess positive charge is accumulated on the outer surface of the outer shell, leaving no electric field inside the shell. Constant motion of the belt causes a build-up of large amounts of positive charge on the shell. Charge will continue to accumulate until the rate of charge leaving the sphere (through leakage and corona discharge) equals the rate at which new charge is being carried into the sphere by the belt.
8(b)(i)	$M_{\text{sphere}} = M_{\text{rod}}$ $\rho 4\pi R_{\text{sphere}}^2 t = \rho 2\pi \left(\frac{D-t}{2}\right) Lt$ $R_{\text{sphere}}^2 = \frac{1}{2}\left(\frac{D-t}{2}\right)L$ $R_{\text{sphere}} = 3.26 \text{ cm}$ $D_{\text{sphere}} = 2R_{\text{sphere}} + t$ $D_{\text{sphere}} = 6.7 \text{ cm}$
8(b)(ii)	$V_{rod} = 2\pi\left(\frac{D-t}{2}\right)Lt$ $V_{rod} = 2\pi\left(\frac{1.0-0.15}{2}\right)(50.0)0.15 = 20.0 \text{ cm}^3$ $m_{rod} = \rho V_{rod} = (2.7)(20.0) = 54 \text{ g}$

8(b)(iii)	Using cosine rule, $L_{spring} = \sqrt{12.00^2 + 5.00^2 - 2(12.00)(5.00)\cos(90.0° + 20.0°)}$ $L_{spring} = 14.49$ cm extension = 14.49 – 14.34 = 0.15 cm = 1.5 mm
8(b)(iv)	First, let's calculate the angle α. $\frac{\sin\alpha}{120.0} = \frac{\sin(90.0+20.0)}{(143.4+1.5)}$ $\alpha = 51.08°$ Taking moments about the base of rod, $M_{sphere}g\left(R+L\right)\sin\left(20.0°\right) + M_{rod}g\left(L/2\right)\sin\left(20.0°\right) = T\left(\sin\alpha\right)0.05$, where $M_{sphere} = M_{rod} = M = 54$ g, R = 0.067/2 m and L = 0.500 m $T = \frac{\left(0.054(9.81)\sin\left(20.0°\right)\right)\left((0.067/2) + 3(0.50/2)\right)}{\sin\left(51.08\right)0.0500}$ $T = 3.649$ N $T = ke \rightarrow k = \frac{T}{e} = \frac{3.649}{0.0015} = 2.43 \times 10^3$ N
8(c)(i)	When the dome is charged up positively, by electrostatic induction, the electrons from the Earth will travel up to the sphere. The sphere will be negatively charged. Since unlike charges attract, an attractive electric force will exert on the sphere and it will be pushed left towards the dome. After sparking happens, the magnitude of charge in the dome will drop significantly, the magnitude of charge in the sphere will drop too. Since the electric force is proportional to the product of the two charges, its magnitude will drop, and the moment produced by the electric force will drop. Since the restoring moment by the spring just before the sparking is now larger, the sphere will be pushed away from the dome.
8(c)(ii)	The extension length in the spring, 14.80 – 14.34 = 0.46 cm $T_{sparking} = ke = 2432.7 \times 0.46 \times 10^{-2} = 11.19$ N Taking moments about the base of rod, $F_E\cos(24.6°)(R+L) + Mg\sin(24.6°)(R+L) + Mg\sin(24.6°)(L/2) = T\sin(47.5°)(0.0500)$

	$T\sin(47.5°)(0.050) = 0.4125$ N m $Mg\sin(24.6°)((0.067/2)+0.50) + Mg\sin(24.6°)(0.50/2) = 0.17278$ N m $F_E\cos(24.6°)((0.067/2)+0.50) = 0.4125 - 0.17278$ $F_E = \dfrac{0.23973}{\cos(24.6°)((0.067/2)+0.50)}$ $F_E = 0.4942$ N $F_E = \dfrac{Q_1Q_2}{4\pi\varepsilon_0 r^2} \rightarrow Q_1Q_2 = F_E 4\pi\varepsilon_0 r^2$ $Q_1Q_2 = 0.4942\left(4\pi\left(8.85\times10^{-12}\right)\left(0.15+0.033+\dfrac{0.067}{2}\right)^2\right)$ $Q_1Q_2 = 2.576\times10^{-12}$ C^2
8(c)(iii)	Since both objects are electrical conductors, the distribution of charges will not be uniform on their surfaces. The more charges are distributed on the right and left of the dome and the sphere respectively. Thus, the effective distance between the centres of charges of the two objects will be smaller and the product of the two charges will be a smaller value. This means the calculation in **(c)(ii)** is an over-estimation.