VICTORIA JUNIOR COLLEGE
JC1 PROMOTIONAL EXAMINATION

H2 MATHEMATICS **9758**

QUESTION PAPER

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer all questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This document consists of **5** printed pages and **3** blank pages.

1 A small garment shop produces dresses, blouses and skirts. The production process includes cutting, sewing and packaging. The table below shows the time in minutes required to produce each piece of garment. The total time spent on cutting, sewing and packaging are 3150, 4300 and 480 minutes per week respectively.

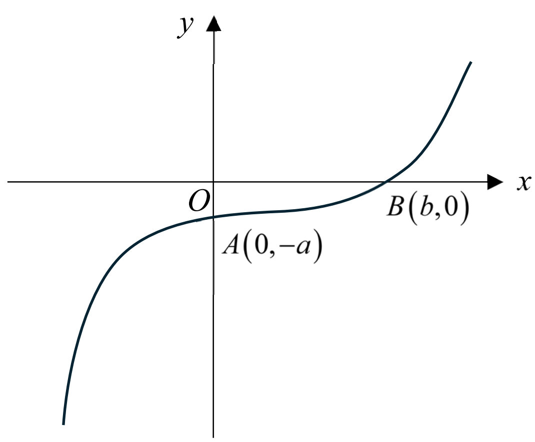
Garment	Time required (min)		
	Cutting	Sewing	Packaging
Dress	45	70	8
Blouse	50	60	6
Skirt	20	25	3

Write down and solve equations to find the number of pieces of each garment the shop produces each week. [4]

2 **(a)** Without using a calculator, solve the inequality $\dfrac{2x^2+3x+10}{x^2-x} \geq 2$. [3]

(b) Hence, find the set of values of x for which $\dfrac{2e^{2x}-3e^{x}+10}{e^{2x}+e^{x}} \geq 2$. [2]

3 The diagram shows the curve $y = f(x)$ which cuts the axes at $A(0,-a)$ and $B(b,0)$, where a and b are positive constants.



(a) State, if it is possible to do so, the coordinates of the points where the following curves cut the axes.

(i) $y = f^{-1}(x)$ [1]

(ii) $y = f(x-2)$ [1]

(iii) $y = f(3x)$ [1]

(b) Sketch the curve $y = \dfrac{1}{f(x)}$, stating the equations of any asymptotes and the coordinates of the point where $y = \dfrac{1}{f(x)}$ crosses the axes. [3]

4 **(a)** The sum, S_n, of the first n terms of a sequence $u_1,\ u_2,\ u_3,\ ...$ is given by $S_n = 75 - \dfrac{3^{n+1}}{5^{n-2}}$.

Show that $u_n = k\left(\dfrac{3}{5}\right)^{n-1}$, where k is a constant to be determined. Find S_∞. [4]

(b) Another sequence of numbers $v_1,\ v_2,\ v_3,\ ...$ is such that $\displaystyle\sum_{i=1}^{n} v_i = \frac{2n}{2n+1}$.

Find the exact value of $\displaystyle\sum_{i=11}^{\infty} v_i$. [3]

5 A curve C has equation $3x^2 + 2xy - 4y^2 = 1$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x+y}{4y-x}$. [2]

(b) Find the acute angle between the tangent to C at the point (1, 1) and the x-axis. [2]

(c) Show that there is no point on C where the tangent to the curve is parallel to the x-axis. [3]

6 **(a)** An infinite sequence is such that $u_1 = p$, where p is a constant, and $u_{n+1} = 12 - \dfrac{24}{u_n}$, where $n \geq 1$.

(i) Showing your working clearly, explain why p cannot be 2.4. [2]

(ii) A constant sequence is a sequence in which every term is the same. Given that $p > 5$, find the value of p for which the sequence is a constant sequence. [2]

(b) An arithmetic series has a positive first term v_1 and common difference d. Given that $3v_{10} = 5v_{19}$, find the largest possible value of the sum of the first n terms, leaving your answer in terms of d. [4]

7 **(a)** **(i)** Showing your working, find the complex numbers v and w which satisfy the following simultaneous equations.

$$v + 2w = 5$$
$$3v - w^* = 1 + 5\mathrm{i}$$ [4]

