Name	( )	Class	

南 华 中 学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2022

Subject	**:**	**Mathematics**
Paper	**:**	**4048/01**
Level	**:**	**Secondary Four Express**
Date	**:**	**17 August 2022**
Duration	**:**	**2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue, correction fluid or correction tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

This paper consists of 19 printed pages.

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** questions.

1 The number of people living in a town is given as 60 000, correct to 2 significant figures. Write down values for the smallest and largest possible number of people who could be in the town.

Smallest possible number is 59500
Largest possible number is 60499

2 **(a)** Use prime factors to explain why 135×200 is a perfect cube.

$$135 \times 200 = 5 \times 3^3 \times 10^2 \times 2$$
$$= 5 \times 3^3 \times 5^2 \times 2^2 \times 2$$
$$= 2^3 \times 3^3 \times 5^3$$

Since the index of each prime factor is multiple of 3, 135×200 is a perfect cube.
OR
$2^3 \times 3^3 \times 5^3 = (2 \times 3 \times 5)^3$

(b) The lowest common multiple of x and 135 is $2 \times 3^4 \times 5 \times 7$.
Find the smallest possible value of x.

$135 = 5 \times 3^3$
Smallest possible value of $x = 2 \times 3^4 \times 7$
$= 1134$

3

Write the following numbers in order of size, starting with the **smallest**.

