Mathematics

If you develop the habits of success, you will make success a habit.

Michael E Angier

Number Systems

Sets of numbers

Set of **whole numbers**,
{ 0, 1, 2, 3, 4, …… }

Set of **integers**,
{ ……, -3, -2, -1, 0, 1, 2, 3, …… }

Set of **rational numbers**,
{ a rational number is of the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$}

Set of **real numbers**,
{all rational and irrational numbers}

Rational and Irrational Numbers

The set of **rational numbers** consists of terminating decimals (eg. $5.625 = 5\frac{5}{8}$) and recurring decimals (eg. $0.2727... = \frac{3}{11}$).

Irrational numbers are numbers which cannot be expressed as a fraction (eg. $\pi, \sqrt{2}$)

$\frac{22}{7}$ is a rational number used to approximate π .

Note the definition of rational and irrational numbers.

Whole numbers

Set of **even numbers** = { 2, 4, 6, 8, …… }
= { set of multiples of 2 }

Set of **odd numbers** = { 1, 3, 5, 7, …… }

Set of **prime numbers** = { 2, 3, 5, 7, 11, …… }

> A prime number, p, is a positive integer that is exactly divisible only by itself (p) and by 1. p cannot be equal to 1.
>
> Hence, 1 and 0 are not prime numbers. They are also not composite numbers

If x is an integer, then $2x$ gives an even number
and $(2x-1)$ gives an odd number.

Memorise prime nos. up to 31 → 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31
Memorise perfect squares up to 17^2 → 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 196, 225, 256, 289

Integers

E.g. The temperature at 0900 is –4°C.
The temperature at 1500 is 14°C.
The temperature at 2100 at –4°C.

Note:
Difference betw highest and lowest = Higher – Lower = 14 – (–4) = 18°C
Increase in temp. from 0900 to 1500 = Higher – Lower = 14 – (–4) = 18°C
Decrease in temp. from 1500 to 2100 = Higher – Lower = 14 – (–4) = 18°C

Mean temperature

$$=\frac{\text{Sum of temp}}{2}=\frac{14+(-4)}{2}=5°C$$

Significant Figures

The first non-zero digit of any number is called the first significant figure.

eg. 236	= 200	(correct to 1 sig. fig.)
0.0236	= 0.024	(correct to 2 sig. fig.)
356	= 360	(correct to 2 sig. fig.)
0.3002	= 0.300	(correct to 3 sig. fig.)
6.049	= 6.05	(correct to 3 sig. fig.)
6.049	= 6.0	(correct to 2 sig. fig.)

Place values cannot change.

Decimal Places

eg. 53.14	= 53.1	(correct to 1 dec. place)
0.7364	= 0.74	(correct to 2 dec. places)
7.1302	= 7.130	(correct to 3 dec. places)
0.00743	= 0.007	(correct to 3 dec. places)

Standard Form

Very large or small numbers can be expressed in standard form $a \times 10^n$, where $1 \le a < 10$, and n is an integer.

eg. 1 350 000 = 1.35×10^6
0.00135 = 1.35×10^{-3}

Can only add and subtract a directly when indices are the same.

Highest Common Factor & Lowest Common Multiple

$168 = 2^3 \times 3 \times 7$
$324 = 2^2 \times 3^4$

or

2	168	324
2	84	162
3	42	81
	14	27

HCF of 168 and 324 = $2^2 \times 3 = 12$ lower indices

LCM of 168 and 324 = $2^3 \times 3^4 \times 7 = 4536$ higher indices

If $168x$ is a perfect square, then $x = 2 \times 3 \times 7 = 42$ indices mult of 2

If $324y$ is a perfect cube, then $y = 2 \times 3^2 = 18$ indices mult of 3

If $168k$ is a multiple of 324, then $k = 3^3 = 27$ 324 is subset of $168k$

Indices

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $a^m \cdot b^m = (ab)^m$
5. $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
6. $a^0 = 1, a \neq 0$
7. $\frac{1}{a^m} = a^{-m}$
8. $a^{\frac{1}{n}} = \sqrt[n]{a}$
9. $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Simple Interest

Interest, $I = \frac{PRT}{100}$, where P = principal
R = rate per year in percentage
T = time

Principal → original sum borrowed

Time → in years

Interest rate → given per annum, meaning per year

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

P: Principal (the original amount of money)

r: Compound interest rate (% per annum)

n: No of Years

Mary deposited $500 in her OCBC account at a compound interest rate of 0.4% per year.

Find the total amount in Mary's account at the end of 3 years.

$$\text{Total amount} = 500\left(1+\frac{0.4}{100}\right)^3$$

$$= 506.024$$

$$= \$506.02$$

Hire Purchase (HP)

Paying by instalments

Eg. A car costs $128 000 (cash price, if buyer pays cash)

Down payment of 20% = $25 600 (no int charged)
Remaining Balance = $102 400 @ 5% simple int p.a over 10 years

Int charged over 10 years = ($102 400 x 0.05 x 10) = $51 200

Total sum paid for the car using HP (more expensive than cash price)
= cash price + int charged
= $128 000 + $51 200
= $179 200

Instalment paid per month = (remaining bal + int) / no. of instalments
= ($102 400 + $51 200) / (10 x 12)
= $1 280

p.a. means per annum

4-4

Discount

A discount is a reduction in price, usually given as a percentage of the list price.

Original selling price → Marked price or List price,
rep. by 100% (base)

Sale price → Price after discount
rep. by (100% – discount %)

$$\% \text{ discount} = \frac{\text{discount}}{\text{original selling price}} \times 100\%$$

4-5

Profit or loss as a percentage of Cost

$$\text{Profit as a \% of cost} = \frac{\text{Profit}}{\text{Cost}} \times 100\%$$

$$\text{Loss as a \% of cost} = \frac{\text{Loss}}{\text{Cost}} \times 100\%$$

Cost price for the seller → rep. by 100% (base)

Selling price → rep. by (100% + profit %)
Selling price → rep. by (100% – loss %)

4-6

In general,

$$\text{Percentage increase} = \frac{\text{Increase}}{\text{Original Amount}} \times 100\%$$

$$\text{Percentage decrease} = \frac{\text{Decrease}}{\text{Original Amount}} \times 100\%$$

Original amount → rep. by 100% (base)

New amount → rep. by (100% + increase %)
New amount → rep. by (100% – decrease %)

5-1

Maps & Scales

The scale of a map is calculated in centimetres and expressed as a ratio.

If 5 cm rep. 3 km, the scale is 1 : 600.

Eg. A map is drawn to a scale of 1 : 400 000.
A city covers an area of 800 km^2. Find in sq. centimetres, the area representing the city on the map.

1 cm rep. 400 000 cm

1 cm rep. 4 km

Sq both sides ( ) Sq both sides

1^2 cm^2 rep. 4^2 km^2

1 cm^2 rep. 16 km^2

$$\text{Area rep. city on map} = \frac{800}{16} = 50 \text{ cm}^2$$

Density

The density of a substance is mass per unit volume. The units of density are gm/cm^3 or kg/m^3

$$Density = \frac{Mass}{Volume}$$

Mass is directly proportional to volume.

Proportion

(i) **Direct Proportion (proportionate or directly proportionate)**
If y varies directly as x, then $y = kx$ where k is a constant.

(ii) **Inverse Proportion (inversely proportionate)**

If y varies inversely as x, then $y = \frac{k}{x}$ where k is a constant.

Direct & Inverse Proportion

Eg.1) p is proportional to q^3. It is known that $p = 24$ for a particular value of q. Find the value of p when this value of q is halved.

When q is halved, $p = (\frac{1}{2})^3 \times 24 = \frac{1}{8} \times 24 = 3$

Eg.2) y is inversely proportional to x^2. $y = 4$ when $x = 3$.
Find y when $x = 10$.

When $x = 3$, $y = \frac{k}{x^2}$

$4 = \frac{k}{3 \times 3}$

$\therefore k = 36$

When $x = 10$, $y = \frac{36}{10 \times 10} = 0.36$

Proportion

(i) Inverse proportion

A unit of flat can be completed by 10 workers in 20 days.

	Men	Flat	Days
	10	1	20
Calculate for 1 man 1 flat	1	1	10 × 20 = 200

No. of days to complete a similar unit by 8 workers

$= \frac{200}{8} = 25$

(ii) Direct proportion

A toy costs \$20. The cost of 5 similar toys = \$20 × 5 = \$100

Prefixes for SI units

Tera	T	10^{12}	Greek 'teras' = monster
Giga	G	10^{9}	Greek 'gigas' = giant
Mega	M	10^{6}	Greek 'megas' = large
Kilo	K, k	10^{3}	Greek 'kilioi' = thousand
hecto	h	10^{2} or 100	Greek 'hekaton' = hundred
deca	da	10	Greek 'deka' = ten
deci	d	10^{-1} or 0.1	Latin 'decima pars' = one tenth
centi	c	10^{-2} or 0.01	Latin 'centesima pars' = one hundredth
milli	m	10^{-3}	Latin 'millesima pars' = one thousandth
micro	μ, u	10^{-6}	Greek 'mikros' = small
nano	n	10^{-9}	Latin 'nanus' = dwarf
pico	p	10^{-12}	Spanish 'pico' = minimal measure

Multiply (↓) Divide (↑)

Conversion of Units

Eg. For length, the SI unit is metre.

×1000 ×1000 ×1000 ×10 ×10 ×10 ×10 ×10 ×10 ×1000 ×1000 ×1000

Tm Gm Mm Km hm dam m dm cm mm μm nm pm

÷1000 ÷1000 ÷1000 ÷10 ÷10 ÷10 ÷10 ÷10 ÷10 ÷1000 ÷1000 ÷1000

Number Patterns

Linear Pattern

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}		
2	5	8	11	14	p	...	...

+3 +3 +3 +3 +3

Since the differences between consecutive terms are the same, the expression for the n^{th} term, it is just $an+k$.

$a =$ difference between consecutive terms
$k =$ first term – difference

E.g. In this case, $a = 3$
$k = 2-(3) = -1$

The expression for the nth term is $3n-1$.

Number Patterns

Quadratic Pattern

1 6 15 28 45 ....

1st difference: +5 +9 +13 +17 +21

2nd difference: +4 +4 +4 +4

a = first term, d_1 = first difference, d_2 = second difference

The n^{th} term is

$$n^{th}\ term = a + d_1(n-1) + \frac{1}{2}(d_2)(n)(n-1)$$

Expansion

Some useful identities

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

Factorisation (Reverse of Expansion)

(i) Common Factors

Consider the expression $4a^2b + 2ab - 6ab^2$.
Each term in the expression contains $2ab$.
$\therefore 4a^2b + 2ab - 6ab^2 = 2ab\,(2a - 3b + 1)$ — Identify HCF

(ii) Grouping

Consider : $3x + 10xy - 5y - 6x^2$
$= (3x - 6x^2) + (10xy - 5y)$
$= 3x(1 - 2x) + 5y(2x - 1)$
$= (3x - 5y)(1 - 2x)$

Four terms, factorise two at a time using HCF

(iii) Difference of two squares — Two terms written as perfect squares, minus sign in between

$25x^4 - 4 = (5x^2)^2 - (2)^2$
$= (5x^2 - 2)(5x^2 + 2)$

(iv) Cross Multiplying (Trial & Error) — Three terms, in the form $ax^2 + bx + c$

To find the factors of $2x^2 + 5x - 3$.

$2x$	-1	$+6x$
x	+3	$-x$
$2x^2$	-3	$+5x$

Hence, $2x^2 + 5x - 3 = (2x - 1)(x + 3)$

Quadratic Equation

An equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation, where a, b and c are constants.

