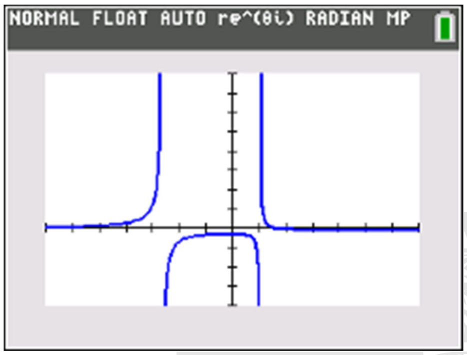
Raffles Institution
H2 Mathematics (9758)
Solution for 2019 A-Level Paper 2

Section A: Pure Mathematics

Question 1

No.	Suggested Solution	Remarks for Student
(i)	$I = -\frac{2}{3}x(1-x)^{\frac{3}{2}} + \frac{2}{3}\int (1-x)^{\frac{3}{2}}\ \mathrm{d}x$ $= -\frac{2}{3}x(1-x)^{\frac{3}{2}} - \frac{4}{15}(1-x)^{\frac{5}{2}} + c$	Don't forget to include the arbitrary constant of integration.
(ii)	$x = 1-u^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -2u$ $I = \int (1-u^2)\sqrt{u^2}(-2u)\ \mathrm{d}u$ $= 2\int (u^2-1)u^2\ \mathrm{d}u$ $= 2\int (u^4-u^2)\ \mathrm{d}u$ $= \frac{2}{5}u^5 - \frac{2}{3}u^3 + d$ $= \frac{2}{5}(1-x)^{\frac{5}{2}} - \frac{2}{3}(1-x)^{\frac{3}{2}} + d$	$x = 1-u^2$ $u = (1-x)^{\frac{1}{2}}$ Final answer in terms of x.
(iii)	$\frac{2}{5}(1-x)^{\frac{5}{2}} - \frac{2}{3}(1-x)^{\frac{3}{2}} + d - \left(-\frac{2}{3}x(1-x)^{\frac{3}{2}} - \frac{4}{15}(1-x)^{\frac{5}{2}} + c\right)$ $= \frac{2}{3}x(1-x)^{\frac{3}{2}} - \frac{2}{3}(1-x)^{\frac{3}{2}} + \frac{4}{15}(1-x)^{\frac{5}{2}} + \frac{2}{5}(1-x)^{\frac{5}{2}} + d - c$ $= \frac{2}{3}(1-x)^{\frac{3}{2}}(x-1) + \frac{10}{15}(1-x)^{\frac{5}{2}} + d - c$ $= -\frac{2}{3}(1-x)^{\frac{3}{2}}(1-x) + \frac{2}{3}(1-x)^{\frac{5}{2}} + d - c$ $= -\frac{2}{3}(1-x)^{\frac{5}{2}} + \frac{2}{3}(1-x)^{\frac{5}{2}} + d - c$ $= d - c$	Show difference in answer in parts (i) and (ii) do not depend on x.

Question 2

No.	Suggested Solution	Remarks for Student
(i)	$y=\dfrac{2-x}{3x^2+5x-8}=\dfrac{2-x}{(3x+8)(x-1)}$  Label Asymptotes: $x=-\dfrac{8}{3}, x=1, y=0$ Label axial intercepts: $\left(0,-\dfrac{1}{4}\right)$, $(2,0)$	
(ii)	$x<-\dfrac{8}{3}$ or $1<x<2$	
(iii)	$\dfrac{x-2}{(3x+8)(x-1)}>0$ $\dfrac{2-x}{(3x+8)(x-1)}<0$ $\therefore -\dfrac{8}{3}<x<1$ or $x>2$	