$$\left(\frac{3}{10}\right)^2,\ -\frac{21}{90},\ 90\%,\ -0.23\ ,\ \sqrt{0.3}$$

$$-\frac{21}{90},\ -0.23\ ,\ \left(\frac{3}{10}\right)^2,\ \sqrt{0.3}\ ,90\%$$

4 Factorise $\dfrac{6a-15a^2+20ab-8b}{25a^3-4a}$ completely.

$$\frac{6a-15a^2+20ab-8b}{25a^3-4a}=\frac{3a(2-5a)+4b(5a-2)}{a(25a^2-4)}$$

$$=\frac{-3a(5a-2)+4b(5a-2)}{a(5a+2)(5a-2)}$$

$$=\frac{(5a-2)(4b-3a)}{a(5a+2)(5a-2)}$$

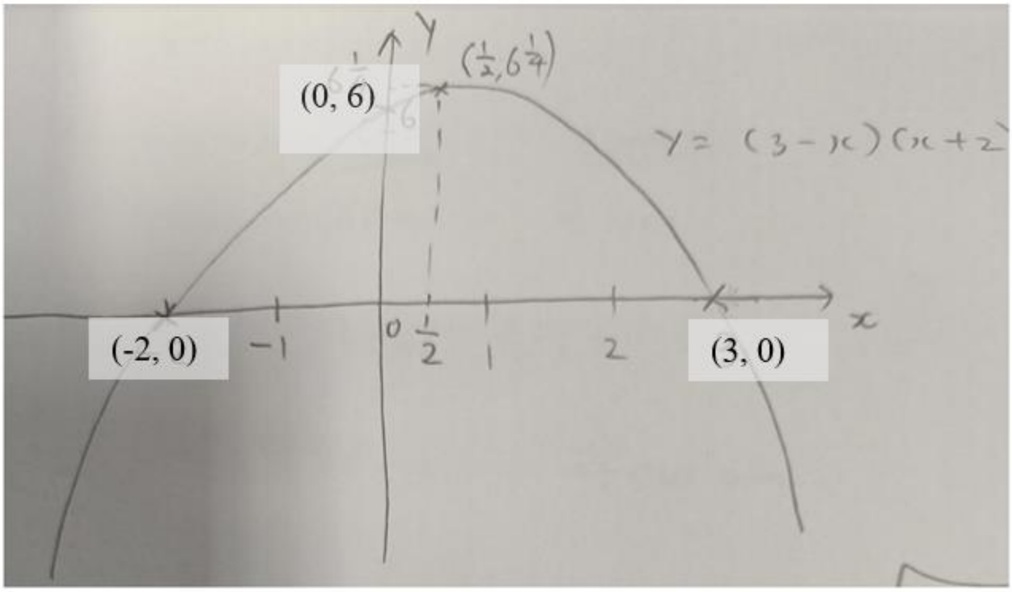
$$=\frac{4b-3a}{a(5a+2)}$$

5 Rearrange the formula $\frac{1}{a}-\frac{1}{2b}=\frac{1}{3c}$ to make b the subject.

5	$\frac{1}{a}-\frac{1}{2b}=\frac{1}{3c}$ $6bc-3ac=2ab$ $6bc-2ab=3ac$ $2b\left(3c-a\right)=3ac$ $b=\frac{3ac}{2\left(3c-a\right)}$ OR $\frac{3ac}{6c-2a}$

6 Sketch the graph of $y=\left(3-x\right)\left(x+2\right)$ on the axes below.

Indicate clearly the coordinates of the points where the graph crosses the axes and the turning point of the curve.



7 The scale of a map is 8 cm : 2 km .

(a) Write this scale in the form 1 : n .

8 cm :2 km

1 cm: 0.25 km

1 cm: 25 000 cm

Scale is 1: 25000

(b) The actual area of a lake is 90 000 m^2.
Calculate the area, in square centimetres, of the lake on the map.

250 m :1 cm

1 m: $\frac{1}{250}$ cm

1 m^2: $\left(\frac{1}{250}\right)^2$ cm^2

90000 m^2: $90000\times\left(\frac{1}{250}\right)^2$

$= 1.44\ cm^2$

8

3.8 is the mean of 5 positive numbers a, b, c, d and e.
The sum of their squares is 360. Each of the numbers is now multiplied by 2.
Find the new standard deviation.

Standard deviation = $\sqrt{\frac{2^2(360)}{5}-(2\times3.8)^2}$

=15.2 (to 3 s.f)

9 One solution of the equation $(k+1)x^2+kx=15$ is $x=-3$.

(a) Find the value of *k*.

Sub x = -3,

$$(k+1)(-3)^2-3k=15$$
$$9k+9-3k=15$$
$$6k=6$$
$$k=1$$

(b) Find the second possible value of *x*.

$2x^2+x=15$

$2x^2+x-15=0$

$(2x-5)(x+3)=0$

$x=2.5$ or -3

The second possible value of x is 2.5

(b) Find angle BCD.

$\angle EBD=180°-60°$ ($\angle$ sum of Δ)

$=120°$

$\angle CBD=\frac{2}{5}\times 120°$

$=48°$

$BC=CD \Rightarrow \Delta BCD$ is an isosceles Δ

$\angle BCD=180°-48°-48°$ ($\angle$ sum of Δ)

$=84°$

12 The first four terms in a sequence of numbers are given below.

(a) $T_1 = 2^2 + 7$

$T_2 = 3^2 + 12$

$T_3 = 4^2 + 17$

$T_4 = 5^2 + 22$

Explain why the value of T_n must be odd for all values of *n*.

$T_n = (n+1)^2 + (5n+2)$

When *n* is odd, $(n+1)^2$ is even and $(5n+2)$ is odd.

When *n* is even, $(n+1)^2$ is odd and $(5n+2)$ is even.

The sum of an even number and an odd number is always odd.

(b) The product of the first *n* terms of a sequence is given by $2n^2 + 3n$.
Find the 12th term of this sequence.

$$\text{Product of first 11 terms} = 2(11)^2 + 3(11) = 275$$

$$\text{Product of first 12 terms} = 2(12)^2 + 3(12) = 324$$

$$\text{12th term} = \frac{324}{275} = 1\frac{49}{275}$$

13 The thickness of a layer of ice in a water body is 0.00205 m.

(a) Write 0.00205 in standard form.

