Candidate Index Number

Anglo - Chinese School (Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2021
YEAR 3 INTEGRATED PROGRAMME
CORE MATHEMATICS
PAPER 1

Friday **1st October 2021** **1 hour 30 minutes**

Candidates answer on the Question Paper.
No additional materials are required.

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

Paper 1 containts more straight-forward questions, but many students made careless mistakes in expansion, simplification and factorisation. They do not check the workings and verify the answers with the knowledge or the facts they learned. Many students still could not give the form of equation properly and they wrote expression instead.

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This paper consists of 15 printed pages and 1 blank page.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions in the spaces provided.*

1. *[Maximum mark: 9]*

(a) Evaluate $4-\dfrac{3}{3-\dfrac{3}{3-\dfrac{3}{4}}}$. *[3 marks]*

(b) Make q the subject of the formula, $p=\sqrt{\dfrac{3q-2p}{q+5}}$. *[3 marks]*

(c) Factorise $12x^3-4x^2y-3xz^2+yz^2$ completely. *[3 marks]*

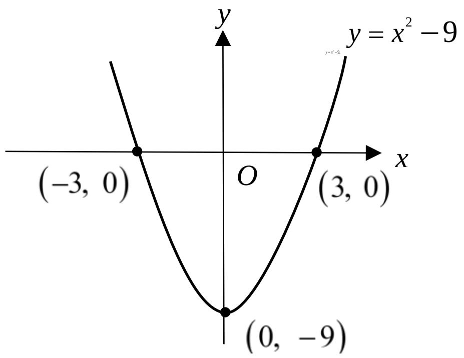
1 (a)	$4-\dfrac{3}{3-\dfrac{3}{3-\dfrac{3}{4}}}=4-\dfrac{3}{3-\dfrac{3}{\frac{9}{4}}}$ $=4-\dfrac{3}{3-\dfrac{4}{3}}$ $=4-\dfrac{3}{\frac{5}{3}}$ $=4-\dfrac{9}{5}$ $=\dfrac{11}{5}$ Students made careless mistake in simplifying the fraction in fraction.
(b)	$p=\sqrt{\dfrac{3q-2p}{q+5}}$ $p^2=\dfrac{3q-2p}{q+5}$ $p^2q+5p^2=3q-2p$ $q\left(p^2-3\right)=-2p-5p^2$ $q=\dfrac{-2p-5p^2}{p^2-3}$ Since the question wants q to be subject, q must not appear on the right hand side of the equation.
©	$12x^3-4x^2y-3xz^2+yz^2$ $=4x^2(3x-y)-z^2(3x-y)$ $=\left(4x^2-z^2\right)(3x-y)$ $=(2x+z)(2x-z)(3x-y)$ Students did not factorise completely the last step.

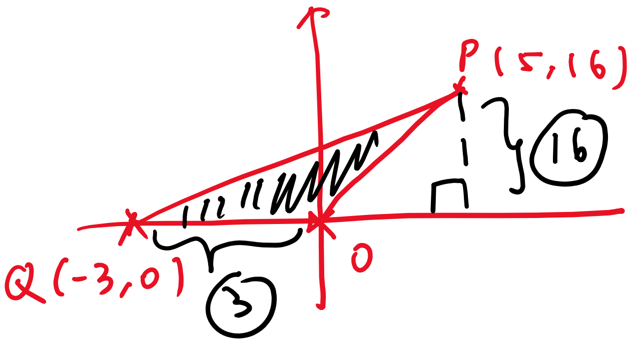
2. *[Maximum mark: 8]*

(a) Sketch the graph of $y = x^2 - 9$ clearly labelling the coordinates of the axes-intercepts and turning point. *[2 marks]*

(b) The curve $y = x^2 - 9$ meets the line $y = 2x + 6$ at points P and Q. Find the coordinates of P and Q. *[4 marks]*