How to solve

(i) **By factorisation**

Solve the equation $2x^2 + 5x - 3 = 0$
We can have $(2x - 1)(x + 3) = 0$
$x = ½$ or -3

(ii) **Using the quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

"Solve" – give the values of the variables or unknowns when $y = 0$
2 methods to solve

7C-1

Rate

Eg. A pool is filled by two pipes A and B working together in 13 hours. Pipe A alone can fill it up in x hours while pipe B alone takes $2x + 2$ hours. Form an equation in x.

In x hr, A fills one pool. ∴ in 1 hr, A fills $\frac{1}{x}$ of the pool.

In $(2x+2)$ hr, B fills one pool. ∴ in 1 hr, B fills $\frac{1}{2x+2}$ of the pool.

In 13 hours, A and B fill the whole pool.

This implies $\frac{13}{x}+\frac{13}{2x+2}=1$

Think in terms of fraction of a pool

This is simplified to $2x^2-37x-26=0$.

7D-1

Inequality

Properties of inequalities:

(i) If $a \geq b$, then

$$a+k \geq b+k$$
$$a-k \geq b-k$$

(ii) If $a \geq b$ and $c \geq 0$, then

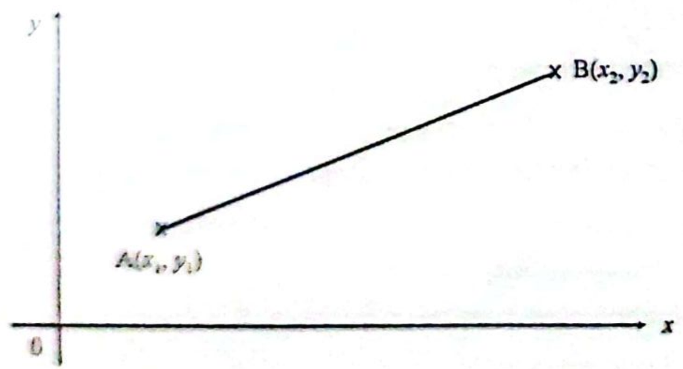
$$ca \geq cb$$
$$\frac{a}{c} \geq \frac{b}{c}$$

(iii) If $a \geq b$ and d is **negative**, then

$$da \leq db$$
$$\frac{a}{d} \leq \frac{b}{d}$$

Dividing or multiplying by a negative number changes the inequality sign

8-1



The **gradient** of the line joining any two points A(x_1, y_1) and B(x_2, y_2) is

$$m=\frac{y_2-y_1}{x_2-x_1}$$

Any two points sitting on the line

8-2

The **distance between two points** A(x_1, y_1) and B(x_2, y_2) is given by

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

8-3

Horizontal Line

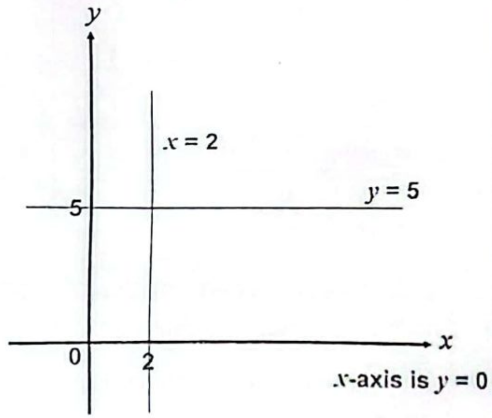
(i) Gradient is zero.

(ii) Equation y = constant

Vertical Line

(i) Gradient is undefined.

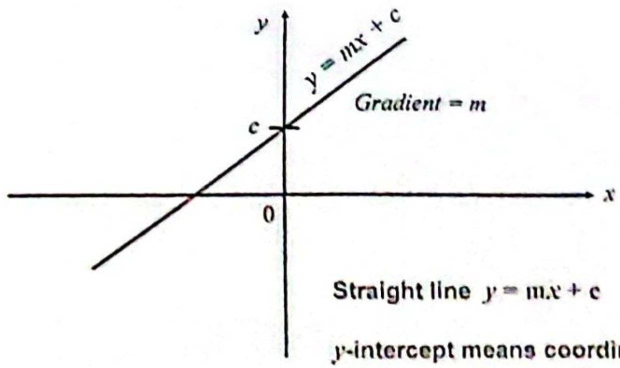
(ii) Equation x = constant



Parallel lines have the same gradient.

8-4

The gradient/intercept form of the equation of a straight line is $y = mx + c$, where m = gradient and c = intercept on y-axis.



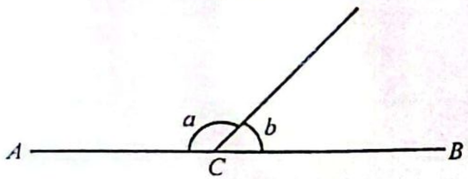
Straight line $y = mx + c$

y-intercept means coordinates $(0, c)$ and c can be zero

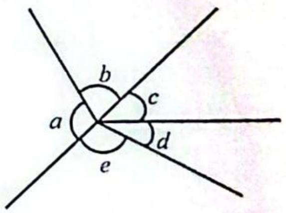
9-1

Basic Angle Properties

$a + b = 180°$ (adj. ∠s on st line)

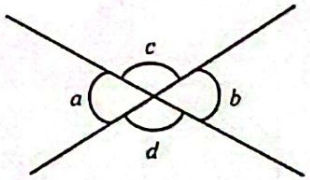


$a + b + c + d + e = 360°$ (∠s at a pt)



$a = b$ (vert. opp. ∠s)

$c = d$ (vert. opp. ∠s)



9-2

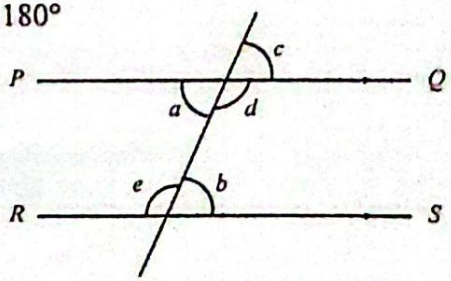
$a = b$ (alternate ∠s)

$c = b$ (corresponding ∠s)

$b + d = 180°$ (interior ∠s between || lines)

$a + e = 180°$

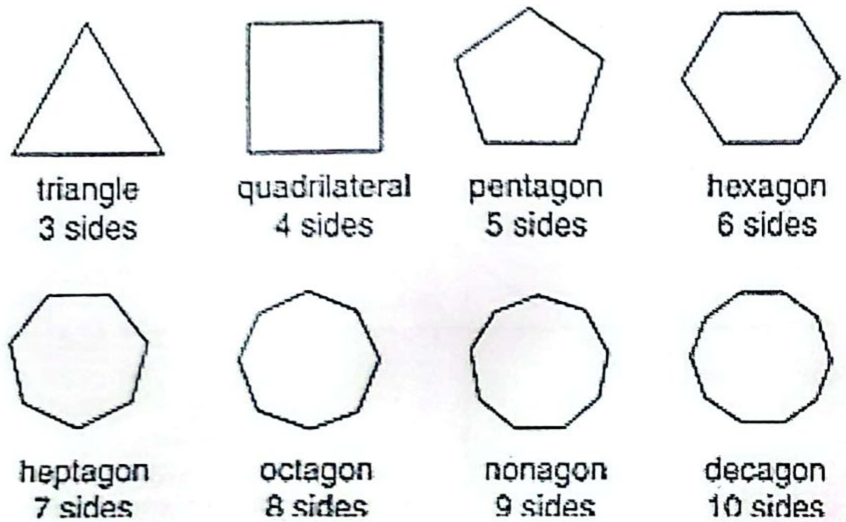
Only for parallel lines



a and b are complementary angles if $a + b = 90°$

a and b are supplementary angles if $a + b = 180°$

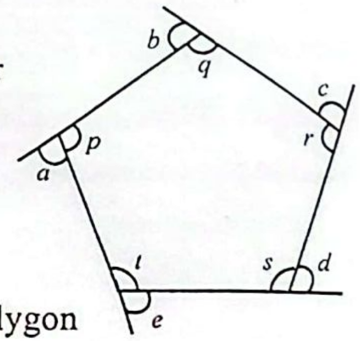
Types of Polygons



Sum of interior angles of a n-sided polygon
$= (n - 2) \times 180°$

Each ~~interior angle~~ of a regular n-sided polygon

$$= \frac{(n-2)\times 180°}{n}$$



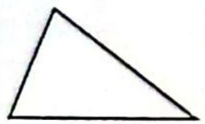
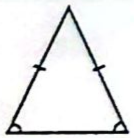
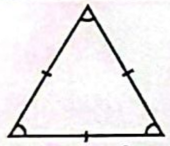
Sum of **exterior angles** of a polygon
$= 360°$ (e.g. $a + b + c + d + e$)

Each exterior angle of a regular n-sided polygon $= \frac{360°}{n}$

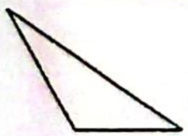
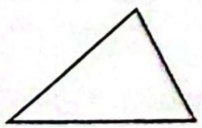
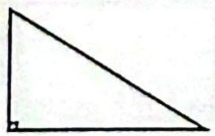
Difference between regular and irregular polygons

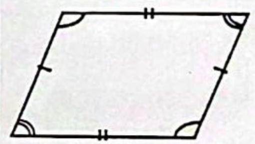
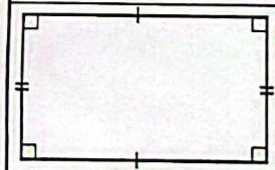
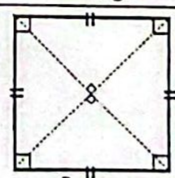
Triangles are classified according to:

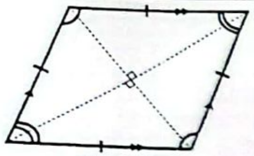
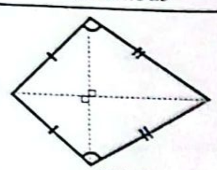
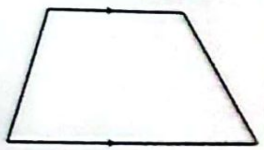
(a) the lengths of their sides, ie.

Scalene	Isosceles	Equilateral
 All sides are of unequal length	 Two sides are of equal length	 All sides are of equal length

(b) the size of their angles, ie.

Obtuse-angled	Acute-angled	Right-angled
 One angle is an obtuse angle	 All angles are acute angles	 One angle is a right angle

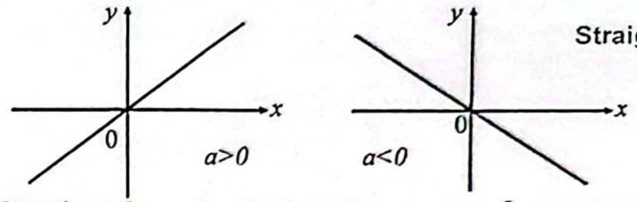
Quadrilateral	Sides	Angles	Diagonals
 Parallelogram	Both pairs of opposite sides are equal & parallel.	Both pairs of opposite angles are equal.	Diagonals bisect each other.
 Rectangle	Both pairs of opposite sides are equal & parallel.	All four angles are 90°.	Diagonals bisect each other & are equal.
 Square	All four sides are equal. Opposite sides are parallel.	All four angles are 90°.	Diagonals are equal bisect each other at right angles

Quadrilateral	Sides	Angles	Diagonals
 Rhombus	All four sides are equal. Both pairs of opposite sides are parallel.	Both pairs of opposite angles are equal.	Diagonals bisect each other at right angles
 Kite	Two pairs of adjacent sides are equal.	One pair of opposite angles are equal.	Longer diagonal bisects shorter diagonal at right angles; bisects interior angles.
 Trapezium	One pair of opposite sides are parallel.	Interior angles add up to 180°	

Graphs of functions

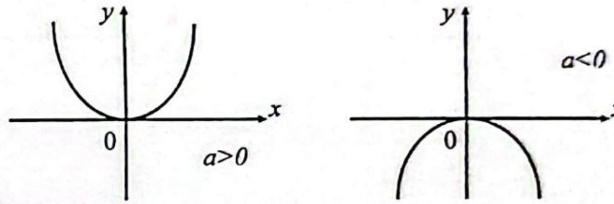
Graphs of $y = ax^n$ where $n = \pm1, \pm2, +3$.

