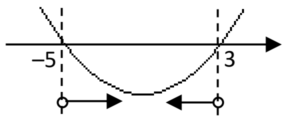
Sec 4 Add Math Preliminary Exam 2024 P2 Marking Scheme

Qn.	Solution	Marks	AO
1	$2(e^x - 3) = e^{\frac{1}{2}x}$ Let $y = e^{\frac{1}{2}x}$, $2(y^2 - 3) = y$ $2y^2 - y - 6 = 0$ $(2y+3)(y-2) = 0$ $y = -\frac{3}{2}$ or $y = 2$ $e^{\frac{1}{2}x} = -\frac{3}{2}$ (rej. as -ve) $e^{\frac{1}{2}x} = 2$ $\frac{1}{2}x = \ln 2$ $x = 2\ln 2$ $= 1.39$ (3 s.f) Therefore, the equation has only one solution where x=1.39	 M1 (substitution) M1 (factorization) A1 (reject $-\frac{3}{2}$) (Did not award marks for students who squared both sides and could not justify why they rejected one answer when they ended up with 2 answers) AG1	3

2	$y = ax^3 + b$ $\frac{dy}{dx} = 3ax^2$ $5y + 2x = 12$ $5y = -2x + 12$ $y = -\frac{2}{5}x + \frac{12}{5}$ Grad of normal = $-\frac{2}{5}$ Grad of tangent = $\frac{5}{2}$ $\therefore \frac{dy}{dx} = \frac{5}{2}$ At $x = 1$, $y = -\frac{2}{5}(1) + \frac{12}{5}$ $y = 2$ At $x = 1, y = 2$ $2 = a(1)^3 + b$ $a + b = 2$ ------ (1) At $x = 1, \frac{dy}{dx} = \frac{5}{2}$ $\frac{5}{2} = 3a(1)^2$ ------ (2) $a = \frac{5}{6}$ Substituting $a = \frac{5}{6}$ into (1) $\frac{5}{6} + b = 2$ $b = \frac{7}{6}$	B1 (find $\frac{dy}{dx}$) B1 (grad of tangent) M1 (find y = 2) M1 (for either (1) or (2)) A1 (for a) A1 (for b)	2

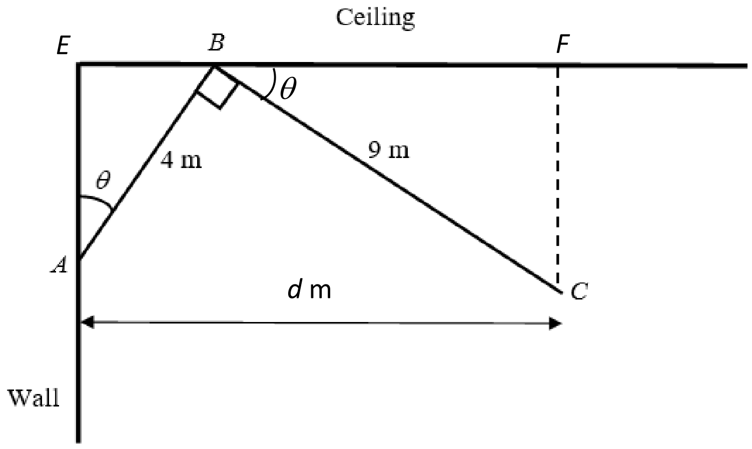
3a	$y = \frac{3\ln(2x)}{x^2}$ $\frac{dy}{dx} = \frac{x^2\left(\frac{3(2)}{2x}\right) - 6x\ln 2x}{\left(x^2\right)^2}$ $= \frac{3x - 6x\ln 2x}{x^4}$ $= \frac{3}{x^3}(1 - 2\ln 2x)$	M1 (Apply quotient rule) M1 (Able to diff $\ln 2x = \frac{2}{2x} = \frac{1}{x}$.) A1	1
3b	$\int\left(\frac{3}{x^3} - \frac{6}{x^3}\ln 2x\right)dx = \frac{3\ln 2x}{x^2} + C$ $\int\left(\frac{1}{x^3} - \frac{2}{x^3}\ln 2x\right)dx = \frac{\ln 2x}{x^2} + C_1$ $\int \frac{2\ln 2x}{x^3}dx = -\frac{\ln 2x}{x^2} - \frac{1}{2x^2} + C_2$ $\int \frac{\ln 2x}{x^3}dx = -\frac{1}{2}\left(\frac{\ln 2x}{x^2} + \frac{1}{2x^2}\right) + C_3$	M1 (reverse differentiation – must include + C) M1 (integrate $\frac{1}{x^3}$) A1(must include + C) (Whole question will only deduct once if they did not put + C)	2

