NAVAL BASE SECONDARY SCHOOL
PRELIMINARY EXAMINATION, 2021

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Name ______________________________( ) **Class** ______________

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **30 August 2021**

Candidates answer on the Question Paper **2 hours 15 minutes**
No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

Item	***For Examiner's Use***
Presentation	
Accuracy	
Units	
Total	
Parent's Signature	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 Express $\dfrac{20-3x-2x^2}{(2x+3)(x-1)^2}$ in partial fractions. [6]

2 The table shows experimental values of two variables x and y.

x	1.5	2	3	4	5
y	10.9	14.1	24.0	41.7	76.4

It is known that x and y are related by an equation $y = ab^x + e$, where a and b are constant.

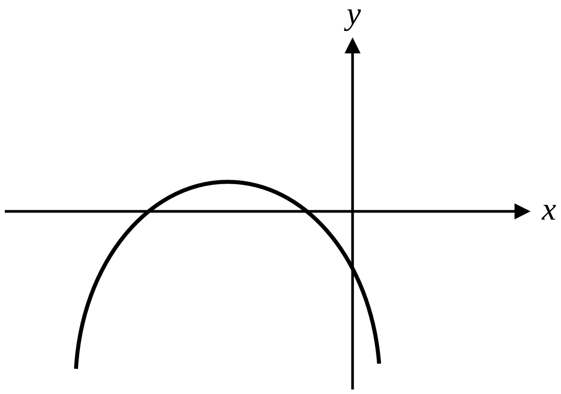
(i) Explain how a straight line graph may be drawn to represent the given data. [2]

(ii) On the grid on **page 5**, draw this graph for the given data and use it to estimate the value of a and of b. [4]

(iii) By inserting another suitable line on your graph, solve the equation $ab^x = 4e^{-\frac{x}{2}}$. [2]

3 **(a)** Find the range of values of k for which the curve $y=(x-2)(x-3)$ and the line $y=kx+5$ do not intersect. [3]

(b)



The above sketch shows part of the graph of $y = px^2 + qx + r$.
For each of the following expressions state, with a reason, whether it is positive, zero or negative.

(i) r, [2]

(ii) $q^2 - 4pr$, [2]

(iii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [2]

4 **(i)** Express $\dfrac{8x}{4x+1}$ in the form $a+\dfrac{b}{4x+1}$, where a and b are integers. [2]

(ii) Given that $y=2x\ln(4x+1)$, find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(iii) Using the results in part **(i)** and **(ii)**, determine $\int \ln(4x+1)\ \mathrm{d}x$. [4]

5 **(a)** The equation of a curve is $y = xe^{-\frac{x}{2}}$.

(i) Find expressions for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$. [5]

(ii) Find the **exact** value of the coordinates of the stationary point. [2]

(b) Find $\int 3\sin\frac{x}{2}\ \mathrm{d}x$. [2]

6 A circle, centre C, has a diameter AB where A is the point $(-3, 7)$ and B is the point $(5, 1)$.

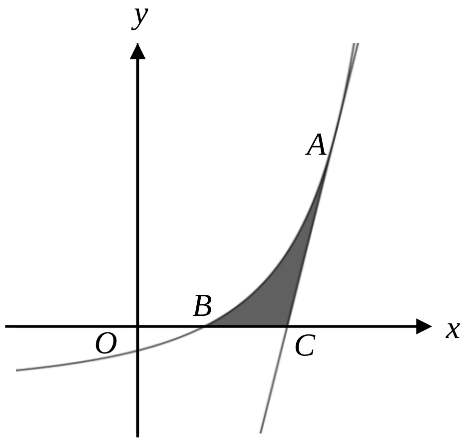
(i) Find the coordinates of C and the radius of the circle. [4]

(ii) Find the equation of the circle. [1]

(iii) Show that the equation of the tangent to the circle at B is $4x - 3y = 17$. [3]

(iv) The lowest point on the circle is D. Find the coordinates of the point at which the tangents to the circle at B and D intersect. [2]

7



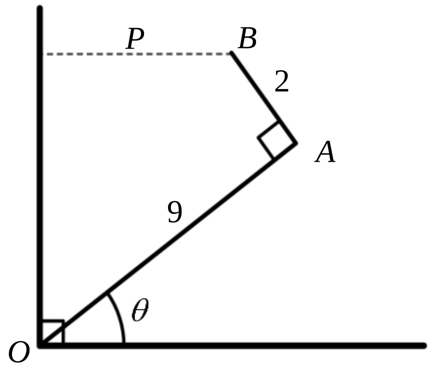
The diagram show part of the curve $y = \frac{9}{(4-x)^2} - 1$ cutting the x-axis at B. The tangent at the point A on the curve cuts the x-axis at C. Given that the gradient of this tangent is 18,

calculate

(i) the coordinates of the point A, [5]

(ii) the area of the shaded region *ABC*. [7]

8



A L-shaped structure *OAB* is hinged at the point *O*. *OA* is 9 m and *AB* is 2 m long.
OA makes an acute angle, θ, with the ground.
Given that *P* is the perpendicular distance from *B* to the wall.

(i) Show that $P = 9\cos\theta - 2\sin\theta$. [2]

(ii) Express *P* in the form $R\cos(\theta + \alpha)$ where $R > 0$, $0° < \alpha < 90°$. [4]

(iii) State the minimum value of P and find the corresponding value of θ. [3]

(iv) Find the value of θ when $P = 5$ m. [2]

(v) Explain why the maximum value of P is not R. [1]

9 **(a)** Using the substitution $u = 5^x$, solve the equation $8 - 5^{2x+1} = 6(5^x)$. [4]

(b) Solve the equation $\log_3(2x+1)+\log_{\frac{1}{3}}3=\log_9(x-2)^4-\log_3(x-1)$. [5]

(c) Given that $\log_3 x = p$ and $\log_9 y = q$, express the following in terms of p and q.

(i) $\log_9 \dfrac{x}{y}$, [2]

(ii) $x^2 y$, [2]

(iii) $\log_x 3y$. [2]

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