$0.00205 = 2.05 \times 10^{-3}$

(b) The ice covers an area of $1.60 \times 10^{5}\text{ m}^{2}$.
Assuming that all the ice melts and ignoring the expansion of volume when water freezes, calculate the volume of water, in litres.

$$\text{Volume of water} = 1.60 \times 10^{5} \times 0.00205$$
$$= 328\text{ m}^{3}$$
$$= 328000\ l$$

Answer ……………………………………*l* [2]

(c) A thunderstorm occurs and rainwater is falling at an average rate of 5×10^{-1} litres per second over the water body.
Calculate the percentage increase in the volume of water in the water body after two hours.

$$\text{Volume of rain water} = 2 \times 3600 \times 5 \times 10^{-1}$$
$$= 3600\ l$$
$$\text{Percentage increase} = \frac{3600l}{328000l} \times 100\%$$
$$= 1.10\%\ (\text{to 3s.f})$$

14 Given that the coordinates of A is $(2,-3)$, $\overrightarrow{AB}=\begin{pmatrix}-5\\7\end{pmatrix}$ and $\overrightarrow{FE}=k\begin{pmatrix}2.5\\-3.5\end{pmatrix}$.

(a) Find the value of k if $ABFE$ is a parallelogram.

$$\overrightarrow{AB}=\overrightarrow{EF}$$
$$\begin{pmatrix}-5\\7\end{pmatrix}=k\begin{pmatrix}-2.5\\3.5\end{pmatrix}$$
$$k=2$$

(b) Find the coordinates of B.

$$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$$
$$\begin{pmatrix}-5\\7\end{pmatrix}=\overrightarrow{OB}-\begin{pmatrix}2\\-3\end{pmatrix}$$
$$\overrightarrow{OB}=\begin{pmatrix}-3\\4\end{pmatrix}$$
$$B\text{ is }(-3,4)$$

(c) C is the point $(6,-10)$.
Justify if *A*, *B* and *C* are collinear.

Answer [2]

$$\overrightarrow{AB}=\begin{pmatrix}-5\\7\end{pmatrix}$$

$$\text{gradient of } AB=\frac{7}{-5}$$

$$\text{gradient of } AC=\frac{-3+10}{2-6}$$

$$=\frac{7}{-4}$$

Since gradient of $AB \neq$ gradient of AC, A, B and C are not collinear

OR

$$\overrightarrow{AC}=\begin{pmatrix}6\\-10\end{pmatrix}-\begin{pmatrix}2\\-3\end{pmatrix}$$

$$=\begin{pmatrix}4\\-7\end{pmatrix}$$

$$\overrightarrow{AB}=\begin{pmatrix}-5\\7\end{pmatrix}$$

Since $\overrightarrow{AB}\neq k\overrightarrow{AC}$, where k is a constant, hence, A, B and C are not collinear

15 A tour agency records the total number of people buying tour packages to Thailand and Vietnam in the months of November and December.
In November, 144 people bought the Thailand tour package and 100 people bought the Vietnam tour package.
In December, 208 people bought the Thailand tour package and 180 people bought the Vietnam tour package.

This information can be represented by the matrix, $\mathbf{M} = \begin{pmatrix} 144 & 100 \\ 208 & 180 \end{pmatrix}$, where the columns are Thailand and Vietnam, and the rows are November and December.

(a) The price of the Thailand and Vietnam package is \$890 and \$750 respectively.
Represent the price of the tour package by a 2×1 column matrix **K**.

$$K = \begin{pmatrix} 890 \\ 750 \end{pmatrix}$$

(b) Evaluate the matrix **R** = **MK**.

$$\begin{aligned} R &= MK \\ &= \begin{pmatrix} 144 & 100 \\ 208 & 180 \end{pmatrix}\begin{pmatrix} 890 \\ 750 \end{pmatrix} \\ &= \begin{pmatrix} 203160 \\ 320120 \end{pmatrix} \end{aligned}$$

(c) State what the elements of **R** represent.

Answer
The cash received for total sales of tour packages sold in the months of November and December respectively.