(ii) Points O, V and W on an Argand diagram represent the complex numbers 0, v and w respectively. Plot these points on an Argand diagram and state the transformation that maps the point W onto V. [2]

(b) The roots of the equation $z^4 - 2z^3 - 3z^2 + az + b = 0$, where a and b are real numbers, are z_1, z_2, z_3 and z_4. It is given that $z_1^{\ 2} + z_2^{\ 2} + z_3^{\ 2} + z_4^{\ 2} < 0$. Explain why at most two of z_1, z_2, z_3 and z_4 are real. [2]

8 The curve C_1 has equation

$$(x-1)^2+4y^2=4.$$

The curve C_2 has parametric equations

$$x=\sec\theta \text{ and } y=2\tan\theta.$$

(a) Show that the cartesian equation of C_2 is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, where a and b are positive integers to be determined. [2]

(b) Sketch C_1 and C_2 on the same diagram, indicating clearly the equations of any asymptotes and coordinates of vertices. [4]

(c) Given that $k>0$, find the range of values of k such that $k(x-1)^2+4y^2=4$ cuts C_2 at most twice. [2]

(d) Describe a sequence of transformations that maps C_1 to a unit circle with centre (0, 0). [2]

9 **(a)** Using standard series from the List of Formula (MF27), expand $\frac{\cos ax}{x^2+a^2}$ as far as the term in x^4, where a is a positive constant. Give your answer in the form $c_1+c_2x^2+c_3x^4$ where c_1, c_2 and c_3 are in terms of a. [6]

(b) Find the range of values of x, in terms of a, for which the expansion in part **(a)** is valid. [1]

(c) Using your answer in part **(a)** and a suitable value of a, show that $\int_0^1 \frac{\cos 2x}{x^2+4}\,dx \approx \frac{119}{960}$. [3]

10 **(a)** **(i)** State the derivative of $e^{\tan x}$. [1]

(ii) Hence, find $\int e^{\tan x}(\sec^3 x)(\sin x)\,dx$. [2]

(b) Write down constants A and B such that, for all values of x, $1-4x=A(3-8x)+B$. Hence, find $\int \frac{1-4x}{\sqrt{1-4x^2+3x}}\,dx$. [5]

(c) Use the substitution $u=\sqrt{1+x}$ to find the exact value of $\int_1^3 \frac{x^2}{\left(\sqrt{1+x}\right)^3}\,dx$. [4]

11 The functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{5x+13}{x^2-1} \quad \text{for } x \in \mathbb{R},\ x \neq \pm 1,$$

$$\mathrm{g}: x \mapsto -\ln(x-1) \quad \text{for } x \geq 2.$$

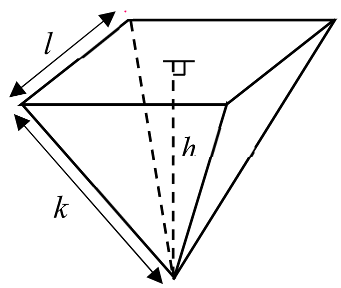
(a) Sketch the graph of $y = \mathrm{f}(x)$, stating the equations of any asymptotes, the coordinates of any turning points and points of intersections with the axes. [4]

(b) Explain why gf does not exist. [1]

(c) Show that g has an inverse and find $\mathrm{g}^{-1}(x)$. [4]

(d) Find the range of fg^{-1}. [2]

12 [The volume of a square-based pyramid is $\frac{1}{3} \times \text{base area} \times \text{height}$.]



A manufacturer makes open containers in the shape of an inverted square pyramid with square base of side length l cm, vertical height h cm and fixed slant edge k cm, as shown above. He decides that the volume of each container, V cm^3, should be as large as possible.

(a) Show that $V^2 = \dfrac{k^2 l^4}{9} - \dfrac{l^6}{18}$. [2]

(b) Use differentiation to find, in terms of k, the exact maximum value of V, proving that it is a maximum. [6]

(c) Water is poured into an empty container with the dimensions found in part **(b)** at a constant rate of $\frac{1}{3}$ cm^3 per second. It is given that at time t seconds, the volume of water in the container is W cm^3 and the water level is p cm. Show that $W = \frac{4}{3}p^3$ and hence, find the rate of increase of p at the instant when $p = \frac{k}{4}$. [4]

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