4a	$\mathrm{f}(x) > x-1$ $x^2 - ax + 3 > x - 1$ $x^2 - (a+1)x + 4 > 0$ Since it is always positive for all real values of *x*, the graph of the curve $y = x^2 - (a+1)x + 4$ lies entirely above the *x*-axis $\Rightarrow$ the equation $x^2 - (a+1)x + 4 = 0$ has no real roots Discriminant, $D < 0$ $(a+1)^2 - 4(1)(4) < 0$ $(a+1)^2 - 4^2 < 0$ $(a+5)(a-3) < 0$  $\therefore -5 < a < 3$	M1 (form inequality) (students who equated both eqns together will not get M1 unless they explain that there are no real roots and lead to *D*<0) M1 (Discriminant less than zero) M1 (Factorization) A1	2
4b	$\begin{cases} y = \mathrm{f}(x) \\ y = a + 4 \end{cases}$ $x^2 - ax + 3 = a + 4$ $x^2 - ax - a - 1 = 0$ Since the line $y = a + 4$ is a tangent to the curve, this equation has equal real roots Discriminant, $D = 0$ $(-a)^2 - 4(1)(-a-1) = 0$ $a^2 + 4a + 4 = 0$ $(a+2)(a+2) = 0$ $\therefore a = -2$	M1 (equate equations together) M1 (D = 0) A1	1
5a	$\sin(A-B) = \sin A \cos B - \cos A \sin B$ $\cos A \sin B = \sin A \cos B - \sin(A-B)$ $= \frac{5}{8} - \frac{3}{8}$ $= \frac{1}{4}$	M1 (make $\cos A \sin B$ the subject) A1	1

5b	$2\sin 2\theta(\sec\theta-\tan\theta)$ $=2(2\sin\theta\cos\theta)\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)$ $=2(2\sin\theta\cos\theta)\left(\frac{1-\sin\theta}{\cos\theta}\right)$ $=4\sin\theta-4\sin^2\theta$	M1 (double angle formula) M1 (bring $\cos\theta$ under same denominator) A1	2
5c	$2\sin 2\theta(\sec\theta-\tan\theta)+3=0$ $4\sin\theta-4\sin^2\theta+3=0$ $4\sin^2\theta-4\sin\theta-3=0$ $(2\sin\theta+1)(2\sin\theta-3)=0$ $\sin\theta=-\frac{1}{2}$ or $\sin\theta=\frac{3}{2}$ (rejected) $\alpha=\frac{\pi}{6}$ $\theta=\pi+\frac{\pi}{6},2\pi-\frac{\pi}{6}$ $\theta=\frac{7\pi}{6},\frac{11\pi}{6}$	M1 (factorize) M1 (show both and must reject one) A1 (must be in terms of π as qn asked for exact solns)	1
6(i)	$y=\int\left(\frac{8}{x^2}-2\right)dx$ $y=-\frac{8}{x}-2x+c$ At $x=1, y=5$ $5=-\frac{8}{1}-2(1)+c$ $c=15$ Equation of curve: $y=-\frac{8}{x}-2x+15$	M1 (with + c) M1 (Sub in values) A1	1
6(ii)	Let $\frac{dy}{dx}=0$ $\frac{8}{x^2}-2=0$ $\frac{8}{x^2}=2$ $x^2=4$ $x=2$ or $x=-2$	M1 A1 (both answers)	1

