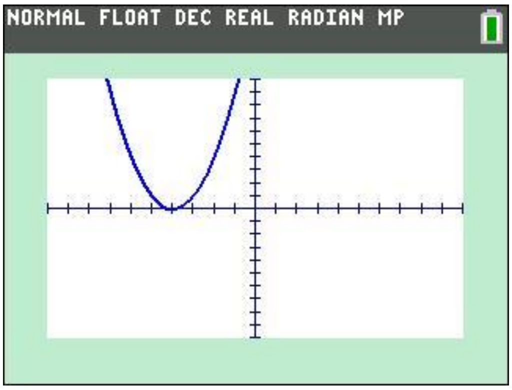
2015 H1 A Level Mathematics Solution

Q1.	For $-2x^2+\left(k-4\right)x+2k$ to be always negative, Discriminant < 0 and coefficient of $x^2=-2<0$ $\left(k-4\right)^2-4\left(-2\right)\left(2k\right)<0$ $(k^2-8k+16)+16k<0$ $k^2+8k+16<0$ $(k+4)^2<0$ But $(k+4)^2\geq 0$ for all real values of *k*. Therefore there is no solution for $(k+4)^2<0$ and there are no values of k for which $-2x^2+\left(k-4\right)x+2k$ is always negative. Graph of $y=(x+4)^2$ 
Q2.	(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{3}{\left(2x-1\right)^4}\right)=3\left(-4\right)\left(2\right)\left(2x-1\right)^{-5}=-\frac{24}{\left(2x-1\right)^5}$ (ii) $\int_{\frac{1}{2}}^{1}\left(x+\frac{2}{x}\right)^2 dx$ $=\int_{\frac{1}{2}}^{1} x^2+4+\frac{4}{x^2}\,dx$ $=\int_{\frac{1}{2}}^{1} x^2+4+4x^{-2}\,dx$ $=\left[\frac{x^3}{3}+4x-\frac{4}{x}\right]_{\frac{1}{2}}^{1}$ $=6\frac{7}{24}$

Q3.

(i)

$$y = 12x + 8e^{-2x}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 12 - 16e^{-2x}$$

For stationary point, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

$$12 - 16e^{-2x} = 0$$

$$16e^{-2x} = 12$$

$$e^{-2x} = \frac{12}{16} = \frac{3}{4}$$

$$\ln e^{-2x} = \ln\left(\frac{3}{4}\right)$$

$$-2x = \ln\left(\frac{3}{4}\right) \Rightarrow x = -\frac{1}{2}\ln\left(\frac{3}{4}\right)$$

(ii)

Area of Region bounded by C =

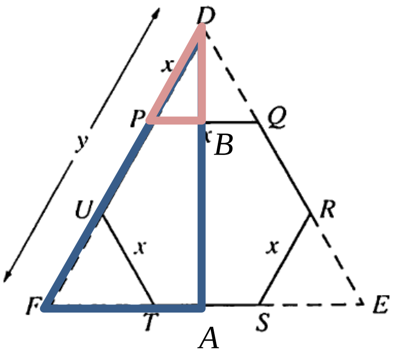
$$\int_0^a 12x + 8e^{-2x}\ dx$$

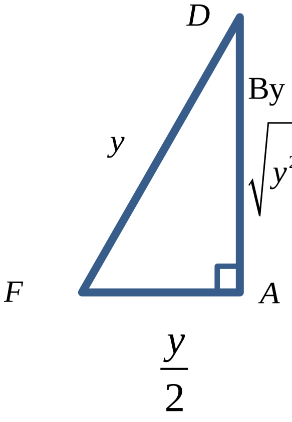
$$= \left[6x^2 - 4e^{-2x}\right]_0^a$$

$$= \left(6a^2 - 4e^{-2a}\right) - \left(0 - 4\right)$$

$$= 6a^2 - 4\mathrm{e}^{-2a} + 4 \text{ units}^2$$

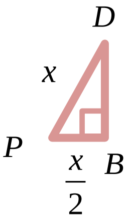
Q4.





