Class	Register Number	Name

Bukit Batok Secondary School
GCE ‘O’ Level Preliminary Examination 2021
Secondary 4 Express

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 24 August 2021

Tuesday

08 00 – 10 15

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

If you need additional writing space, you may request for writing/graph papers from the invigilators.
At the end of the examination, you will insert the additional writing/graph papers into your Question Paper.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **90**.

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For Examiner’s Use

This document consists of **19** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ⊗ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 \quad 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions

1 The quadratic function $3 - 2m + mx - x^2$ is always negative for $p < m < q$.
Determine the value of p and of q. [4]

2 Given that $y=(1+x)e^{3x}$, prove that $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=0$. [4]

3 A watermelon is assumed to be spherical in shape while it is growing. Its mass, M kg, and radius r cm, are related by the formula $M = kr^3$, where k is a constant. It is also assumed that the radius is increasing at a constant rate of 0.1 cm per day. On a particular day the radius is 10 cm and the mass is 3.2 kg.

(i) Find the value of k. [2]

(ii) Find the rate at which the mass is increasing on this day. [3]

4 The reflex angle θ is such that $\cos\theta = k$, where $0 < k < 1$.

(i) Find an expression, in terms of k, for

(a) $\sin\theta$, [2]

(b) $\tan\theta$. [1]

(ii) Explain why $\sin 2\theta$ is negative for $0 < k < 1$. [2]

5 **(a)** Find the term independent of x in the expansion of $\left(2x - \frac{1}{3x}\right)^6$. [2]

(b) In the expansion of $(3 - 2x)\left(1 + \frac{x}{2}\right)^n$, the coefficient of x is 7.

Find the value of the constant n and hence find the coefficient of x^2. [5]

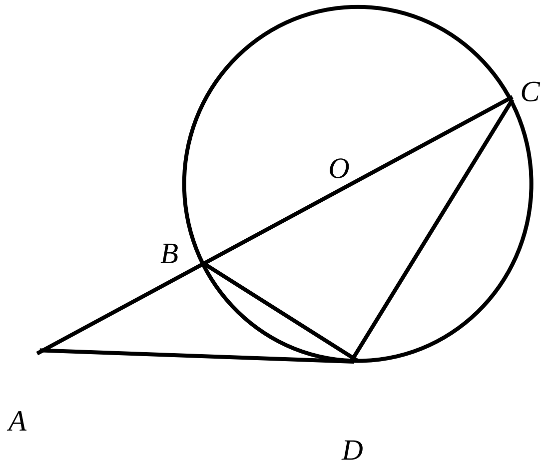
6 The population *P*, in millions, of a country is given by $P = A \times b^t$, where *t* is the number of years after January 2000 and *A* and *b* are constants. In January 2010 the population was 40 million and had increased to 45 million by January 2013.

(i) Show that *b* = 1.04 correct to 2 decimal places and find *A* correct to the nearest integer. [4]

(ii) Find the population in January 2020, giving your answer to the nearest million. [1]

(iii) Find the year when the population will be over 100 million for the first time in the month of January. [3]

7 **[NOT TO SCALE]**



In the diagram, *BC* is a diameter of the circle, *ABC* is a straight line and *AD* is a tangent to the circle at *D*. $AD = DC$ and $AB : BC = 1 : 2$.

(i) Explain why ΔABD is similar to ΔADC . [3]

(ii) Hence, or otherwise, show that $AD = \sqrt{3}AB.$ [3]

(iii) Hence find $\angle ADB.$ [3]

8 **(a)** The equation of a curve is $y = 2 + \dfrac{3}{2x-1}$.

(i) Obtain an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Explain why the curve has no stationary points. [1]

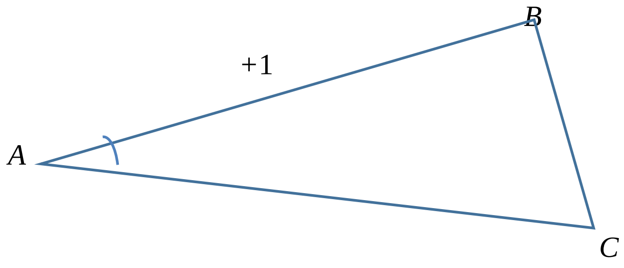
(b) A curve has equation $= (2x-1)\sqrt{4x+3}$, $x > -\frac{3}{4}$.