(i) Graphs of linear function: $y = ax$, when $n = +1$.



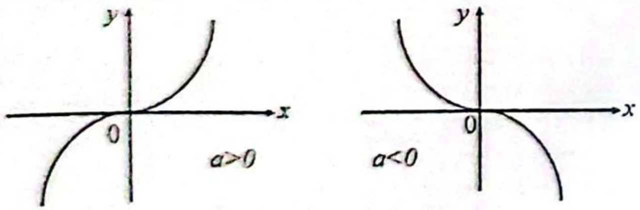
Straight line graphs

(ii) Graphs of quadratic function: $y = ax^2$, when $n = +2$.



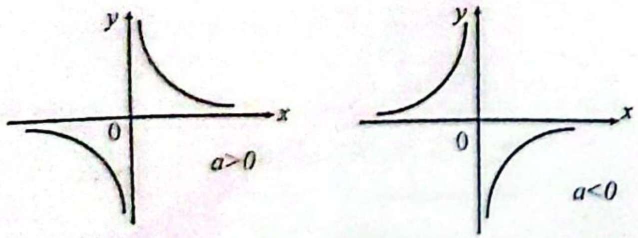
In x^2 graphs, y-intercept works the same way as straight lines

(iii) Graphs of $y = ax^3$, when $n = +3$.



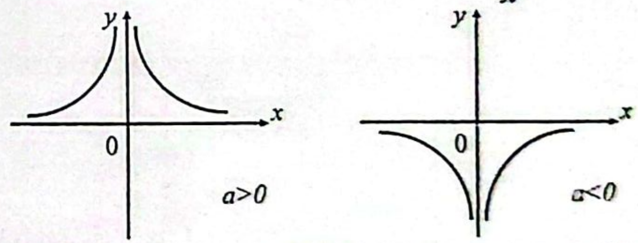
x^3 graphs

(iv) Graphs of reciprocal function: $y = \frac{a}{x}$, when $n = -1$.



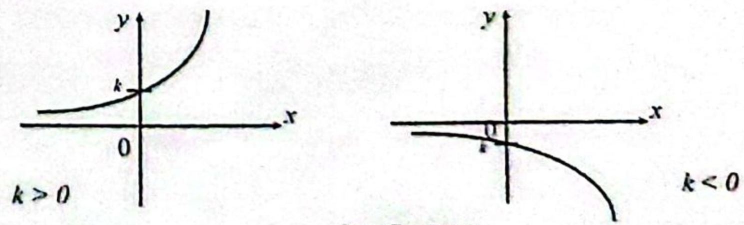
$1/x$ graphs

(v) Graphs of reciprocal function $y = \frac{a}{x^2}$, when $n = -2$.



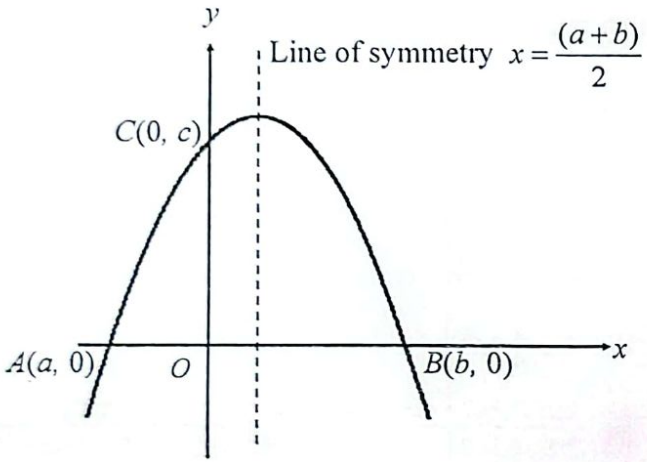
$1/x^2$ graphs

(vi) Graphs of exponential function: $y = ka^x$ where a is a positive integer



10-4

E.g. Graph of $y=(x+2)(3-x)$



The roots are -2 and 3.

y-intercept is 6.

10-5

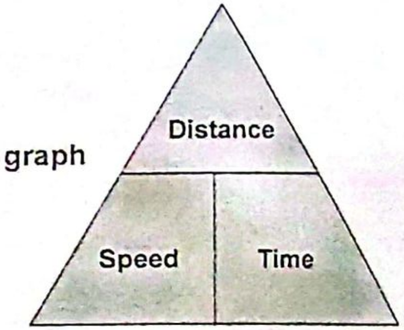
Average Speed

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Total time include rest times.

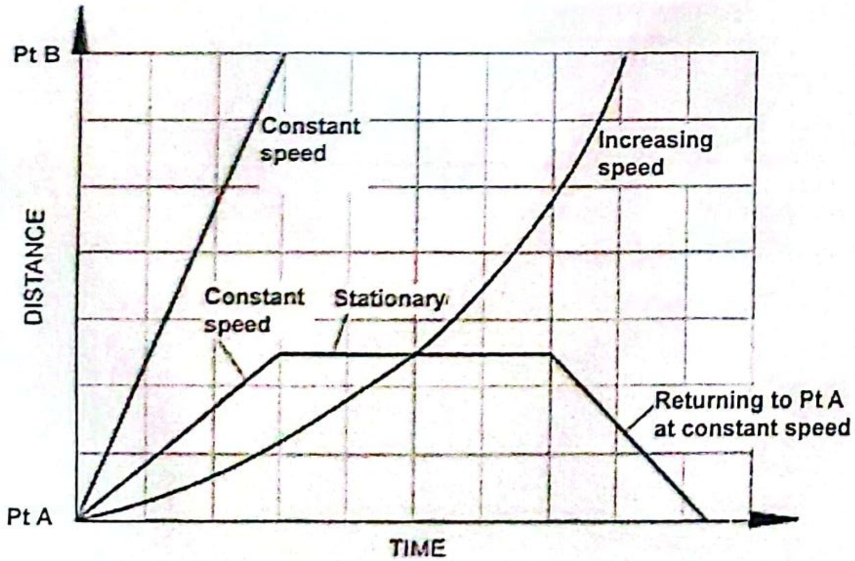
Distance-time graph
distance = add from the lines of the graph
gradient = speed

Speed-time graph
distance = area under graph
gradient = acceleration



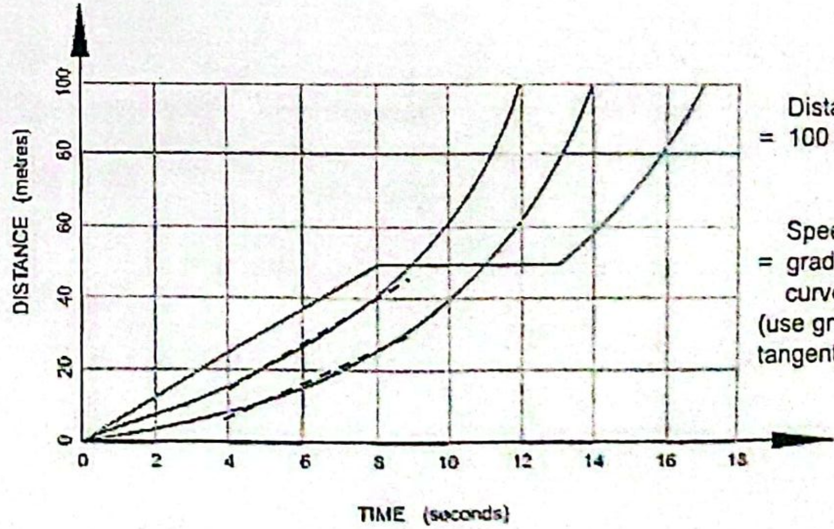
10-6

Distance-Time Graph



10-7

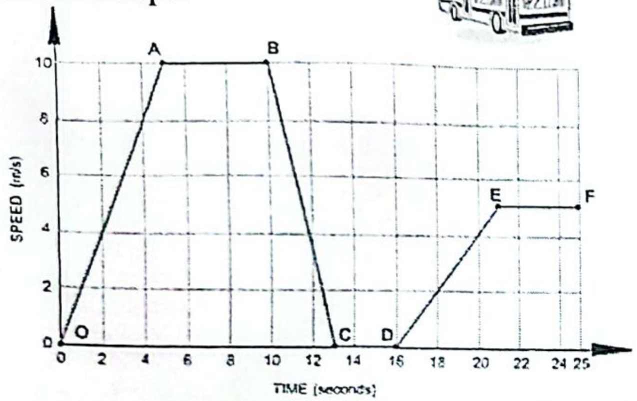
Distance-Time Graph



Distance = 100 m each

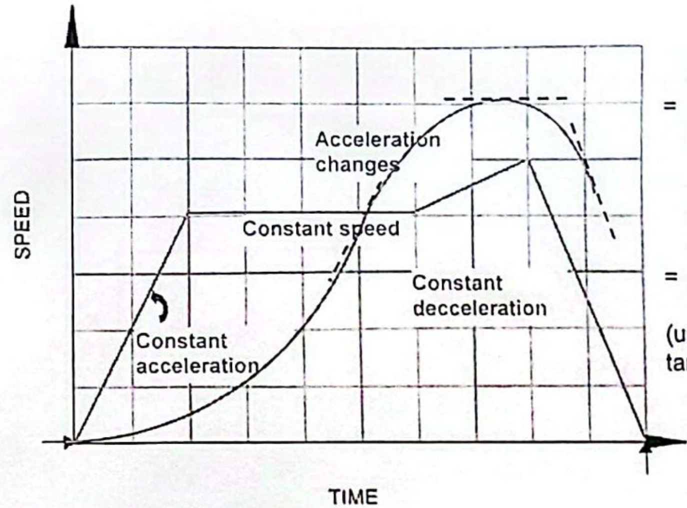
Speed at time t = gradient of curve at t (use gradient of tangent at t)

Speed-Time Graph



0-A Acceleration. Speed changes from 0 to 10 m/s in 5 seconds.
A-B Steady speed of 10 m/s for 5 seconds.
B-C Deceleration. Slows down from 10 m/s to rest in 3 seconds.
C-D Stationary. Speed is zero.
D-E Acceleration. Gradually increasing in speed from zero.
E-F Moves at a steady speed of 5 m/s.

Speed-Time Graph



Distance = area under graph

Acceleration or Deceleration at time *t* = gradient of curve at *t* (use gradient of tangent at *t*)

Probability

$$P(E) = \frac{\textit{No. of outcomes favourable to the occurence of } E}{\textit{Total number of equally likely outcomes}}$$

Note that $0 \leq P(E) \leq 1$.

What does it mean if $P(E) = 0$?

The event cannot possibly happen.

What does it mean if $P(E) = 1$?

The event will certainly happen.