By Pythagoras' Theorem,

$$\sqrt{y^2 - \left(\frac{y}{2}\right)^2} = \sqrt{\frac{3y^2}{4}}$$



By Pythagoras' Theorem,

$$\sqrt{x^2 - \left(\frac{x}{2}\right)^2} = \sqrt{\frac{3x^2}{4}}$$

Perimeter of the remaining shape *PQRSTU* = 30 cm

$$\Rightarrow 3\left(y-2x\right)+3x=30$$
$$\Rightarrow y-x=10$$
$$\Rightarrow y=10+x$$

Method 1:
Area of the remaining shape *PQRST*, A

= Area of triangle FDE – 3 (Area of triangle PDQ)

$$=\frac{1}{2}y^2\sin 60^o-3\left(\frac{1}{2}x^2\sin 60^o\right)$$
$$=\frac{1}{2}y^2\frac{\sqrt{3}}{2}-3\left(\frac{1}{2}x^2\frac{\sqrt{3}}{2}\right)$$
$$=\frac{\sqrt{3}}{4}\left(y^2-3x^2\right)$$
$$=\frac{\sqrt{3}}{4}\left[\left(10+x\right)^2-3x^2\right]$$
$$=\frac{\sqrt{3}}{4}\left(100+20x+x^2-3x^2\right)$$
$$=\frac{\sqrt{3}}{2}\left(50+10x-x^2\right)$$

Method 2:

Area of the remaining shape *PQRSTU*, A

= Area of triangle FDE – 3 (Area of triangle PDQ)

$$=\frac{1}{2}\left(y\right)\left(\sqrt{\frac{3y^2}{4}}\right)-3\left[\left(\frac{1}{2}\right)\left(x\right)\left(\sqrt{\frac{3x^2}{4}}\right)\right]$$
$$=\frac{1}{2}\left(10+x\right)^2\left(\frac{\sqrt{3}}{2}\right)-\left[\left(\frac{3}{2}\right)\left(x\right)\left(\frac{\sqrt{3}}{2}x\right)\right]$$
$$=\frac{\sqrt{3}}{2}\left[\frac{1}{2}\left(100+20x+x^2\right)-\frac{3}{2}x^2\right]$$
$$=\left(\frac{1}{2}\sqrt{3}\right)\left(50+10x-x^2\right)\text{ (shown)}$$

$$A=\left(\frac{1}{2}\sqrt{3}\right)\left(50+10x-x^2\right)$$

$$\frac{\mathrm{d}A}{\mathrm{d}x}=\left(\frac{1}{2}\sqrt{3}\right)\left(10-2x\right)$$

For stationary value of A, $\frac{\mathrm{d}A}{\mathrm{d}x}=0$

$$10-2x=0$$

$$x=5$$

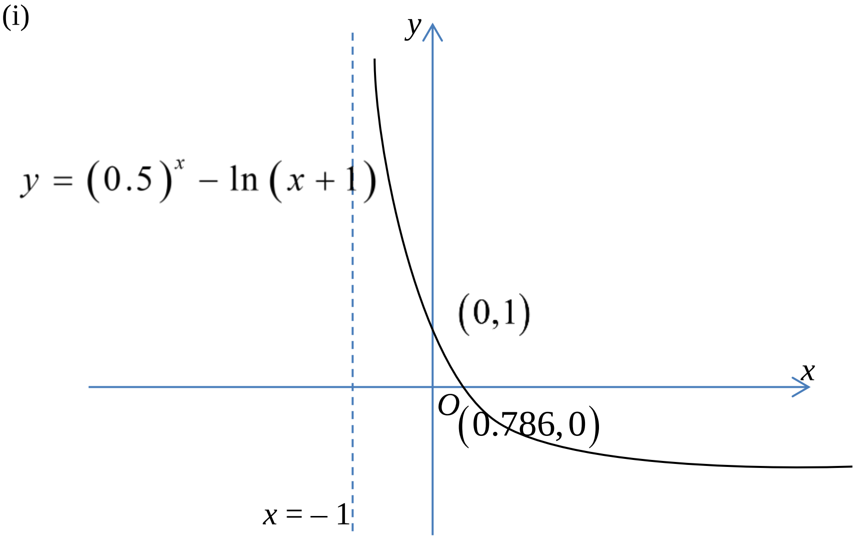
$$\frac{\mathrm{d}^2A}{\mathrm{d}x^2}=-2<0$$

$\therefore$ A is maximum when x = 5.