(c) Hence, find the area of ΔPOQ where O is the origin. *[2 marks]*

(a)		The two x-intercepts and turning point must be clearly labelled. Since the question asked for a sketch, students SHOULD NOT plot the graph to scale!
(b)	When $y = x^2 - 9$, $x^2 - 9 = 2x + 6$ $x^2 - 2x - 15 = 0$ $(x-5)(x+3) = 0$ $x - 5 = 0$ or $x + 3 = 0$ $x = 5$ $\quad x = -3$ $y = 2(5) + 6$ $\quad y = 2(-3) + 6$ $= 16$ $\quad = 0$ The coordinates of P and Q are (5, 16) and (-3, 0).	This question is meant to be a simultaneous equation. Solution to simultaneous equation = point(s) of intersection between the two equations. However, there are some students who tried to solve them by plotting the two graphs to scale. This is time consuming and not making use of the right concept.
(c)	Area of ΔPOQ $= \frac{1}{2}\begin{vmatrix} 5 & -3 & 0 & 5 \\ 16 & 0 & 0 & 16 \end{vmatrix}$ $= \frac{1}{2}\lvert 0 + 0 + 0 - (-48) - 0 - 0 \rvert$ $= 24$ units^2	Alternate solution: Area $= \frac{1}{2} \times 3 \times 16 = 24$



3. *[Maximum mark: 8]*

(a) Solve $5y \geq y^2 + 6$. *[2 marks]*

(b) Solve $\frac{2x+1}{2} < -4(x+2) \leq 5x+19$ and hence state the integer values of *x* that satisfies the inequality.

[6 marks]

3 (a)	$y^2 + 6 \leq 5y$ $y^2 - 5y + 6 \leq 0$ $(y-3)(y-2) \leq 0$ $2 \leq y \leq 3$		
(b)	$\frac{2x+1}{2} < -4(x+2) \leq 5x+19$ $\frac{2x+1}{2} < -4(x+2)$ $2x+1 < -8x-16$ $10x < -17$ $x < -1.7$ $\therefore -3 \leq x < -1.7$ The integer values are -3 and -2.	$-4(x+2) \leq 5x+19$ $-4x-8 \leq 5x+19$ $-27 \leq 9x$ $x \geq -3$	Students must state the integer values which satisfies the final inequality.

4. *[Maximum mark: 5]*

A cuboid has a square base of length $1+\sqrt{2}$ units and the volume is $7+5\sqrt{2}$ units^3. Express the height of the cuboid, *H*, in the form $a+b\sqrt{2}$, where *a* and *b* are constants. What do you notice about this cuboid?

[5 marks]

4	$(1+\sqrt{2})(1+\sqrt{2})H = (7+5\sqrt{2})$ $H = \frac{7+5\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$ $= \frac{21-14\sqrt{2}+15\sqrt{2}-10(2)}{9-4(2)}$ $= 21-20+\sqrt{2}(15-14)$ $= 1+\sqrt{2}$ units The height of the cuboid is equal to the length of its square base. Hence it is a cube.	It is preferred that students state the cuboid is a cube. However, students MUST compare the lengths and not merely state the lengths of the sides.

5. *[Maximum mark: 5]*

Given that A is an acute angle and $\cos A = \frac{1}{3}$,

(a) find the value of $\sin A$, *[2 marks]*

(b) hence, show that $\frac{2\tan A - 1}{3\sin A} = 2 - \frac{\sqrt{2}}{4}$. *[3 marks]*

5 (a)	$\sin A = \frac{\sqrt{3^2 - 1}}{3}$ $= \frac{\sqrt{8}}{3} / \frac{2\sqrt{2}}{3}$	Angle A is acute and it is in Quadrant 1. Hence all the trigo ratios are positive. Some students gave sin A as a negative value which don't make sense.
(b)	$\frac{2\tan A - 1}{3\sin A} = \frac{4\sqrt{2} - 1}{3\left(\frac{2\sqrt{2}}{3}\right)}$ $= 2 - \frac{1}{4\sqrt{2}}$ or $2 - \frac{\sqrt{2}}{4}$ (shown)	

When asked to "show", students SHOULD not cross multiply with the right-hand side of the equation.

Students are expected to manipulate the left-hand side of the equation until it looks like the expression on the right hand side.

6. *[Maximum mark: 9]*

Given that the roots of the quadratic equation $3x^2 - 2x - 3 = 0$ are α and β.