Probability

For two mutually exclusive events A and B,

- P(*A* occurs or *B* occurs) = P(*A* or *B*)
  = P(*A*) + P(*B*)
- P(**not** *E*) = 1 – P(*E*)

For two events *A* and *B* that can occur together,

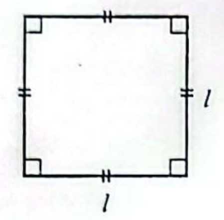
- P(*A* occurs and *B* occurs) = P(*A* and *B*)
  = P(*A*) × P(*B*)

12-1

Square

Area = l^2

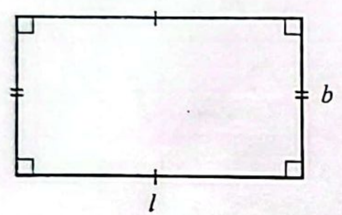
Perimeter = $4l$



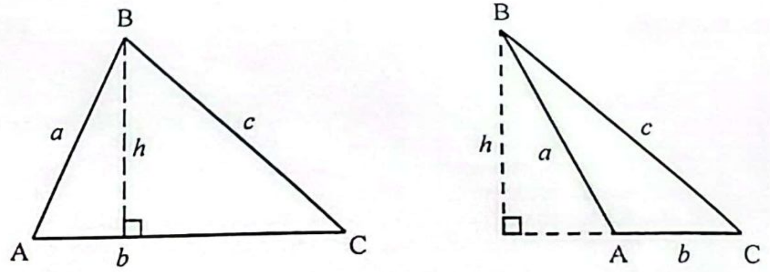
Rectangle

Area = $l \times b$

Perimeter = $2(l + b)$



12-2



Triangle

Area = $\frac{1}{2}$ x base x height

$= \frac{1}{2} bh$

$= \frac{1}{2} ab \sin C$

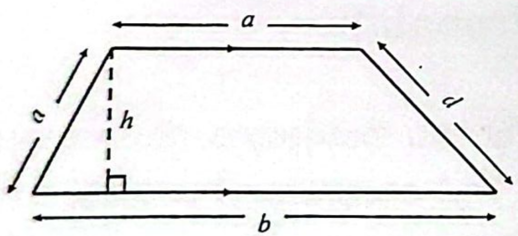
Perimeter = $a + b + c$

12-3

Trapezium

Area = $\frac{1}{2}(a + b)h$

Perimeter = $a + b + c + d$



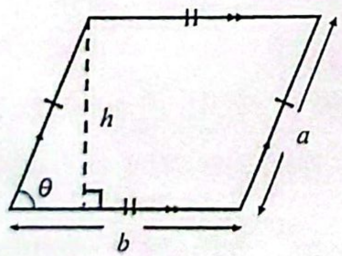
Parallelogram

Area = base x height

$= bh$

$= ab \sin\theta$

Perimeter = $2(a + b)$



12-4

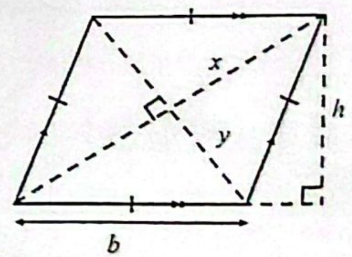
Rhombus

Area = bh

$= \frac{1}{2} xy$

where x and y are diagonals.

Perimeter = $b + b + b + b$



12-5

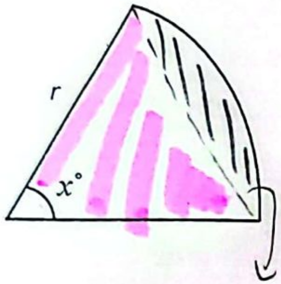
Angle is measured in degrees

Sector

Area = $\frac{x^\circ}{360^\circ} \times \pi r^2$

Arc Length = $\frac{x^\circ}{360^\circ} \times 2\pi r$

Perimeter = $r + r + \frac{x^\circ}{360^\circ} \times 2\pi r$



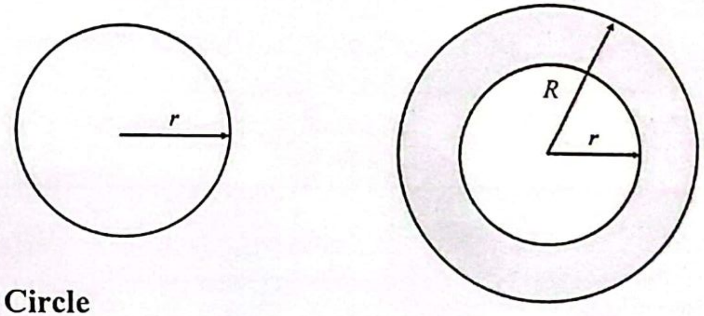
Perimeter → external outline

$= \frac{x^\circ}{360} \times \pi r^2 - \frac{1}{2}ab\sin C$

Sec 4Exp/2016

big - small = ※

12-6



Circle

Area = πr^2

Perimeter = $2\pi r$

Area of annulus = $\pi R^2 - \pi r^2$

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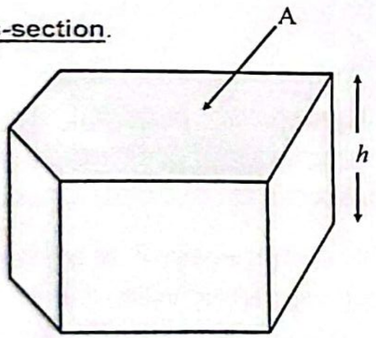
12-7

A prism has a <u>uniform cross-section</u>.



Prism

Volume = (Area of cross-section) x height

= $A \times h$

Total surface area = Sum of all sides

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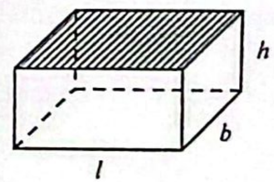
12-8

Cuboid

Volume = $l \times b \times h$

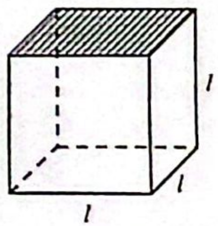
Total surface area = $2(lb + bh + lh)$



Cube

Volume = l^3

Total surface area = $6(l \times l)$



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12-9

Volume of solids

Solids

Right Prisms	Right Pyramids	Right Circular Cones	Sphere
base area x ht	⅓ x base area x ht	⅓ x base area x ht	⅔ x base area x ht
		$\frac{1}{3}\pi r^2 h$	$\frac{4}{3}\times \pi r^3$

12-10

Total surface area of solids

Solids

Right Prisms	Right Pyramids	Right Cones	Sphere
Sum of areas of base & rectangular (or sq) sides	Sum of areas of base & triangular sides	Sum of areas of circular base & curved surface area	Surface area
		$\pi r^2 + \pi rl$	$4\times \pi r^2$

12-11

Conversion of Units

Length, $1\ cm = 10\ mm$
$1\ m = 100\ cm$
$1\ km = 1000\ m$

Area, $1\ cm^2 = 10^2\ mm^2$
$1\ m^2 = 10^4\ cm^2$
$1\ hectare\ (ha) = 10^4\ m^2$
$1\ km^2 = 100\ hectares$

Volume, $1\ litre = 1000\ ml$
$= 1000\ cm^3$

$1\ m^3 = 1\ 000\ 000\ cm^3$

Mass, $1\ g = 1000\ mg$
$1\ kg = 1000\ g$
$1\ tonne = 1000\ kg$

13A-1

Congruency

Congruent figures have the **same shape and size**.
(i) All corresponding angles are equal.
(ii) All corresponding sides are equal.

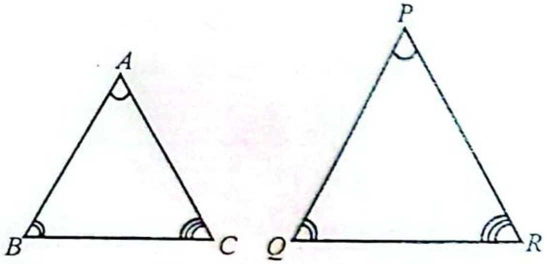
Four tests for congruency of triangles:
(a) **SSS** – corresponding sides are equal
(b) **SAS** – 2 sides & angle "in-between" are equal
(c) **ASA** – 2 angles & any 1 side
(d) **RHS** – 1 right angle, equal hypotenuse & 1 side

Memorise the te[illegible]
Corresponding → im[illegible]rtant point

Similarity

13A-2

Similar figures have the **same shape but differ in size**.
(i) All corresponding angles are equal.
(ii) Corresponding sides are in the same ratio.



If $\Delta ABC \equiv \Delta PQR$, then

$\angle A = \angle P$

$\angle B = \angle Q$ $\quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$

$\angle C = \angle R$

There are 3 tests for similarity of triangles:
(a) Corresponding angles are equal (AAA)
(b) Corresponding sides are in the **same ratio** (SSS)
(c) Two pairs of corresponding sides are in the same ratio and a pair of included angles are equal (SAS)

13A-3

For two similar plane figures, the ratio of their areas (A) is equal to the square of the ratio of any two corresponding lengths (l) $\quad \left(\frac{A_1}{A_2}\right) = \left(\frac{l_1}{l_2}\right)^2$

For two similar solids,

(a) the ratio of their surface areas (A) is equal to the square of the ratio of any two corresponding lengths (l) $\quad \left(\frac{A_1}{A_2}\right) = \left(\frac{l_1}{l_2}\right)^2$

(b) ☺ the ratio of their volumes (V) is equal to the cube of the ratio of any two corresponding lengths (l) $\quad \frac{V_1}{V_2} = \frac{M_1}{M_2} = \left(\frac{l_1}{l_2}\right)^3$

☺ the ratio of their masses (M) is equal to the cube of the ratio of any two corresponding lengths (l)

13B-1

Right-angled Triangles

In a triangle *ABC* right-angled at *B*, the longest side *AC* (opposite the right angle) is the *hypotenuse*.

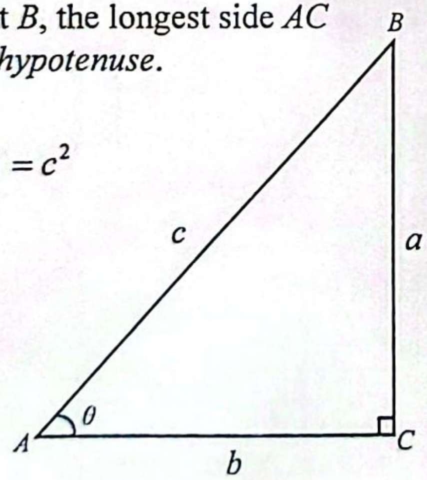
You can apply:
•**Pythagoras' Theorem** $a^2 + b^2 = c^2$

•**Trigo Ratios**

$\tan\theta = \frac{\text{opp side}}{\text{adj side}} = \frac{BC}{AC}$

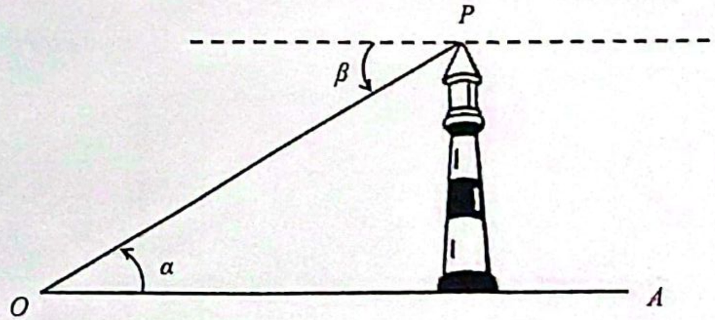
$\cos\theta = \frac{\text{adj side}}{\text{hyp}} = \frac{AC}{AB}$

$\sin\theta = \frac{\text{opp side}}{\text{hyp}} = \frac{BC}{AB}$



13B-2

Angles of Elevation and Depression



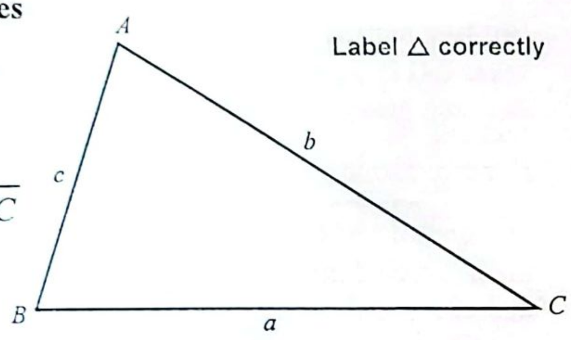
The angle α, measured from a horizontal reference level *OA*, is called the angle of elevation of point *P* from *O*; the angle β, is the angle of depression of *O* from *P*.