Maximum value of $A=\left(\frac{1}{2}\sqrt{3}\right)\left(50+10(5)-(5)^2\right)=\left(\frac{75}{2}\sqrt{3}\right)\text{cm}^2$

Q5.

(i)



At x axis intercept (y = 0)	At y axis intercept (x = 0)
$(0.5)^x-\ln(x+1)=0$ $(0.5)^x=\ln(x+1)$ Using GC, $x=0.786$	$y=(0.5)^0-\ln(1)=1$

The coordinates of points of intersection with the axes are (0 , 1) and (0.786, 0).
Asymptote : x = -1

(ii) When x = 0.5, y = 0.30164

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0.5}=-1.1568\approx-1.157\text{ (3 dec. places)}$$

(iii) **Equation of normal is not in syllabus. But we can change the question to the following:**

The tangent to *C* at *P* meet the *x*-axis at *A* and *y*-axis at *B*. Find the length *AB*. [5]

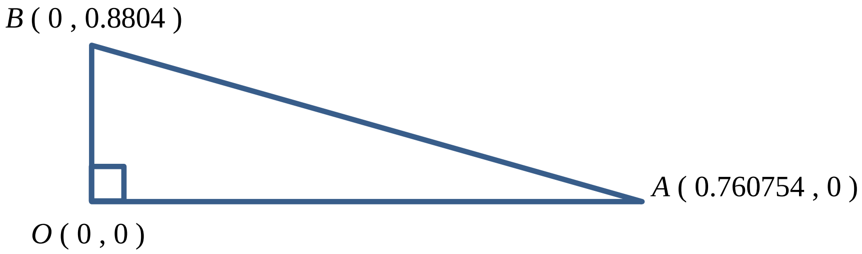
Equation of tangent to *C* at *P*

$$y-(0.30164)=-1.1568\left[x-(0.5)\right]$$

$$y=-1.1568x+0.88004$$

When $y = 0$, $x = 0.760754$, point *A* is (0.760754 , 0)

When $x = 0$, $y = 0.8804$, point *B* is (0 , 0.8804)



By Pythagoras' Theorem,

Length of *AB*

$$=\sqrt{0.88004^2+0.760754^2}$$

$$=1.16328$$

$$\approx 1.16\ (3\text{ sig. fig})$$

Q6

Let *X* be the r.v the masses of peaches sold by a shop.

$$X \sim N(\mu,\sigma^2)$$

$$\mathrm{P}(X<40)=0.2$$

$$\mathrm{P}(Z<\frac{40-\mu}{\sigma})=0.2$$

$$\frac{40-\mu}{\sigma}=-0.8416212335$$

$$40-\mu=-0.8416212335\sigma------(1)$$

$$\mathrm{P}(X>60)=0.25$$

$$\frac{60-\mu}{\sigma}=0.674487495$$

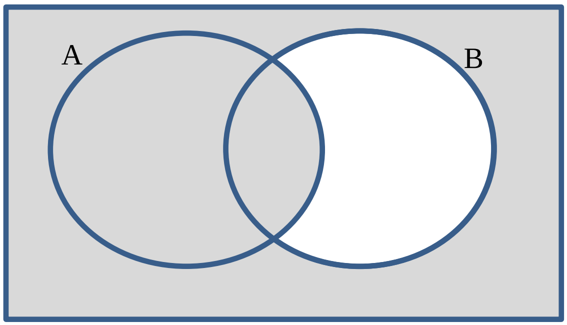
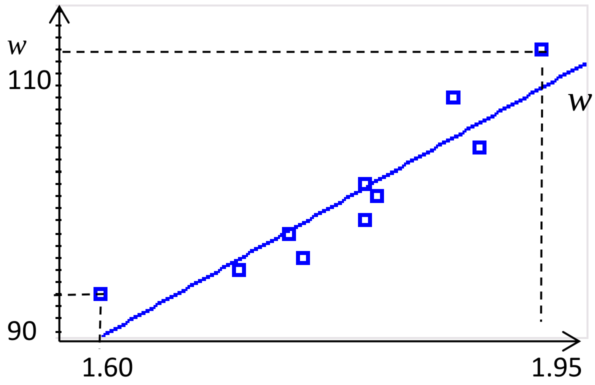
$$60-\mu=0.674487495\sigma--------(2)$$