Question 3

No.	Suggested Solution	Remarks for Student
(i)	$900 = 2\pi r^2 + 2\pi rh$ $450 = \pi r^2 + \pi rh$ $h = \dfrac{450 - \pi r^2}{\pi r}$ $V = \pi r^2 h = \pi r^2\left(\dfrac{450 - \pi r^2}{\pi r}\right) = 450r - \pi r^3$ $\dfrac{dV}{dr} = 450 - 3\pi r^2 = 0 \quad \Rightarrow \pi r^2 = 150$ $\Rightarrow r = \sqrt{\dfrac{150}{\pi}}\ (r > 0)$ $\dfrac{d^2V}{dr^2} = -6\pi r < 0$ for all $r > 0$, that is, $r = \sqrt{\dfrac{150}{\pi}}$ gives max V. $\max V = \pi r^2 h$ $= 150\left(\dfrac{450 - \pi r^2}{\pi r}\right)$ $= 150\left(\dfrac{450 - 150}{\pi\sqrt{\dfrac{150}{\pi}}}\right)$ $= \dfrac{300\sqrt{150}}{\sqrt{\pi}}$ $h = \dfrac{450 - \pi r^2}{\pi r}$ $\dfrac{1}{h} = \dfrac{\pi r}{450 - \pi r^2}$ $\dfrac{r}{h} = \dfrac{\pi r^2}{450 - \pi r^2} = \dfrac{150}{450 - 150} = \dfrac{1}{2}$ $\therefore r : h = 1 : 2$	

Question 4

No.	Suggested Solution	Remarks for Student
(i)	$\mathrm{f}'(x)=2\sec 2x\tan 2x$ $\mathrm{f}''(x)=2(2\sec 2x\tan 2x)\tan 2x+2(2\sec^2 2x)\sec 2x$ $\quad\quad=4\sec 2x\tan^2 2x+4\sec^3 2x$ $\mathrm{f}(0)=1$ $\mathrm{f}'(0)=0$ $\mathrm{f}''(0)=4$ $\mathrm{f}(x)\approx 1+2x^2$	
(ii)	$\int_0^{0.02}(1+2x^2)\,\mathrm{d}x = 0.0200053333 \approx 0.02001$ (5 d.p.)	
(iii)	$\int_0^{0.02}\sec 2x\ \mathrm{d}x = 0.0200053355 \approx 0.02001$ (5 d.p.)	
(iv)	The approximation is good for small values of x. Part (ii) uses Maclaurin series with polynomial of degree 2 to approximate the integral with x = 0.02 which only differ from actual value from 9th d.p. This already provides a good approximation.	
(v)	We would need g and all derivatives of g to be defined in order to apply Macluarin series. However $\operatorname{cosec}2x=\dfrac{1}{\sin 2x}$ is undefined at x = 0.	

Question 5

No.	Suggested Solution	Remarks for Student
(i)	$\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{BX}$ $= \underset{\sim}{b} + \lambda\overrightarrow{BD}$ $= \underset{\sim}{b} + \lambda\left(\overrightarrow{OD} - \overrightarrow{OB}\right)$ $= \underset{\sim}{b} + \lambda\left(\underset{\sim}{b} + 5\underset{\sim}{a} - \underset{\sim}{b}\right)$ $= \underset{\sim}{b} + 5\lambda\underset{\sim}{a}$ $\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$ $= \underset{\sim}{a} + \mu\overrightarrow{AC}$ $= \underset{\sim}{a} + \mu\left(\overrightarrow{OC} - \overrightarrow{OA}\right)$ $= \underset{\sim}{a} + \mu\left(2\underset{\sim}{a} + 4\underset{\sim}{b} - \underset{\sim}{a}\right)$ $= (1+\mu)\underset{\sim}{a} + 4\mu\underset{\sim}{b}$ $\underset{\sim}{b} + 5\lambda\underset{\sim}{a} = (1+\mu)\underset{\sim}{a} + 4\mu\underset{\sim}{b}$ $4\mu = 1 \Rightarrow \mu = \frac{1}{4}$ $5\lambda = 1 + \mu = \frac{5}{4} \Rightarrow \lambda = \frac{1}{4}$ $\overrightarrow{OX} = \underset{\sim}{b} + \frac{5}{4}\underset{\sim}{a}$	
(ii)	$\overrightarrow{OY} = \alpha\overrightarrow{OD} + (1-\alpha)\overrightarrow{OC}$ $= \alpha\left(\underset{\sim}{b} + 5\underset{\sim}{a}\right) + (1-\alpha)\left(2\underset{\sim}{a} + 4\underset{\sim}{b}\right)$ $\overrightarrow{OY} = \beta\overrightarrow{OX}$ $= \beta\left(\underset{\sim}{b} + \frac{5}{4}\underset{\sim}{a}\right)$ $\alpha\left(\underset{\sim}{b} + 5\underset{\sim}{a}\right) + (1-\alpha)\left(2\underset{\sim}{a} + 4\underset{\sim}{b}\right) = \beta\left(\underset{\sim}{b} + \frac{5}{4}\underset{\sim}{a}\right)$ $\beta = \alpha + 4(1-\alpha) \Rightarrow \beta + 3\alpha = 4 \quad ...(1)$ $\frac{5}{4}\beta = 5\alpha + 2(1-\alpha) \Rightarrow \frac{5}{4}\beta - 3\alpha = 2 \quad ...(2)$ $(1)+(2): \quad \frac{9}{4}\beta = 6$ $\therefore \beta = \frac{24}{9} = \frac{8}{3}$ $\overrightarrow{OY} = \beta\overrightarrow{OX} = \frac{24}{9}\left(\underset{\sim}{b} + \frac{5}{4}\underset{\sim}{a}\right) = \frac{8}{3}\underset{\sim}{b} + \frac{10}{3}\underset{\sim}{a}$ $OX : OY = 3 : 8$	