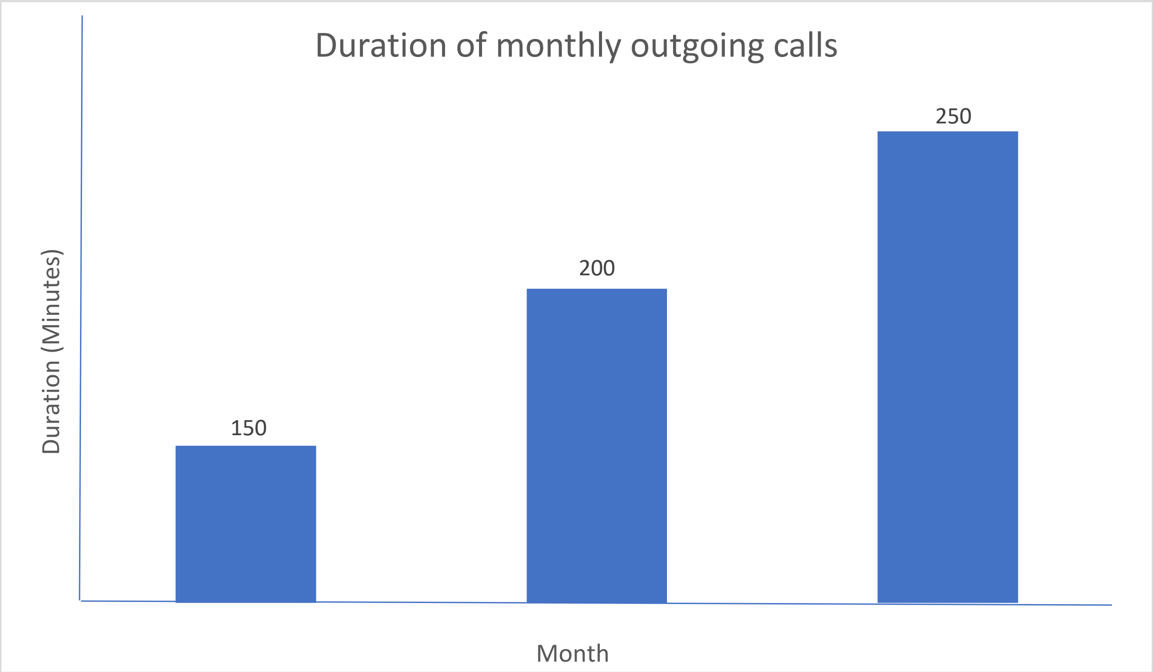
(d) Evaluate $\frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{R}$ and explain what the answer represents.

$$\frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 203160 \\ 320120 \end{pmatrix} = \begin{pmatrix} 261640 \end{pmatrix}$$

Answer

The average sales of tour packages sold in the months of November and December.

16 Peter draws this graph to show the duration of his monthly outgoing calls (in minutes) for the last three months.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Vertical axis did not start from zero OR vertical height of the bars are not in proportion. It gives readers the wrong impression that the second month's duration of outgoing calls is twice that of first month's duration OR the third month's duration of outgoing calls is thrice that of first month's duration. OR The horizontal axis did not state the order of months, whether was the latest month the bar furthest on the right. It doesn't allow readers to make any conclusions on its trend whether is duration increasing or decreasing across the months.

17 $\xi = \{\text{integer } x : 0 < x \leq 20\}$
$P = \{\text{perfect square}\}$
$Q = \{\text{even number which solves } 3x > 11\}$
$R = \{\text{multiple of } 4\}$

(a) (i) List all the elements in *P*.

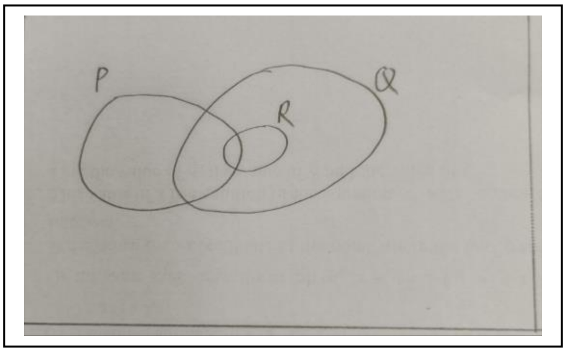
$P = \{1, 4, 9, 16\}$

(ii) Find $n(P' \cap Q)$.