(i) Find $\frac{dy}{dx}$ in the form $\frac{4(Ax+B)}{\sqrt{4x+3}}$ where *A* and *B* are integers. [3]

(ii) Hence write down the x-coordinate of the stationary point of the curve. [1]

(iii) Determine the nature of this stationary point. [2]

9 **(a)**



Given that the area of the triangle ABC is 5.5 cm^2, and length of AB is $(2\sqrt{3}+1)$cm, find the exact value of AC in the form $+b\sqrt{3}$, where a and b are integers. [4]

(b) Solve the following simultaneous equations.

$$3^x \times 9^{y-1} = 243$$

$$8 \times 2^{y-\frac{1}{2}} = \frac{2^{2x+1}}{4\sqrt{2}}$$

[5]

10 A particle P moves in a straight line so that its displacement, s metres, from a fixed point O is given by $s = t^3 - 9t^2 + 24t + 2$, where t is the time taken in seconds after the start of the motion.

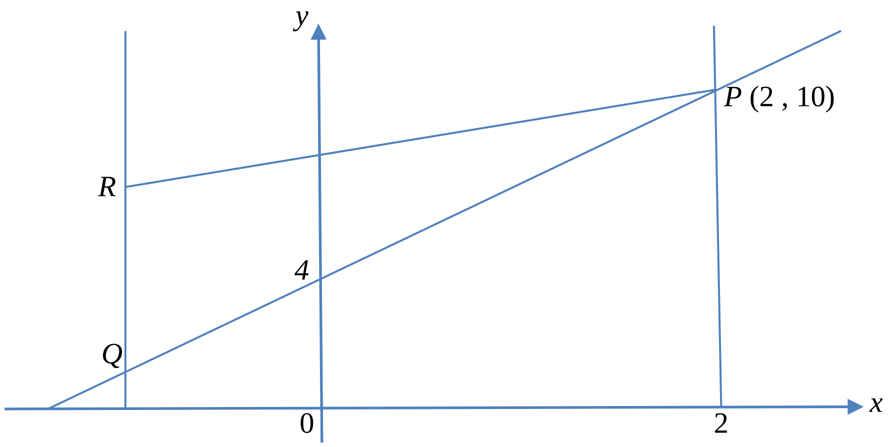
(i) Find the initial velocity and acceleration. [2]

(ii) Find the values of t when P is instantaneously at rest. [2]

(iii) Find the total distance travelled in the first 6 seconds. [3]

(iv) Show that P will never return to its starting point. [3]

11 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows the line $y = mx + 4$ meets the lines $x = 2$ and $x = -1$ at the points P and Q respectively.

The point R is such that QR is parallel to the y-axis and the gradient of RP is 1.

The point P has coordinates (2 , 10).

(i) Find the value of m. [1]

(ii) Find the y-coordinate of Q. [1]

(iii) Find the coordinates of R. [2]

(iv) Find the equation of the line through P, perpendicular to PQ, giving your answer in the form $ax + by = c$, where a, b and c are integers. [3]

(v) Find the coordinates of the midpoint, M, of PQ. [1]

(vi) Find the area of triangle MRP. [2]

12 A function f is defined by $f(x) = 3\cos 2x - 2$ for $0 \le x \le 2\pi$ and a function g is defined by $g(x) = 3\ tanx - 4$ for $0 \le x \le 2\pi$.

(i) State the amplitude and the period of f(x). [2]

(ii) State the period of g(x). [1]

(iii) Explain why g(x) has no amplitude. [1]

(iv) Solve f(x) = 0. [2]

(v) On the same axes, for $0 \le x \le 2\pi$, sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$, indicating the intercepts on the y-axis. [4]

--END OF PAPER 1--

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