6(iii)	$\frac{d^2y}{dx^2}=-\frac{16}{x^3}$ At $x=2$, $\frac{d^2y}{dx^2}=-\frac{16}{2^3}=-2<0$ $\therefore$ Maximum point at $x=2$ At $x=-2$, $\frac{d^2y}{dx^2}=-\frac{16}{(-2)^3}=-\frac{16}{-8}=2>0$ $\therefore$ Minimum point at $x=-2$	M1 (2nd derivative) A1 A1 (for students who got part (ii) wrong, maximum mark is 1M if M1 shown)	1
7a	$\left(x^2+\frac{m}{x}\right)^9=x^{18}+\binom{9}{1}\left(x^2\right)^8\left(\frac{m}{x}\right)^1+\binom{9}{2}\left(x^2\right)^7\left(\frac{m}{x}\right)^2+....$ $=x^{18}+9\,mx^{15}+36m^2x^{12}+....$	B2 (3 terms all correct) B1(2 terms correct)	1
7b(i)	For $\left(x^2+\frac{m}{x}\right)^9$, general term is $(r+1)^{th}$ term $=\binom{9}{r}\left(x^2\right)^{9-r}(m)^r\left(x^{-1}\right)^r$ $=\binom{9}{r}(m)^r\left(x^{18-2r}\right)\left(x^{-r}\right)$ $=\binom{9}{r}(m)^r\left(x^{18-3r}\right)$ $18-3r=3$ $r=5$ $\therefore -126=\binom{9}{5}m^5$ $-126=126m^5$ $m^5=-1$ $m=-1$	M1 (general term) A1 ($r=5$) AG1	3

7b(ii)	For $\left(x^2+\frac{m}{x}\right)^9$, the term independent of *x*: $18-3r=0$ $r=6$ $\therefore \binom{9}{6} m^6 = 84(-1)^6 = 84$ For $\left(2-\frac{1}{x^3}\right)\left(x^2+\frac{a}{x}\right)^9$, the term independent of *x:* $\left(2-\frac{1}{x^3}\right)(....-126x^3+84+....)^9$ $= 2\times 84+126$ $= 294$	 M1 A1 M1 A1	1
8a	<table><tr><td>*t* (years)</td><td>6</td><td>9</td><td>12</td><td>15</td><td>18</td><td>25</td><td>33</td></tr><tr><td>*P*</td><td>274</td><td>203</td><td>151</td><td>112</td><td>83</td><td>41</td><td>18</td></tr><tr><td>$\ln P$</td><td>5.61</td><td>5.31</td><td>5.02</td><td>4.72</td><td>4.42</td><td>3.71</td><td>2.89</td></tr></table>	P2 – Plot points accurately. L1 – Plot straight line graph *(See graph attached.)*	1
8b	$P = Ae^{-kt}$ $\ln P = \ln Ae^{-kt}$ $\ln P = \ln A + \ln e^{-kt}$ $\ln P = \ln A - kt$ $\ln P = -kt + \ln A$ Grad = $\frac{5.61-4.72}{6-15}$ $= -0.0989$ (3 s.f) $-k = -0.0989$ $k = 0.1$(1 d.p) $\ln A = 6.2$ $A = 492.75$ $A = 500$ (nearest 100) $\because P = 500e^{-0.1t}$	M1 (product law or if evidence shown in transformation from eqn of graph to $P = Ae^{-kt}$) M1 (gradient) A1 B1 (If students did not use the gradient of line to solve for k and A, maximum mark is 1M as question mentioned hence.)	2

8c	$\ln 100 = 4.6$ When $\ln P = 4.6$ $t = 16$ (nearest year)	M1 A1	1
9a	 $EB = 4\sin\theta$ $BF = 9\cos\theta$ $d = 4\sin\theta + 9\cos\theta$	} M1 AG 1	3
9b	$R = \sqrt{81+16}$ $R = \sqrt{97}$ $9\cos\theta + 4\sin\theta = R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ $9 = R\cos\alpha$ $4 = R\sin\alpha$ } $\tan\alpha = \frac{4}{9}$ $\alpha = 23.962°(3d.p)$ $\alpha = 24.0°(1d.p)$ $9\cos\theta + 4\sin\theta = \sqrt{97}\cos(\theta - 24.0°)$	M1 (Find R) M1 (No M1 given if student do not show this) M1 (Find α) A1	1
9c	$\sqrt{97}\cos(\theta - 23.96°) = 6$ $\cos(\theta - 23.96°) = \frac{6}{\sqrt{97}}$ $\theta = 52.467° + 23.96°$ $\theta = 76.427° \approx 76.4°$	M1 A1 (no A1 if 76.5)	1