Usually uses tangent of angle to solve

13B-3

For all types of Triangles

Label △ correctly



Sine rule

$$\frac{a}{\sin\angle A}=\frac{b}{\sin\angle B}=\frac{c}{\sin\angle C}$$

Cosine Rule

$a^2 = b^2 + c^2 - 2bc\cos\angle A$

$b^2 = a^2 + c^2 - 2ac\cos\angle B$

$c^2 = a^2 + b^2 - 2ab\cos\angle C$

Sine and cosine rules also work with right-angled triangles but not the other way round.

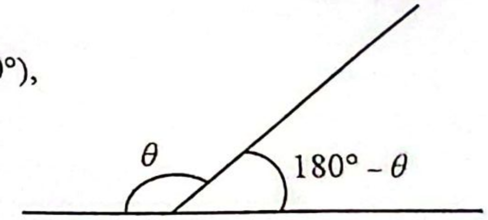
13B-4

Sine and Cosine of Obtuse Angles

When θ is obtuse ($90° < \theta < 180°$),

$\sin\theta = \sin(180° - \theta)$

$\cos\theta = -\cos(180° - \theta)$

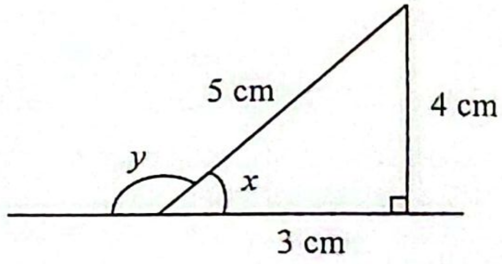


E.g. since $x + y = 180°$

$\sin x = \sin y = \frac{opp}{hyp} = \frac{4}{5}$

$\cos x = \frac{adj}{hyp} = \frac{3}{5}$

whereas $\cos y = -\frac{adj}{hyp} = -\frac{3}{5}$



Cosine – just remember to multiply by -1

13B-5

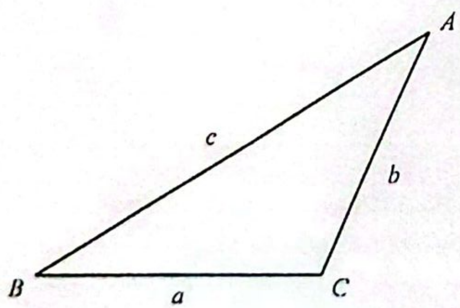
Area of a triangle

Area of $\Delta = \frac{1}{2}$ x base x height

$= \frac{1}{2} bc\sin A$

$= \frac{1}{2} ab\sin C$

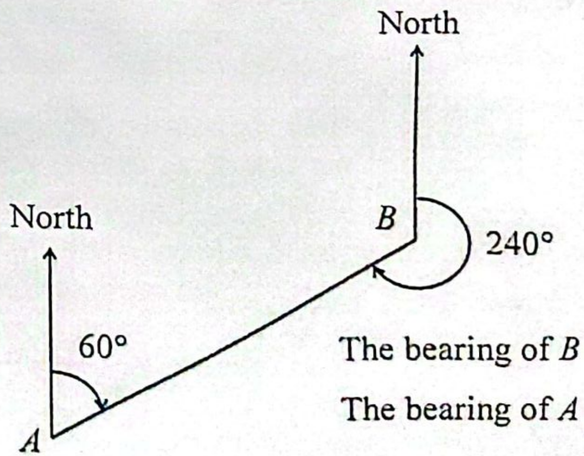
$= \frac{1}{2} ac\sin B$



13B-6

Bearings

Bearings are always measured from the North and in a clockwise direction and stated as a three-digit number.



The bearing of B from A is 060°.

The bearing of A from B is 240°.

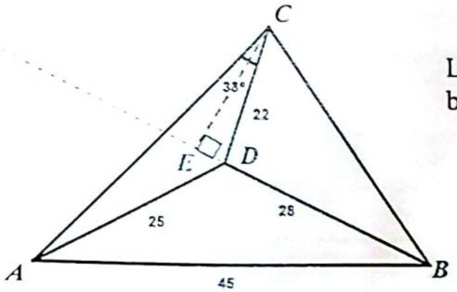
Trigonometry Problem Solving

Eg. $ABCD$ lie on a horizontal plane. The line BD is produced beyond D.

$\angle CDB = 125.54°$.

Calculate the shortest distance from C to this extended line.

Shortest distance is ⊥ to line



Let the intersection on the extended line be E.

$\angle CDE = 180° - 125.54° = 54.46°$

$\sin \angle CDE = \dfrac{CE}{CD}$

$\sin 54.46° = \dfrac{CE}{22}$

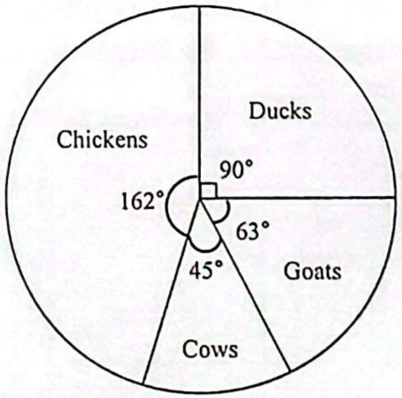
$CE = 22 \sin 54.46°$

≈ 17.9 m

Pie Chart

The angle of each sector is calculated by: $\dfrac{\text{Quantity of item}}{\text{Total quantity}} \times 360°$

The pie chart shows the various animals in the farm is as follows:



Animals	No. of animals	Angle of sector
Goats	14	$\frac{14}{80} \times 360° = 63°$
Cows	10	$\frac{10}{80} \times 360° = 45°$
Chickens	36	$\frac{36}{80} \times 360° = 162°$
Ducks	20	$\frac{20}{80} \times 360° = 90°$
Total	80	360°

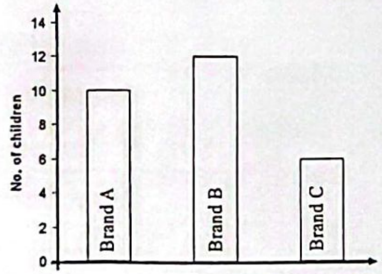
Bar Chart

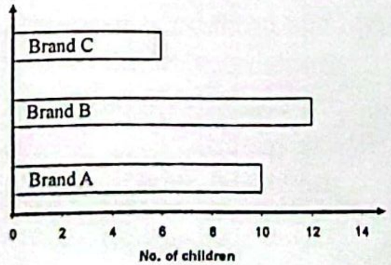
(i) Bar charts can be vertical or horizontal. A vertical bar chart is also called a column graph.

(ii) Bars or columns must be of equal width with gaps in between.

(iii) The quantities of various items are represented by the length of the bars.

In the example, 10 children like Brand A, 12 like Brand B and, 6 like Brand C.

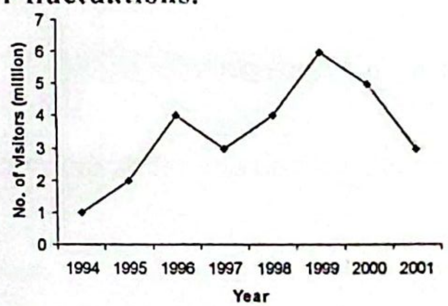




Line Graph

In a line graph, plotted points are joined by line segments to show trends or fluctuations.

e.g.



The line graph shows the number of visitors to a country from 1994 to 2001. There was a steady increase in the number of visitors for the periods from 1994 to 1996 and from 1997 to 1999. However, for the periods from 1996 to 1997 and from 1999 to 2001, there was a steady decrease in the number of visitors.

Histogram

(i) A histogram is very similar to a vertical bar chart but there are no gaps between the columns.

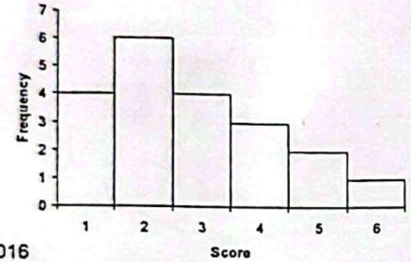
(ii) The area of each column represents the frequency of each class.

(iii) However, when the column are of equal width, the height of the column represents the frequency.

e.g. The results, when a dice is thrown 20 times, are shown in the frequency table below:

Score	1	2	3	4	5	6
Frequency	4	6	4	3	2	1

The histogram representing the above data is as follows:



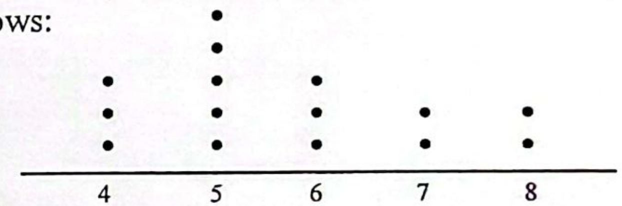
Dot Diagram

In a dot diagram, dots are used to display information.

e.g. The marks scored by 15 students are as follows:

4 6 5 5 4 8 5 7 7 6 6 5 5 4 8

The dot diagram representing the above information is as follows:



The dots above the number line represents the values in the set of data.
It shows that the lowest score is 4 and the highest score is 8.
The most common score is 5.

Stem-and-Leaf Diagram

In a stem-and-leaf diagram, each data is split into two parts, namely a stem and a leaf.

e.g. The length (in mm) of 12 pea pods are 65, 68, 70, 72, 55, 58, 63, 65, 70, 60, 65, 68.

The ordered stem-and-leaf diagram is as follows:

Stem	Leaf
5	5 8
6	0 3 5 5 5 8 8
7	0 0 2

Key: 3 | 2 stands for 32 mm

Mean (Average)

$$\text{Mean} = \frac{\text{Sum of values}}{\text{Number of values}} = \frac{\text{Sum of (frequency} \times \text{value)}}{\text{Total frequency}}$$

For grouped data, take the middle of each class to multiply by frequency

Mode

(i) The mode of a distribution is the value with the highest frequency

e.g. The mode of 1, 1, 2, 3, 5, 5, 5 is 5.