$n(P' \cap Q) = 7$

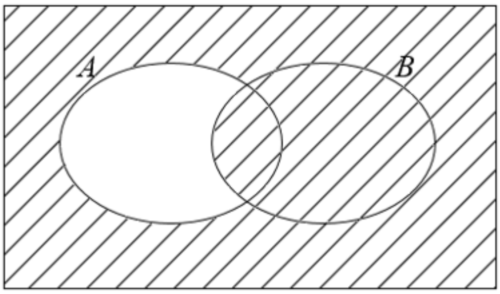
(iii) On the answer space provided, draw the Venn diagram to illustrate the relationship between sets *P*, *Q* and *R*. [2]

ξ



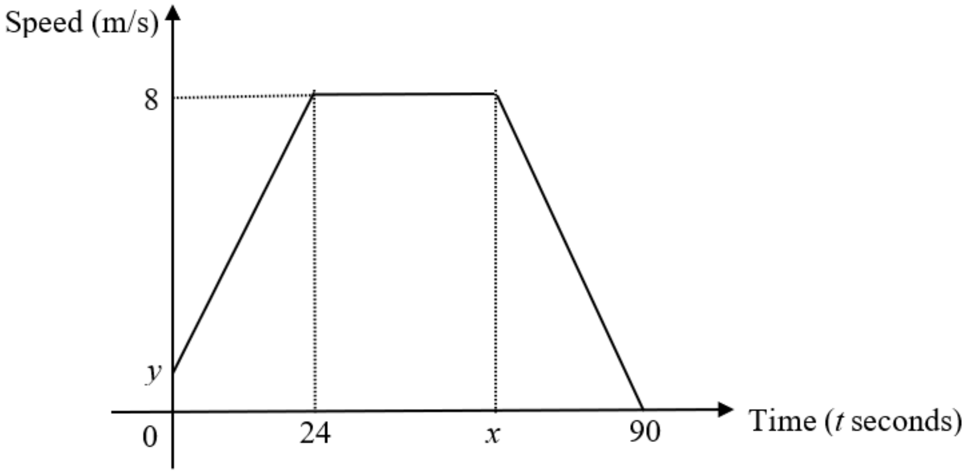
(b) Use set notation to describe the set shaded in the Venn diagram below.

ξ



$A' \cup B$

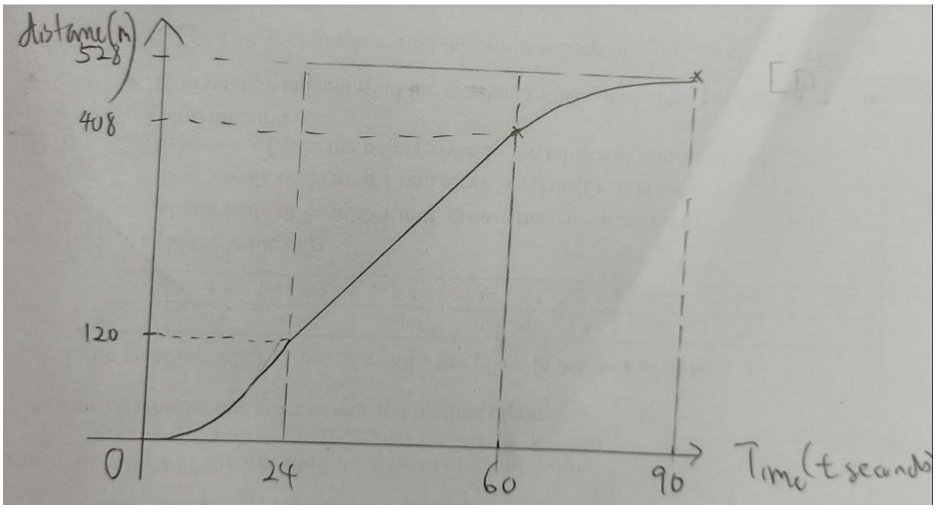
18 The diagram shows the speed-time graph for a cyclist's journey for a period of 90 seconds.
The cyclist accelerates at 0.25 m/s^2 in the first 24 seconds.
He then travels at a constant speed of 8 m/s for a distance of 288 m.