9d	Maximum value of *d* = $\sqrt{97}$ $\cos(\theta - 24.0°) = 1$ $\theta = 24.0$ Maximum value of *d* = $\sqrt{97}$ and occurs when $\theta = 24.0°$	B1 B1	1
10a	$v = 6\cos 4t$ When $t = 0, v = 6$ Initial velocity of the particle is 6m/s.	B1	1
10b	$a = \frac{dv}{dt} = -24\sin 4t$ $-24\sin 4t = 8$ $\sin 4t = -\frac{1}{3}$ $4t = 3.4814$ $t = 0.870$ (3 s.f)	M1 ($\frac{dv}{dt}$) A1	2
10c	$s = \int 6\cos 4t\ dt$ $s = \frac{6}{4}\sin 4t + c$ $s = \frac{3}{2}\sin 4t + c$ When $t = 0$, $s = 0$, $c = 0$ $s = \frac{3}{2}\sin 4t$ When $t = 4$, $s = \frac{3}{2}\sin 16$ $s = -0.432$ Displacement = -0.432m (3 sf)	M1 (integration with + *c*) A1 (conclude c=0) (students who used definite integral must indicate when t = 0, s=0) B1	1
10d	At instantaneous rest, $v = 0$ $6\cos 4t = 0$ $\cos 4t = 0$ $4t = \frac{\pi}{2}, \frac{3\pi}{2}$ $t = \frac{\pi}{8}, \frac{3\pi}{8}$	M1 (*v*= 0) M1 (values of *t*)	2

	When $t=\frac{\pi}{8}$, $s=\frac{3}{2}\sin\frac{\pi}{2}=1.5\text{m}$ When $t=\frac{3\pi}{8}$, $s=\frac{3}{2}\sin\frac{3\pi}{2}=-1.5\text{m}$ Total distance travelled $=(1.5\times 2)+1.5$ $=4.5\text{m}$	M1 M1 A1	
11a	**<u>Solution 1</u>** Grad E$F=\frac{2-(-6)}{-1-1}=-4$ Grad D$F=\frac{4-2}{7-(-1)}=\frac{1}{4}$ Since Grad $DF\times$ Grad $EF=-4\times\frac{1}{4}=-1$ $\therefore DF\perp EF$ $\angle DFE=90^\circ$ (Right angle in semi-circle) $\therefore DE$ is the diameter of C_1 Since PQ is the diameter of C_1 Centre of $C_1=\left(\frac{1+7}{2},\frac{-6+4}{2}\right)$ $=(4,-1)$	M1 (Find grad) B1 (Show – 1) B1(Conclude 90°) AG 1 (State reason angle in semi-circle) B1	3
11a	**<u>Solution 2</u>** $DF=\sqrt{(7+1)^2+(4-2)^2}=\sqrt{68}$ $EF=\sqrt{(1+1)^2+(-6-2)^2}=\sqrt{68}$ $DE=\sqrt{(7-1)^2+(4+6)^2}=\sqrt{136}$ $DF^2+EF^2=\left(\sqrt{68}\right)^2+\left(\sqrt{68}\right)^2=136$ $DE^2=\left(\sqrt{136}\right)^2=136$ $\because DF^2+EF^2=DE^2$ By converse of Pythagoras theorem, triangle DFE is a right angled triangle. Therefore $\angle DFE=90^\circ$.	M1 (Find distance) B1 (Show this statement) B1(Conclude 90° - must state Pythagoras thm)	3

	Since $\angle DFE = 90°$ By converse of right angle in semi-circle $\therefore DE$ is the diameter of C_1 Since PQ is the diameter of C_1 Centre of $C_1 = \left(\frac{1+7}{2}, \frac{-6+4}{2}\right)$ $= (4,-1)$	AG 1 (State reason angle in semi-circle) B1	
11b	Radius of $C_1 = \sqrt{(4-1)^2+(-1-(-6))^2} = \sqrt{34}$ Equation of C_1: $(x-4)^2+(y+1)^2 = 34$ $x^2+y^2-8x+2y-17=0$	B1 M1 A1	1
11c	Centre of $C_2 = (-8,-1)$ Radius = $\sqrt{34}$ Equation of C_2: $(x+8)^2+(y+1)^2 = 34$	B1 (centre) B1	2
11d	Distance of E from centre of C_1 $= \sqrt{(3-4)^2+(4+1)^2}$ $= \sqrt{26}$ < radius of C_1 Distance of E from centre of C_2 $= \sqrt{(3+8)^2+(4+1)^2}$ $= \sqrt{146}$ > radius of C_2 $\therefore$ (3 , 4) lies within C_1 only	M1 A1	2