$(2)-(1)$

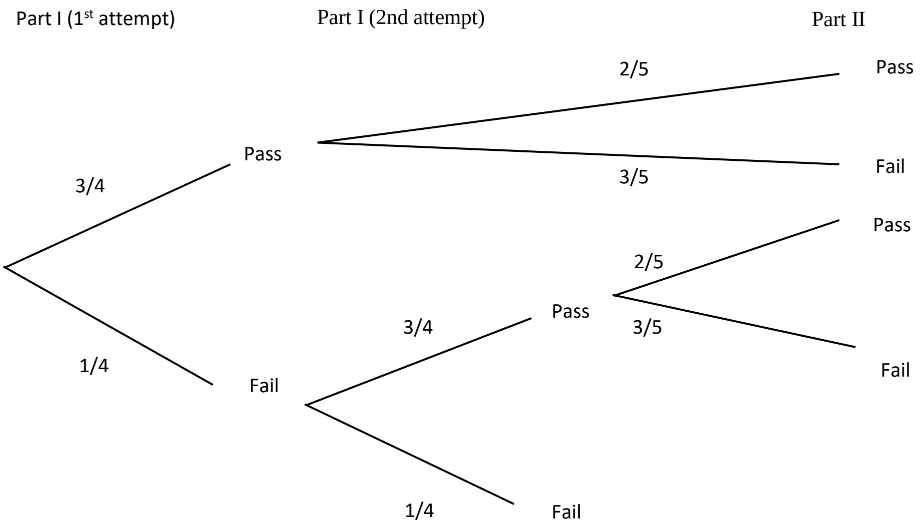
$$20=1.51611\sigma$$

$$\sigma=13.2$$

$$\mu=51.1$$

	Mean of distribution = 51.1, Variance of distribution = $13.2^2 = 174$
Q7.	**Not in syllabus**
Q8	(i) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $0.42 = p + 2p - 0.03$ $p = 0.15$ (shown) (ii) $\mathrm{P}(A \cup B') = \mathrm{P}(A) + \mathrm{P}(A \cup B)'$ $= 0.15 + (1 - 0.42)$ $= 0.73$  $\mathrm{P}(A \cup B')$ (iii) $\mathrm{P}(A \cap B') = \mathrm{P}(A) - \mathrm{P}(A \cap B) = 0.15 - 0.03 = 0.12$ $\mathrm{P}(A) \times \mathrm{P}(B') = 0.15(1 - 2(0.15)) = 0.105$ $\mathrm{P}(A \cap B') \neq \mathrm{P}(A) \times \mathrm{P}(B')$ They are not independent events.
Q9.	Let X be the random variable of the number of 6's out of 8 throws of a fair die. $X \sim B(8, \frac{1}{6})$ (i) $\mathrm{P}(X = 3) = 0.104$ (ii) $\mathrm{P}(X < 4) = \mathrm{P}(X \leq 3) = 0.969$ (iii) **Not in syllabus**
10(i)& (iii)	**NOT in 2020 'A' Level**  $w = 58.0325h - 6.3005$
(ii)	$r = 0.922$