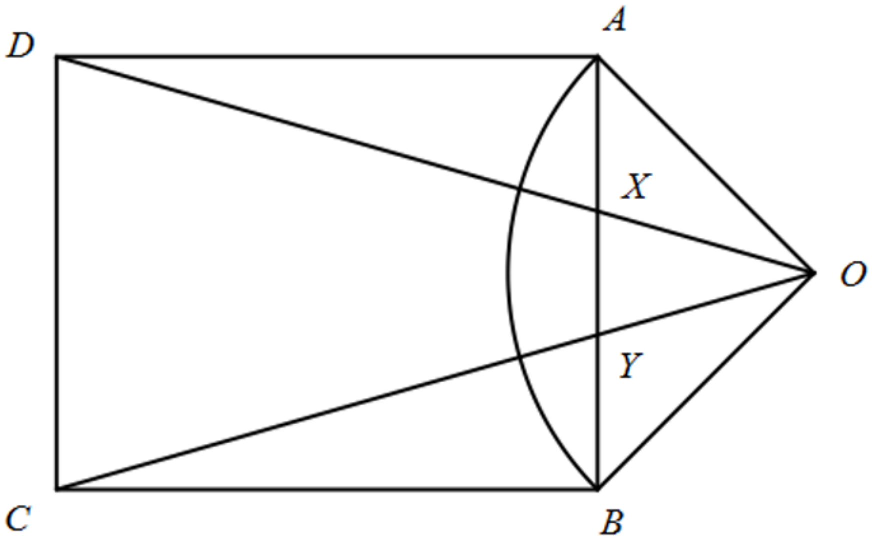
(a) Find the values of *x* and *y*.

$$\frac{8-y}{24}=0.25$$
$$8-y=6$$
$$y=2$$
$$288\div 8=36$$
$$x=24+36$$
$$=60$$

(b) On the grid provided, complete the distance-time graph of the journey from $t = 0$ to $t = 90$ seconds.



19 In the diagram, *ABCD* is a rectangle.
OAB is a sector of a circle, centre *O*.
OXD and *OYC* are straight lines.



(a) Show that triangle *OAD* is congruent to triangle *OBC*.
Give a reason for each statement you make. [3]

$OA = OB$ (radius of circle)
$DA = CB$ (length of rectangle)
Since $\angle DAX = \angle CBY$ (right angle of rectangle),
$\angle OAX = \angle OBY$ (base $\angle$s of isosceles Δ),
$\angle DAO = \angle DAX + \angle OAX$
$= \angle CBY + \angle OBY$
$= \angle CBO$
$\therefore \Delta OAD \equiv \Delta OBC$ (SAS congruency)

(b) Show that triangle *OXY* is similar to triangle *ODC*.
Give a reason for each statement you make. [2]

$\angle DOC = \angle XOY$ (common $\angle$)
$\angle OXY = \angle ODC$ (corresponding $\angle s$, $AB // DC$)
ΔOXY is similar to ΔODC (AA similarity)

(c) The area of triangle *ODC* is 36 times that of the area of triangle *OXY*.
Find the ratio of the area of quadrilateral *DCYX* to area of triangle *OAB*.
Ans: 35:6

20 The ages of 18 swimmers and 11 cyclists in a sports carnival race were recorded.
The results are shown in the stem-and-leaf diagram.

Swimmers	Stem	Cyclists
4 4 3 2 1 1 0	2	1 1 2 2 3 3
9 *x* 5 5	2	5 7
4 1	3	
9 8 8	3	6 9
3 2	4	0

Key (Swimmers)
1 | 2 means 21 years old

Key (cyclists)
2 | 3 means 23 years old

(a) Given that the median age of the swimmers is 26 years old, find the value of *x*.

Median of swimmers' age = 26

$$\frac{25+\text{data}}{2}=26$$

$\text{data}=27$

$x = 7$

(b) Find the interquartile range of the cyclists' age.

lower quartile = 22

upper quartile = 36

interquartile range = 36 - 22

=14

(c) Make two comments comparing the ages of the swimmers and the cyclists.

Median of cyclist' age = 23

interquartile range for swimmers' age = 38 – 23

= 15

The cyclists are generally younger than the swimmers as they have a lower median age.