(ii) For grouped data, the class with the highest frequency is called the modal class

Mode – sounds like most

Median

The median of a set of n numbers is the middle value (if n is odd) or the mean of the two middle values (if n is even) when the values are arranged in ascending or descending order.

e.g. (a) For a set of 7 values (odd) arranged in order

1, 1, 2, (3), 4, 5, 5 — 3 numbers below, 3 numbers below

$\therefore$ Median = 3 (i.e. the 4th value)

(b) For a set of 8 values (even) arranged in order

1, 2, 2, (3), (4), 4, 5, 6 — 4 numbers below, 4 numbers below

Median – middle term $\frac{3+4}{2}$

$\therefore$ Median $= \frac{3+4}{2}$ $\left(\text{i.e. } \frac{4^{th} \text{ value} + 5^{th} \text{ value}}{2}\right)$ $= 3.5$

Standard Deviation

- Measures the spread of each data item from the mean of the data set

$$SD = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{\sum fx^2}{\sum f} - (mean)^2}$$

Old calc: mode, statistics, 1 (2)
shift, 1, 4
AC, >standard deviation
AC, opt, 2
1st = mean

New calc: shift, menu, 2 (statistic), menu 1, menu, 3, 1 →
option, 3

Comparing data using mean/median and SD/IQR

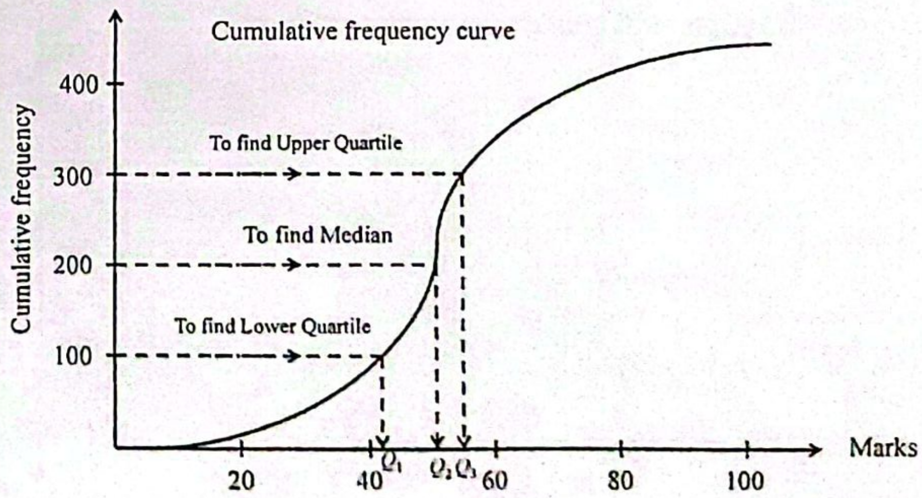
- When comparing data
  - compare mean/median for overall or average performance
  - compare SD/IQR for consistency of performance or spread of data

E.g. if we are comparing test scores,
On average/On the whole, A **performed better** than B if the **median/mean result for class A is higher.**

Class A performed **less consistently** than B since A has a **larger** IQR/SD. In other words, the **spread** of results for class A is wider because of its **larger SD/IQR.**

Cumulative frequency curve

The following figure shows a cumulative frequency curve.



The *range* is the difference between the highest and lowest values.

Q_1 is called the ***lower quartile*** or 25th percentile.

Q_2 is called the middle quartile or ***median*** or 50th percentile.

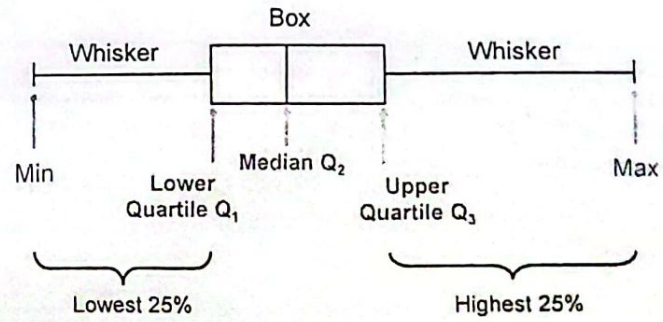
Q_3 is called the ***upper quartile*** or 75th percentile.

$Q_3 - Q_1$ is called the ***interquartile range***

You must read Q_1, Q_2 and Q_3 from the horizontal x-axis

Box-and-whiskers plot

- shows the range of a set of data, including the Minimum value, Maximum value, Lower Quartile, Median and Upper Quartile.



Matrices

Two matrices A and B can be added or subtracted from each other only when they are of the same order.

Let $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$.

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} a-e & b-f \\ c-g & d-h \end{pmatrix}$$

Matrices

Scalar multiplication, where k is a scalar.

$$k\mathbf{A} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

$$\frac{1}{k}\mathrm{A} = \begin{pmatrix} \frac{a}{k} & \frac{b}{k} \\ \frac{c}{k} & \frac{d}{k} \end{pmatrix}$$

Matrices

m rows, *n* columns   *p* rows, *q* columns

Two matrices A (with order $m \times n$) and B (with order $p \times q$) can be multiplied AB when $n = p$, and the result will be a matrix of order $(m \times q)$.

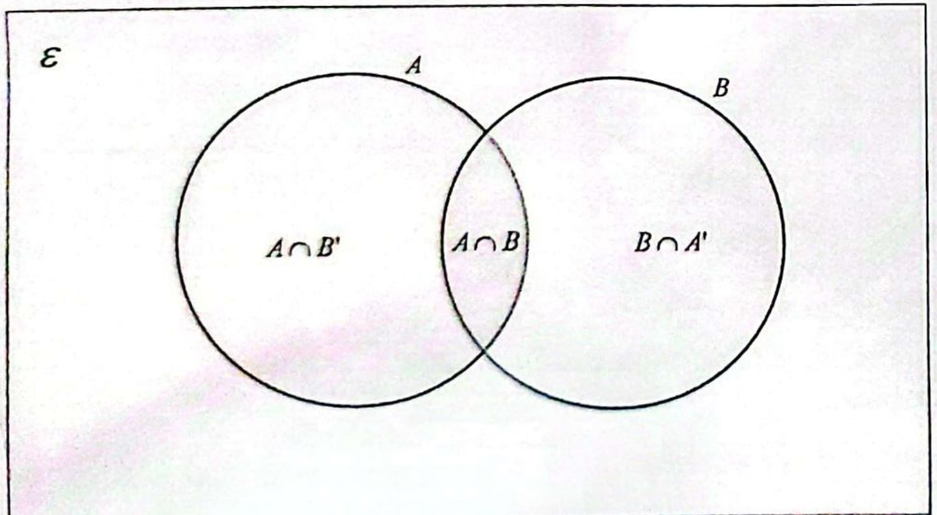
$$(a)(e) = (ae)$$
$$(a)(e \quad f) = (ae \quad af)$$
$$(a \quad b)\begin{pmatrix} e \\ g \end{pmatrix} = (ae + bg)$$
$$(a \quad b)\begin{pmatrix} e & f \\ g & h \end{pmatrix} = (ae + bg \quad af + bh)$$

$$\begin{pmatrix} a \\ c \end{pmatrix}(e) = \begin{pmatrix} ae \\ ce \end{pmatrix}$$
$$\begin{pmatrix} a \\ c \end{pmatrix}(e \quad f) = \begin{pmatrix} ae & af \\ ce & cf \end{pmatrix}$$
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e \\ g \end{pmatrix} = \begin{pmatrix} ae + bg \\ ce + dg \end{pmatrix}$$
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

Sets & Sets Notation

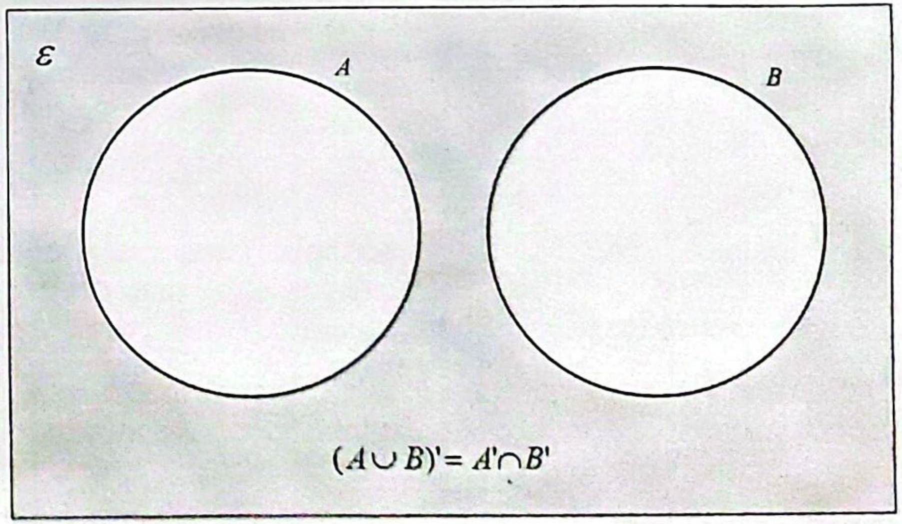
Universal set (all elements)	ε
Union of A and B	$A \cup B$
Intersection of A and B	$A \cap B$
Number of elements in set A	$n(A)$
Complement of A	A'
Empty or null set	{ } or $\varnothing$
C is a proper subset of B	$C \subset B$
A is not a subset of B	$A \not\subset B$

Venn Diagrams



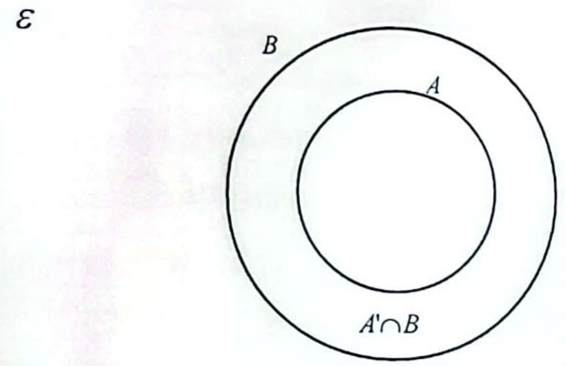
Venn Diagrams

$A \cap B = \varnothing$



Venn Diagrams

16-4



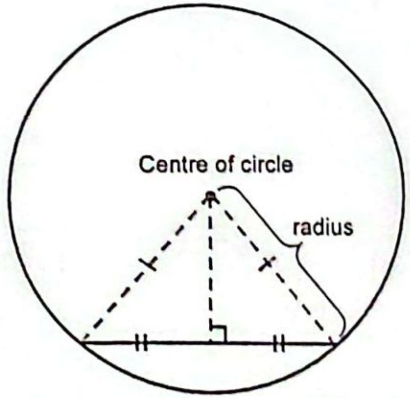
$A \cap B$
$= A$

$A \cup B$
$= B$

17-1

Symmetry Properties of a Circle

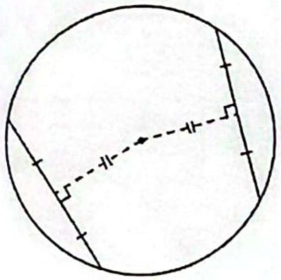
The perpendicular bisector of a chord passes through the centre of the circle

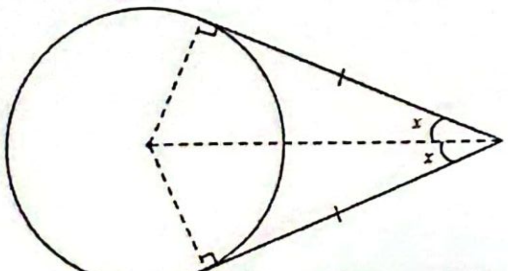


Look out for isosceles triangle.

17-2

Equal chords are equidistant from the centre.





Look out for isosceles triangle.

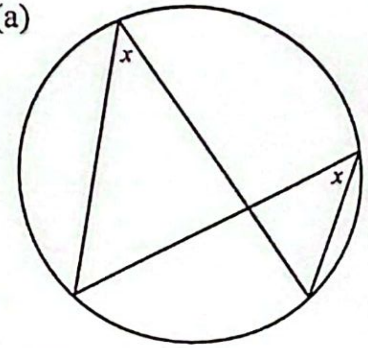
A tangent forms a right angle with the radius at point of contact.

Tangents to a circle from an external point are equal in length.