(a) State the value of $\alpha + \beta$ and of $\alpha\beta$. *[2 marks]*

(b) Show that $\alpha^2 + \beta^2 = \frac{22}{9}$. *[2 marks]*

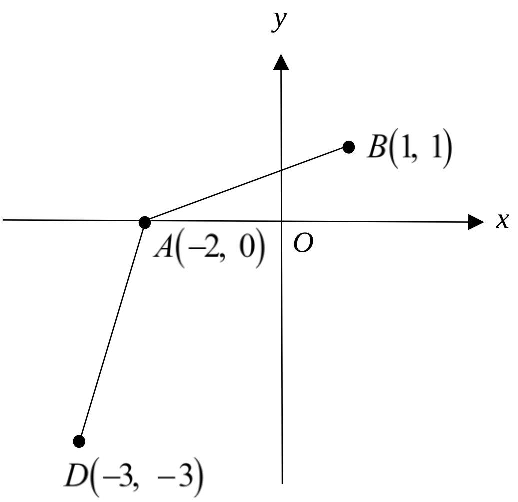
(c) Find the quadratic equation with roots $\alpha + 2\beta$ and $2\alpha + \beta$.. *[5 marks]*

(a)	$3x^2 - 2x - 3 = 0$ $x^2 - \frac{2}{3}x - 1 = 0$ $\therefore \alpha + \beta = \frac{2}{3}, \quad \alpha\beta = -1.$
(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= \left(\frac{2}{3}\right)^2 - 2(-1)$ $= \frac{22}{9}$ (shown)
(c)	$(\alpha + 2\beta) + (2\alpha + \beta) = 3\alpha + 3\beta$ $= 3\left(\frac{2}{3}\right)$ $= 2$ $(\alpha + 2\beta)(2\alpha + \beta) = 2\alpha^2 + \alpha\beta + 4\alpha\beta + 2\beta^2$ $= 2(\alpha^2 + \beta^2) + 5(\alpha\beta)$ $= 2\left(\frac{22}{9}\right) + 5(-1)$ $= -\frac{1}{9}$ Quadratic equation, $x^2 - 2x - \frac{1}{9} = 0 / 9x^2 - 18x - 1 = 0$

NOTE:
It is a MUST for students to equate the expression to 0!

7. *[Maximum mark: 8]*

The points $A(-2,0)$, $B(1,\ 1)$ and $D(-3,-3)$ are the three vertices of a rhombus *ABCD*. *E* is a point at the foot of the perpendicular from *A* to *BD*.



(a) Find the equation of *CD*. *[3 marks]*

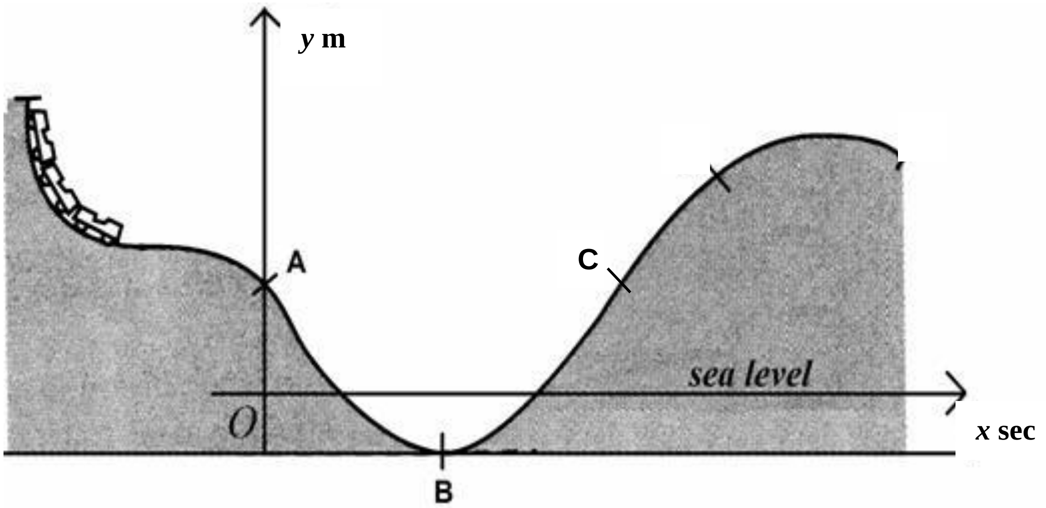
(b) Find the length of *AE*, leaving your answer in surd form. *[3 marks]*