The ages of swimmers have a larger spread of age than that of the cyclists due to its larger interquartile range.

*only award if median of cyclists is correct and IQR of swimmers and cyclists correct

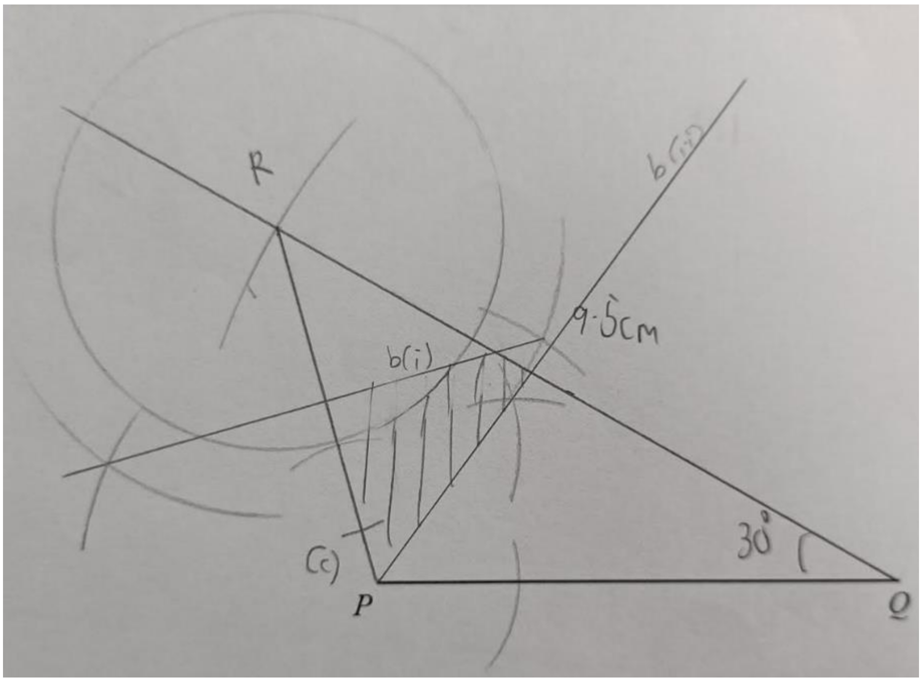
21 The plan of a triangular-shaped garden, *PQR*, is such that $QR = 9.5\text{cm}$ and $\angle PQR = 30°$. *PQ* has been drawn for you.

(a) Construct triangle *PQR* in the space provided. [2]

(b) (i) Construct the perpendicular bisector of *R* and *P*. [1]

(ii) Construct the bisector of angle QPR. [1]

(c) A bench *S* needs to be built inside the garden such that it is nearer to *PR* than to *PQ* and $SR \geq 3$ cm. Shade the region where *S* could be possibly built. [2]



22 Adam invested \$$P$ in a savings account X with interest compounded quarterly at the rate of 1.5% per annum.

Ben invested \$$P$ in a savings account Y, paying simple interest at the rate of x% per year.

At the end of 5 years, Ben made 10% more than Adam.

Mary would like to invest \$$P$ for 20 years.
She believes that savings account Y is better as Ben made more money than Adam.
Justify with clear mathematical working, whether Mary is correct. [4]

$$\text{amount of compound interest} = \left[P\left(1+\frac{1.5}{400}\right)^{20}\right] - P$$

$$= P\left[\left(1+\frac{1.5}{400}\right)^{20} - 1\right]$$

$$\text{amount of simple interest} = P\times\frac{x}{100}\times 5$$

$$= \frac{5Px}{100}$$

$$\frac{5Px}{100} = 1.1P\left[\left(1+\frac{1.5}{400}\right)^{20} - 1\right]$$

$$x = 1.71013$$

If compounded using X,

$$\text{interest amount} = P\left[\left(1+\frac{1.5}{400}\right)^{80} - 1\right]$$

$$= 0.34910P*$$

If simple interest using Y,

$$\text{interest amount} = P\times\frac{1.71013}{100}\times 20$$

$$= 0.34203P*$$

Mary is not correct.

– End of Paper –