17-3

Angle properties of circle

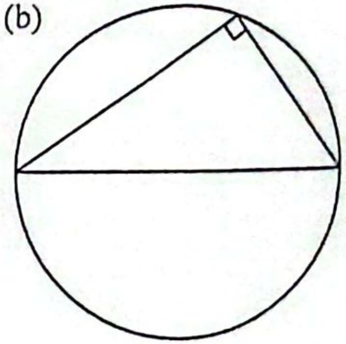
(a)



Angles in the same segment are equal

Ensure that they share the same arc.

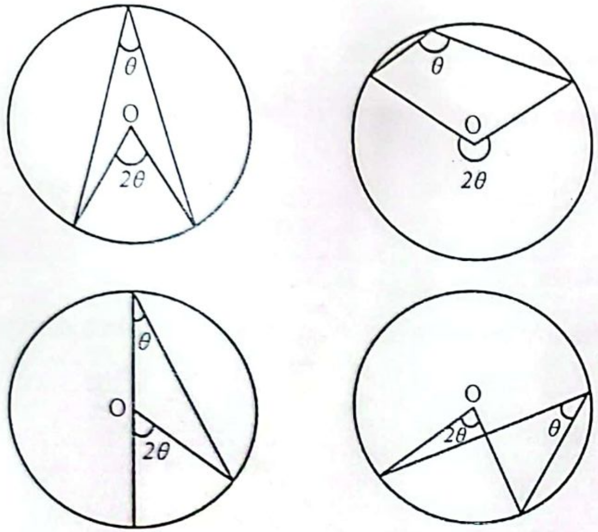
(b)



Angle in a semicircle = 90°

Look out for a diameter (must pass through centre of circle).

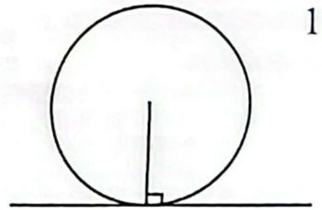
(c)



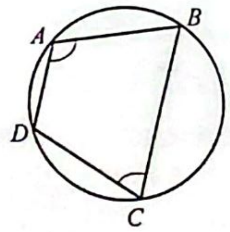
For reflex angle at centre

Angle at centre, 2θ, is twice the angle at circumference, θ.

(d) Angle between tangent and radius of a circle is a right angle



(e) Angles in the opposite segment are supplementary i.e. they add up to 180°



$\angle A + \angle C = 180^\circ$

$\angle B + \angle D = 180^\circ$

Angle is measured in radians

Degrees vs Radians

$360^\circ = 2\pi$ radians

$180^\circ = \pi$ radians

$90^\circ = \frac{\pi}{2}$ radians

$60^\circ = \frac{\pi}{3}$ radians

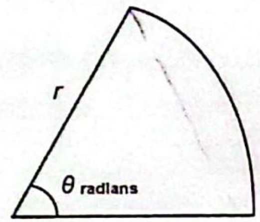
$45^\circ = \frac{\pi}{4}$ radians

$1^\circ = \frac{2\pi}{360}$ radian

$1 \text{ radian} = \frac{360}{2\pi}$ degree

Angle, θ, is measured in radians

Sector



Area = $\frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2} r^2 \theta$

Arc length = $r\theta$

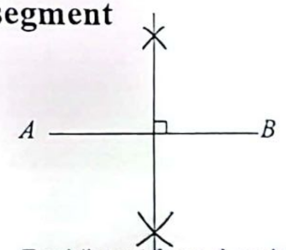
Perimeter = $2r + r\theta$

Area of shaded segment
$= \frac{1}{2} r^2 \theta - \frac{1}{2} ab \sin C$

Perimeter → external outline

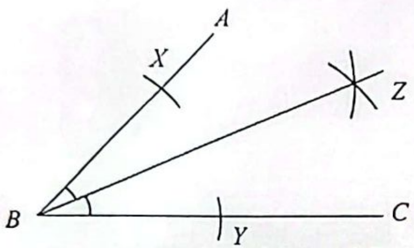
19-1

Perpendicular Bisector of a segment



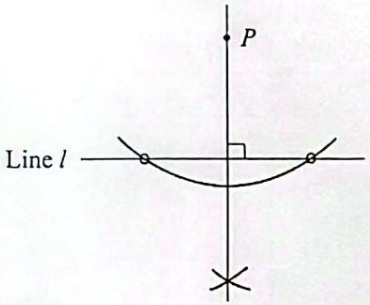
Equidistant from A and B

Bisector of an angle



Equidistant from BA and BC

Perpendicular line from a point to a line



20-1

(a) A **vector** is a quantity that has **both magnitude and direction.**

A vector may be represented in the form $\overrightarrow{AB}$, or u, or in components $\begin{pmatrix} a \\ b \end{pmatrix}$

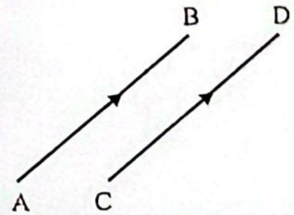
(b) **Equal vectors**

Two vectors are equal when they have the same direction and magnitude.

$\overrightarrow{AB} = \overrightarrow{CD}$ means

(i) AB || CD and

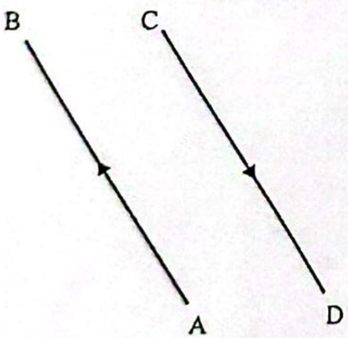
(ii) $|\overrightarrow{AB}| = |\overrightarrow{CD}|$



20-2

(c) **Negative vectors**

$\overrightarrow{AB}$ and $\overrightarrow{CD}$ have the same magnitude but in opposite directions, we write $\overrightarrow{CD} = -\overrightarrow{AB}$



20-3

(d) Addition

Vectors are added by the triangle law or parallelogram law.

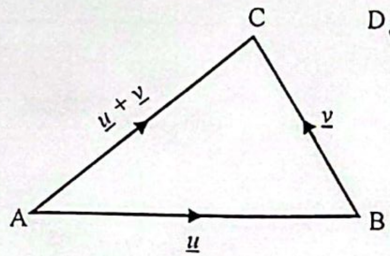


Fig. (i)

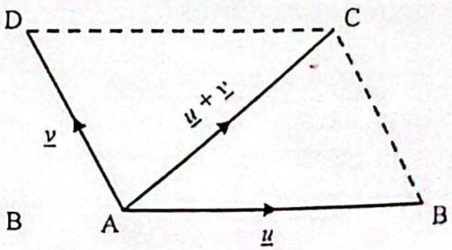


Fig. (ii)

In fig. (i), $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

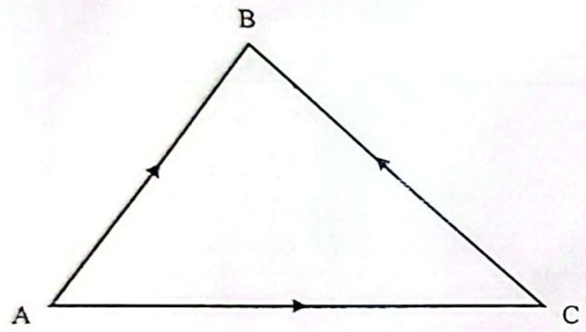
In fig. (ii), $\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

(d) Subtraction

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\therefore \overrightarrow{AB} = \overrightarrow{AC} - \overrightarrow{BC}$ or $\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

also $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$ or $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$



(e) **Position vectors**

A vector whose initial point is the origin, O is called a position vector.

If P is the point (h, k) and O the origin, then the position vector of P with respect to O is

$$\underset{\sim}{p} = \overrightarrow{OP} = \begin{pmatrix} h \\ k \end{pmatrix}$$

(f) Magnitude of a vector

If $\overrightarrow{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$, the magnitude of $\overrightarrow{AB}$, $\left|\overrightarrow{AB}\right| = \sqrt{x^2 + y^2}$.

(g) Mid-Point

If M is the mid-point of AB, then $\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$

(h) Multiplication of $\begin{pmatrix} a \\ b \end{pmatrix}$ by a scalar k,

$$k\underset{\sim}{u} = k\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ka \\ kb \end{pmatrix}$$

(i) If k is positive, $k\underline{a}$ is a vector having the same direction as that of $\underline{a}$ and magnitude equal to k times and magnitude of $\underline{a}$.

If k is negative, $k\underline{a}$ is a vector having the opposite direction to that of $\underline{a}$ and magnitude equal to k times the magnitude of $\underline{a}$.

$$\underline{a} = k\underline{b} \rightarrow |\underline{a}| = k|\underline{b}| \text{ and } \underline{a} \parallel \underline{b}.$$

(ii) If vector $\underline{a}$ is parallel to vector $\underline{b}$ then $\underline{a} = k\underline{b}$

(iii) If vector $\underline{a}$ is not parallel to vector $\underline{b}$ and $h\underline{a} = k\underline{b}$, then $h = 0$ and $k = 0$

(iv) If $n\underline{a} + m\underline{b} = h\underline{a} + k\underline{b}$ and $\underline{a} \nparallel \underline{b}$, then $n = h$ and $m = k$

Units

1. 1 m = 100 cm
2. 1 km = 1000 m
3. 1 cm = 10 mm = 0.01 m
4. 1 min = 60 s
5. 1 hr = 60 mins = 3600 s
6. 1 year = 365 days = 52 weeks
7. 1 kg = 1000 g
8. 1 ton = 1000 kg
9. 1 litre = 1000 ml = 1000cm^3
10. \$1 = 100¢

Prime Factorisation

2	24
2	12
2	6
3	3
	1

$\therefore 24 = 2^3 \times 3$

LCM (Lowest Common Multiple)

2	24, 42
3	12, 21
2	4, 7
2	2, 7
7	1, 7
	1,1

$\therefore$ LCM $= 2^3 \times 3 \times 7 = 168$

HCF (Highest Common Factor)

2	24, 42
3	12, 21
	4, 7

$\therefore$ HCF $= 2 \times 3 = 6$

Alternatively,
$24 = 2^3 \times 3$
$42 = 2 \times 3 \times 7$
LCM $= 2^3 \times 3 \times 7$ (Take the **higher** index)
HCF $= 2 \times 3$ (Take the **lower** index)

Simple Interest

$I = \frac{PRT}{100}$

where

$P = principal$

$R = rate\ (\%)$

$T = time\ (year)$

Note:
Total amount $= P + I$

Compound Interest

Total amount

$= P\left(1+\frac{r}{100}\right)^n$

where

$P = principal$

$R = rate(\%)$

$n = no\ of\ period$

Note:
Compound Int. = Total amount – P

Algebraic Fractions

1. $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$
2. $\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$
3. $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$
4. $\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$
5. $\frac{a}{b}-\frac{c}{d}=\frac{ad}{bd}-\frac{bc}{bd}=\frac{ad-bc}{bd}$
6. $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$

Inequalities

If $a > b$, then

1. $a + c > b + c$,
2. $a - c > b - c$,
3. $ac > bc, if\ c > 0$

*4. $ac < bc, if\ c < 0$

5. $\frac{a}{c} > \frac{b}{c}, if\ c > 0$

*6. $\frac{a}{c} < \frac{b}{c}, if\ c < 0$

change inequality sign only when multiply or divided by negative number

Factorisation

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $a^2 - b^2 = (a+b)(a-b)$
4. $ax + bx + ay + by$
   $= x(a+b) + y(a+b)$
   $= (x+y)(a+b)$