(c) Find the area of the rhombus. *[2 marks]*

(a)	Coordinates of *C* are (0, -2) Equation of *CD*, $y-(-3)=\left(\frac{0-1}{-2-1}\right)\left[x-(-3)\right]$ $y=\frac{1}{3}x-2$	Use the wrong formula to find gradient Use the wrong point.
(b)	Method 1 Coordinates of *E* $=\left(\frac{-3+1}{2},\ \frac{-3+1}{2}\right)$ $=(-1,\ -1)$ Distance of *AE* $=\sqrt{\left[-1-(-2)\right]^2+(-1-0)^2}$ $=\sqrt{2}$ units Sum up the 2 values instead of minus – wrong formula. Could not identify the point E correctly.	Method 2 Area of ΔABD $=\frac{1}{2}\begin{vmatrix}1 & -2 & -3 & 1\\ 1 & 0 & -3 & 1\end{vmatrix}$ $=\frac{1}{2}\lvert 0+6+(-3)-(-2)-0-(-3)\rvert$ $=4\text{ units}^2$ Area of $\Delta ABD=\frac{1}{2}\times AE\times BD$ $4=\frac{1}{2}\times AE\times\sqrt{(-3-1)^2+(-3-1)^2}$ $AE=\frac{8}{\sqrt{32}}$ $=\frac{2}{\sqrt{2}}$ or $\sqrt{2}$ units
(c)	Area of rhombus $=\frac{1}{2}\begin{vmatrix}1 & -2 & -3 & 0 & 1\\ 1 & 0 & -3 & -2 & 1\end{vmatrix}$ $=\frac{1}{2}\lvert 0+6+6+0-(-2)-0-0-(-2)\rvert$ or $=8\text{ units}^2$	$=2\times$ Area of triangle *ABD* $=2\times\frac{1}{2}\times\sqrt{2}\times\sqrt{2(-3-1)^2}$ $=8\text{ units}^2$

8. *[Maximum mark: 9]*

A roller coaster at an amusement park goes through an underwater tunnel. The track traveled by the roller coaster is shown in the diagram below. The height, y metres of the roller coaster above the sea level as it travels from point A to point C can be modelled by the equation $y=\frac{1}{10}x^2-4x+5,$ where x is the time in seconds.



(a) Express $y=\frac{1}{10}x^2-4x+5$ in the form of $y=a(x-h)^2+k$. *[3 marks]*

(b) (i) State the height of the roller coaster above sea level at point A. *[1 mark]*

(ii) State the time when the roller coaster reaches the lowest point at B. *[1 mark]*

(c) Hence, solve $y=0$, leaving your answers in the surd form and state what the answers represent. *[4 marks]*

(a)	$y=\frac{1}{10}x^2-4x+5$ $=\frac{1}{10}(x^2-40x)+5$ $=\frac{1}{10}(x-20)^2-\frac{1}{10}(-20)^2+5$ $=\frac{1}{10}(x-20)^2-35$	Did not factorise the expression but change the expression to equation. Missing step in leading to the final answer. (b)(i) can use the original expression given to find
(bi)	The height of the roller coaster above sea level at point A is 5 m.	
(ii)	The time when the roller coaster reaches the lowest point at B is 20 seconds.	

(c)	$y=0$ $\frac{1}{10}(x-20)^2-35=0$ $(x-20)^2=350$ $x-20=\pm\sqrt{350}$ $x=20\pm\sqrt{350}$ or $x=20\pm5\sqrt{14}$ $20\pm\sqrt{350}$ or $20\pm5\sqrt{14}$ seconds are the time when the roller coaster is at the sea level. Context in the questions as in the x values found are referring to the time at the sea level **not** time taken or points or x-intercepts.

9. *[Maximum mark: 10]*

(a) Find $\log_2 r$ if $r^x=16$ and $3^x=81$. *[3 marks]*

(b) Given that $3^p=8$ and $8^q=81$, find the value of pq. *[3 marks]*

(c) Solve for x if $(\ln x)^2-3(\ln 5)^2=2(\ln 5)(\ln x)$. *[4 marks]*

(a)	$\because 3^x=81$ $3^x=3^4$ $\therefore x=4$ $\because r^x=16$ $r^4=2^4$ $r=\pm2$ $\log_2 r=\log_2 2$ $\quad$ $\log_2 r=\log_2(-2)$ (NA) $=1$ Did not answer the question by finding the value of $\log_2 r$ Square-root any square must have 2 answers
(b)	Given that $3^p=8$ and $8^q=81$, $(3^p)^q=3^4$ $3^{pq}=3^4$ $\therefore pq=4$ or $p=\frac{\lg 8}{\lg 3}, q=\frac{\lg 81}{\lg 8}$ $pq=\frac{\lg 8}{\lg 3}\times\frac{\lg 81}{\lg 8}$ $=\frac{\lg 3^4}{\lg 3}$ $=4$ Did not show the step that $81=3^4$ or simplify the logarithm wrongly (penalize 1 method mark)
(b)	$(\ln x)^2-3(\ln 5)^2=2(\ln 5)(\ln x)$ $(\ln x)^2-2(\ln 5)(\ln x)-3(\ln 5)^2=0$ $(\ln x-3\ln 5)(\ln x+\ln 5)=0$ $\ln x-3\ln 5=0$ or $\ln x+\ln 5=0$ $\ln x=3\ln 5$ $\quad$ $\ln x=-1\ln 5$ $\ln x=\ln 5^3$ $\quad$ $\ln x=\ln 5^{-1}$ $x=125$ $\quad$ $x=\frac{1}{5}$ Solving the equation as Quadratic equation is the key to solve this question. Method marks only awarded to logical and useful steps or workings shown. Common mistakes: $2\ln x-6\ln 5=2\ln(5+x)$ $\ln^2 x-3\ln^2 5=\ln(2\times5x)$