	The value of product moment correlation, r , shows a strong positive linear correlation between the height and weight of members of a rowing club : as height increase, the weight of members of a rowing club increases linearly (at a constant rate).
(iii)	$w = 58.0325h - 6.3005$ $w = 58.0h - 6.30$
(iv)	When $h = 1.66$, $w = 58.03250082(1.66) - 6.300468972$ = 90.0 (3s.f) The estimated weight is 90.0 kg. The estimate is reliable as the value of r is close to 1 and 1.66 metres is within given data range of the height of the members of the rowing club, i.e. the estimate is an interpolation.
11	Let M and W be the random variables denoting the mass of a man and the mass of a woman in kg $M \sim N(77, 9.8^2)$ $W \sim N(62, 10.6^2)$ (i) $\mathrm{P}(\mu - 2 < M < \mu + 2) = \mathrm{P}(77 - 2 < M < 77 + 2) = \mathrm{P}(75 < M < 79) = 0.162$ *OR* $\mathrm{P}(\lvert M - \mu \rvert < 2) = \mathrm{P}(-2 < M - 77 < 2) = \mathrm{P}(75 < M < 79) = 0.162$ (ii) Let $X = (M_1 + M_2 + M_3) - (W_1 + ... + W_4)$ $E(X) = 3 \times 77 - 4 \times 62 = -17$ $Var(X) = 3 \times 9.8^2 + 4 \times 10.6^2 = 737.56$ $X \sim N(-17, 737.56)$ $\mathrm{P}(M_1 + M_2 + M_3 > W_1 + W_2 + W_3 + W_4)$ $= \mathrm{P}((M_1 + M_2 + M_3) - (W_1 + W_2 + W_3 + W_4) > 0)$ $= \mathrm{P}(X > 0) = 0.266$ (iii) Let $Y = (M_1 + M_2 + M_3) + (W_1 + ... + W_4)$ $E(Y) = 3 \times 77 + 4 \times 62 = 479$ $Var(Y) = 3 \times 9.8^2 + 4 \times 10.6^2 = 737.56$ $Y \sim N(479, 737.56)$ $\mathrm{P}(Y \le 460) = 0.242$

12	(i) `  (ii) P (student will succeed in acct qualification) = $\frac{3}{4}\times\frac{2}{5}+\frac{1}{4}\times\frac{3}{4}\times\frac{2}{5}=\frac{3}{8}$ or 0.375 (iii) $\mathrm{P(succeed} \mid \text{failed at 1st attempt}) = \frac{\mathrm{P}\left(\text{succeed} \cap \text{failed at 1st attempt}\right)}{\mathrm{P}\left(\text{failed at 1st attempt}\right)}$ $= \frac{\frac{1}{4}\times\frac{3}{4}\times\frac{2}{5}}{\frac{1}{4}} = \frac{3}{10}$ or 0.3 (iv) Let *X* be the random variable denoting the number of students who qualify out of 5 students. $X \sim B(5, \frac{3}{8})$ $P(X \geq 2) = 1 - P(X \leq 1) = 0.619$
13(i)	Let *X* be the random variable denoting the length of fish in a particular lake in cm and μ be the population mean length of fish. $H_0 : \mu = 15.2$ $H_1 : \mu \neq 15.2$ Under H_o , $\overline{X} \sim N(15.2, \frac{2.1^2}{30})$ Using a two-tailed z-test at 5% level of significance, $\bar{x} = 14.5$ gives $z_{cal} = -1.83$ and p-value $= 0.0679 > 0.05$ We do not reject Ho and conclude that there is insufficient evidence at 5% level of significance that the mean length of fish in the lake is not 15.2cm.
(ii)	Unbiased estimate of the population mean , $\bar{x} = \frac{-32}{40} + 18 = 17.2$

	Unbiased estimate of the population variance, $s^2 = \frac{1}{39}\left(325 - \frac{(-32)^2}{40}\right) = 7.6769 = 7.68$
(iii)	A sample statistic T is an unbiased estimator of a population parameter θ if E(T) = θ .i.e. $\mathrm{E}(\overline{X}) = \mu$ and $\mathrm{E}(S^2) = \sigma^2$ Or: A sample statistic is an unbiased estimate if it contains no systematic bias i.e. it does not tend to overestimate or underestimate the population parameter.
(iv)	Let Y be the random variable denoting the length of fish from the second lake in cm and μ be the population mean length of fish. $H_0 : \mu = 18$ $H_1 : \mu < 18$ Under H_o , $\overline{Y} \sim N(18, \frac{7.6769}{40})$ approximately by Central Linit Theorem since $n = 40$ is large. Using a 1-tailed z test at $\alpha\%$ level of significance, $\overline{x} = 17.2$ gives p-value $= 0.033917$ Since Ho is rejected, p-value $\leq \alpha\%$ $0.033917 \leq \frac{\alpha}{100}$ $\alpha \geq 3.39$ The set values of α is $\{\alpha \in \mathbb{R} : 3.39 \leq \alpha < 100\}$