Quadratic Equation

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Indices

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $a^0 = 1$
5. $(a \times b)^n = a^n \times b^n$
6. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
7. $a^{-n} = \frac{1}{a^n}$
8. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
9. $\sqrt[n]{a^m} = a^{\frac{m}{n}}; \sqrt[n]{a} = a^{\frac{1}{n}}$

Standard form

$A \times 10^n$, *where*

$1 < A < 10, n = integer$

Common Prefixes

Tetra (*trillion*) $= 10^{12}$

Giga (*billion*) $= 10^9$

Mega (*million*) $= 10^6$

Kilo (*thousand*) $= 10^3$

Milli (*thousandth*) $= 10^{-3}$

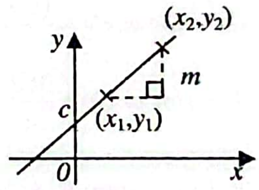
Micro (*millionth*) $= 10^{-6}$

Nano (*billionth*) $= 10^{-9}$

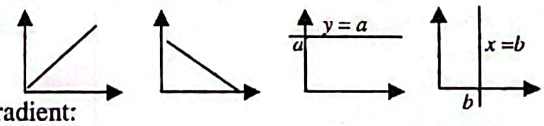
Pico (*trillionth*) $= 10^{-12}$

Coordinate Geometry

1. $gradient, m = \frac{y_2 - y_1}{x_2 - x_1}$
2. $length = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
3. *Equation of straight line*:
   $y = mx + c$
   where $m = gradient, c = y-intercept$



4.



gradient: *positive* *negative* *zero* *undefined*

Direct Variation

$y \propto x$

$\Rightarrow y = kx$

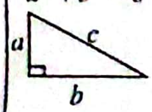
Inverse Variation

$y \propto \frac{1}{x}$

$\Rightarrow y = \frac{k}{x}$

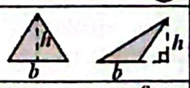
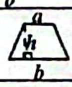
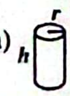
Pythagoras' Theorem

$a^2 + b^2 = c^2$

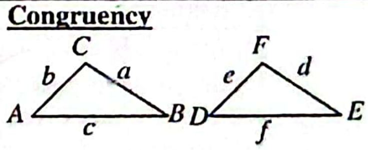


c = hypotenuse
a and b are the two shorter sides.

Mensuration

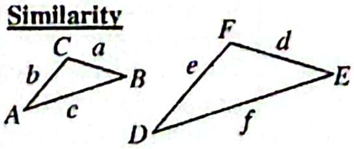
Figure	Area	Volume
Circle 	πr^2	-
Triangle 	$\frac{1}{2}bh$	-
Trapezium 	$\frac{1}{2}(a+b)h$	-
Pyramid 	Find area of all sides and add	$\frac{1}{3} \times base\ area \times h$
Cylinder (curved surface area) (2 closed ends) 	$2\pi rh$ $2\pi r^2$	$\pi r^2 h$
Cone (curved surface area) (base area) 	πrl πr^2	$\frac{1}{3}\pi r^2 h$
Sphere 	$4\pi r^2$	$\frac{4}{3}\pi r^3$

Congruency



Four Tests for congruency:
SSS, SAS, ASA and RHS

Similarity



$\angle A = \angle D; \angle B = \angle E; \angle C = \angle F$

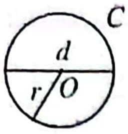
$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}; \frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

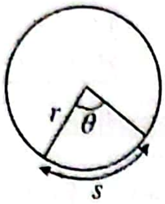
$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 = \frac{m_1}{m_2}$

To prove similar triangles, just find two pairs of equal angles.

Circular Measure

1. $C = 2\pi r = \pi d$; $A = \pi r^2$
2. $\pi\ rad = 180°$
3. $Area\ of\ sector,\ A = \frac{\theta}{360} \times \pi r^2$ or $A = \frac{1}{2} r^2 \theta$
4. $Arc\ length,\ s = \frac{\theta}{360} \times 2\pi r$ or $s = r\theta$
5. Area of segment $= 0.5r^2(\theta - \sin\theta)$

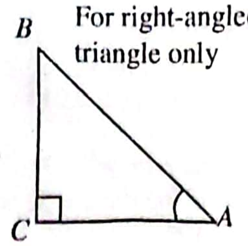




Trigonometry

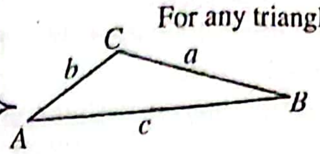
1. $\sin A = \frac{BC}{AB} = \frac{opp}{hyp}$ (SOH)
2. $\cos A = \frac{AC}{AB} = \frac{adj}{hyp}$ (CAH)
3. $\tan A = \frac{BC}{AC} = \frac{opp}{adj}$ (TOA)

For right-angled triangle only



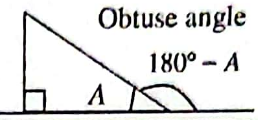
4. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
5. $a^2 = b^2 + c^2 - 2bc\cos A$
6. $Area\ of\ \Delta ABC = \frac{1}{2}ab\sin C$

For any triangle



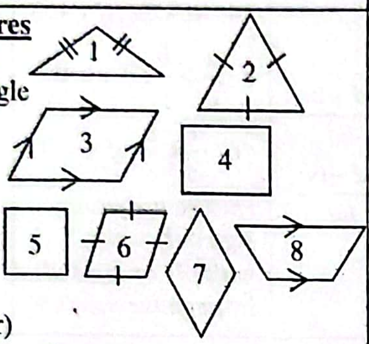
7. $\sin A = \sin(180° - A)$
8. $\cos A = -\cos(180° - A)$

Obtuse angle



Geometrical Figures

1. Isoceles triangle
2. Equilateral triangle
3. Parallelogram
4. Rectangle
5. Square
6. Rhombus
7. Kite
8. Trapezium
9. Polygon (regular)



[$interior\ angle = \frac{(n-2) \times 180°}{n}$

$Exterior\ angle = \frac{360}{n}$

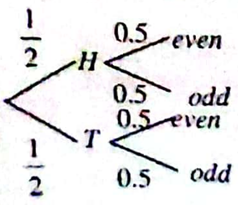
where n = no of sides]

Sets and Venn Diagram

ε = universal set; ϕ or { } = empty set;
$A \cap B = A$ intersect B; $A \cup B = A$ union B;
$A \subset B = A$ is a proper subset of B;
$A \subseteq B = A$ is a subset of B;
A' = complement of set A;
n(A) = no of element in set A;
$x \in A = x$ is an element of set A.

Probability

1. Probability of any event A occurring, P(A) = $\frac{Number\ of\ outcomes\ favourable\ to\ A}{Total\ number\ of\ possible\ outcomes}$
2. If A is an **impossible** event, then P(A) = 0.
3. If A is a **sure** event, then P(A) = 1.
4. $0 \le P(A) \le 1$.
5. For any event A, P(A) + P(not A) = 1, which implies P(not A) = 1 − P(A)
6. For **mutually exclusive** events that cannot happen together, P(A or B) = P(A) + P(B)
7. If A and B are **independent** events, then P(A and B) = P(A) × P(B)
8. Probability can be found using the **possibility diagram** or the **tree diagram.**

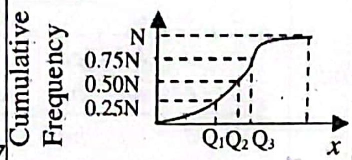


	1	2	3	4
H	H,1	H,2	H,3	H,4
T	T,1	T,2	T,3	T,4

P(H and even) = $\frac{2}{8}$ = 0.25

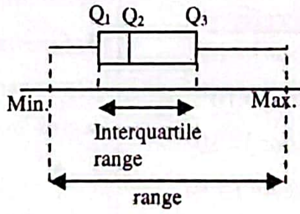
P(H and even) = 0.5 × 0.5 = 0.25

Cumulative Frequency Curve



Q_1 = lower quartile
Q_2 = median
Q_3 = upper quartile
Interquartile range $= Q_3 - Q_1$

Box-and-Whisker Plot

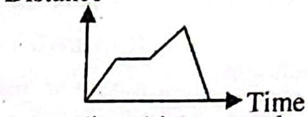


Range =maximum − minimum
Interquartile range $= Q_3 - Q_1$

Statistics

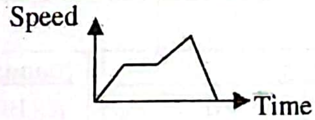
1. Mean = average = $\frac{\sum fx}{\sum f}$
2. Mode = most frequent
3. Median = the middle value (data arranged in ascending order)
4. Standard deviation = $\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$

Distance-Time (*d-t*) Graph



1. Gradient (d-t) = speed

Speed-Time(*s-t*) Graph



2. Gradient(s-t) = acceleration
3. Area under speed time graph = distance traveled
4. Average speed $= \frac{\text{Total distance traveled}}{\text{Total time taken}}$

Matrix Addition

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} a+p & b+q \\ c+r & d+s \end{pmatrix}$$

Subtraction

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} a-p & b-q \\ c-r & d-s \end{pmatrix}$$

Multiplication

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw+by & ax+bz \\ cw+dy & cx+dz \end{pmatrix}$$

Scalar Multiplication

$$k\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

Identity Matrix

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

A = AI = IA

Null Matrix

(0); $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$; $(0\ \ 0\ \ 0)$

Note:

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then

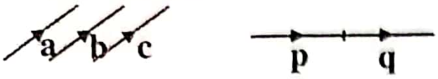
$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$

not $\begin{pmatrix} a^2 & b^2 \\ c^2 & d^2 \end{pmatrix}$

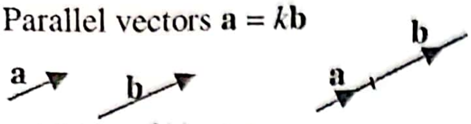
Vectors

1. A vector is a quantity with both magnitude and direction.

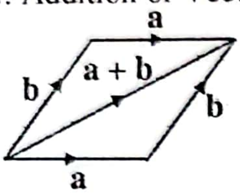
2. Equal vectors



3. Parallel vectors $\mathbf{a} = k\mathbf{b}$



4. Addition of Vectors



Triangular Law and Parallelogram Law

5. Null Vector and Negative of a Vector $\mathbf{a} + \mathbf{b} = \mathbf{0}$.
0 = null or zero vector.

$\mathbf{b} = -\mathbf{a}$

6. Negative and scalar multiplication of vectors

7. Magnitude of vector

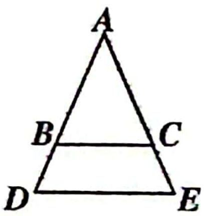
The magnitude of a column vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is given by

$$\left|\begin{matrix} x \\ y \end{matrix}\right| = \sqrt{x^2 + y^2}.$$

8. To find ratio of area of <u>similar</u> triangles ΔABC to ΔADE, use <u>similar triangle method</u>;

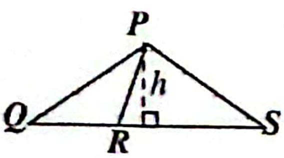
$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{A_{ABC}}{A_{ADE}} = \left(\frac{BC}{DE}\right)^2$$



9. To find ratio of area of <u>non-similar</u> ΔPQR to ΔPQS use <u>common height method</u>.

$$\frac{A_{PQR}}{A_{PQS}} = \frac{\frac{1}{2}(QR)h}{\frac{1}{2}(RS)h} = \frac{QR}{RS}$$



Graphs ($y = ax^n$, where n = -3, -2, -1, 0, 1, 2)