10. *[Maximum mark: 9]*

(a) Find the range of values of k such that the curve $y=-(x-1)^2-k^2-4k$ is always negative for all real values of x. *[4 marks]*

(b) Given that $a-b=22$ and $\sqrt{a}+\sqrt{b}=11$, find the value of $\sqrt{ab}$. *[5 marks]*

(a)	$-(x-1)^2-k^2-4k$ $=-(x^2-2x+1)-k^2-4k$ $=-x^2+2x-k^2-4k-1$ Discriminant < 0 $2^2-4(-1)(-k^2-4k-1)<0$ $4+4(-k^2-4k-1)<0$ $4-4k^2-16k-4<0$ $4k^2+16k>0$ $(4k)(k+4)>0$ $k<-4$ or $k>0$ Method 2 For the negative curve to be always negative, $(1,-k^2-4k)$ must be the maximum point and $-k^2-4k<0$ $-k(k+4)<0$ $k(k+4)>0$ $k<-4$ or $k>0$	Without expansion, students could not look for the value of *a*, *b* and *c* to substitute into the discriminant formula. Some students are unable to link the concept – discriminant. By using graph, it will be easier to locate the range of *k* instead of separating into 2 factors. **[Common mistakes]** Only a few students managed to answer using Method 2.

(b)	$a-b=(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$ $22=11(\sqrt{a}-\sqrt{b})$ $\sqrt{a}-\sqrt{b}=2$ Method 1 $(\sqrt{a}+\sqrt{b})+\left(\sqrt{a}-\sqrt{b}\right)=2\sqrt{a}$ $11+2=2\sqrt{a}$ $\sqrt{a}=\frac{13}{2}$ $\therefore \sqrt{ab}=\left(\sqrt{a}\right)\left(\sqrt{b}\right)$ $=\frac{13}{2}\times\frac{9}{2}$ $=\frac{117}{4}$ $(\sqrt{a}+\sqrt{b})-\left(\sqrt{a}-\sqrt{b}\right)=2\sqrt{b}$ $11-2=2\sqrt{b}$ $\sqrt{b}=\frac{9}{2}$ Complicated questions, Yes! Method marks only awarded to logical and useful steps or workings shown. If you stuck, definitely need to check the workings and try other methods to solve further. Method 2 $\sqrt{a}+\sqrt{b}=11$ $\sqrt{a}=11-\sqrt{b}$.................(1) $(1)^2$, $a=121-22\sqrt{b}+b$ When $a=22+b$, $22+b=121-22\sqrt{b}+b$ $22\sqrt{b}=99$ $\sqrt{b}=\frac{9}{2}$ When $\sqrt{b}=\frac{9}{2}$, $\sqrt{a}=11-\frac{9}{2}$ $=\frac{13}{2}$ $\therefore \sqrt{ab}=\left(\sqrt{a}\right)\left(\sqrt{b}\right)$ $=\frac{9}{2}\times\frac{13}{2}$ $=\frac{117}{4}$ or $\sqrt{a}+\sqrt{b}=11$ $\left(\sqrt{a}+\sqrt{b}\right)^2=11^2$ $a+b+2\sqrt{ab}=121$ When $a=22+b$, $22+b+b+2\sqrt{(22+b)b}=121$ $2\sqrt{22b+b^2}=121-22-2b$ $4\left(22b+b^2\right)=(99-2b)^2$ $88b+4b^2=9801-396b+4b^2$ $484b=9801$ $b=\frac{81}{4}$ When $b=\frac{81}{4}$, $a=22+\frac{81}{4}$ $=\frac{169}{4}$

End of Paper