Section B: Statistics

Question 6

No.	Suggested Solution	Remarks for Student
(i)	These 22 clubs form the population as they are ALL the clubs in Division One whose approaches to training she is interested to find out.	
(ii)	**How** Assuming no special treatment with regard to facilities for supporters of different divisions, he could randomly pick a certain number of clubs out of the 100, say 10, to do a thorough investigation. **Why** This is to avoid bias and it would also be more cost effective and manageable.	
(iii)	${}^{22}C_5 \times {}^{24}C_5 \times {}^{26}C_5 \times {}^{28}C_5 = 7.24\times10^{18}$	

Question 7

No.	Suggested Solution	Remarks for Student
(i)	The event that one mug is faulty is independent of each other. The probability of a mug being faulty remains constant at 0.08.	
(ii)	F ~ B(50, 0.08) $\mathrm{P}(F \geq 7) = 1-\mathrm{P}(F \leq 6)$ $= 0.10187$ ≈ 0.102	
(iii)	Let W denote number of days out of 5 with at least 7 faulty mugs. W ~ B(5, 0.10187) $\mathrm{P}(W \leq 2) = 0.99098$ ≈ 0.991	
(iv)	${}^{10}C_2 p^2 (1-p)^8 = 45p^2(1-p)^8$	
(v)	P(no fault) + P(one fault) $= 0.92^2(1-p)^2 + \left[\mathrm{P}(\text{fault with one mug}) + \mathrm{P}(\text{fault with one saucer})\right]$ $= 0.8464(1-p)^2 + \left[2(0.92)(0.08)(1-p)^2 + 2(0.92^2)p(1-p)\right]$ $= 0.8464(1-p)^2 + 0.1472(1-p)^2 + 1.6928p(1-p)$ $= 0.9936(1-p)^2 + 1.6928p(1-p)$ $0.9936(1-p)^2 + 1.6928p(1-p) = 0.97$ $p = 0.0689$	

Question 8

<table>
<tr><th>No.</th><th>Suggested Solution</th><th>Remarks for Student</th></tr>
<tr><td></td><td>
<table>
<tr><td></td><td>Orange</td><td>Yellow</td><td>Green</td><td>White</td><td>Total</td></tr>
<tr><td>Horse</td><td>1</td><td>1</td><td>3</td><td>4</td><td>9</td></tr>
<tr><td>Rider</td><td>1</td><td>1</td><td>7</td><td>5</td><td>14</td></tr>
<tr><td>Dog</td><td>3</td><td>7</td><td>1</td><td>6</td><td>17</td></tr>
<tr><td>Bird</td><td>4</td><td>5</td><td>6</td><td>1</td><td>16</td></tr>
<tr><td>Total</td><td>9</td><td>14</td><td>17</td><td>16</td><td>56</td></tr>
</table>
</td><td>You can just create the “total” column on the question booklet itself in the A level.</td></tr>
<tr><td>(i)
(a)</td><td>$\frac{9+14}{56}=\frac{23}{56}$</td><td>Just count relevant cells from the table above</td></tr>
<tr><td>(b)</td><td>$\frac{17+16-7}{56}=\frac{13}{28}$</td><td></td></tr>
<tr><td>(ii)
(a)</td><td>$\frac{8\times7}{56\times55}=\frac{1}{55}$</td><td></td></tr>
<tr><td>(b)</td><td>Case 1: Dog is yellow, the other item is a non-yellow Horse/Rider/Bird
Probability = $\frac{7}{56}\times\frac{32}{55}\times2$
Case 1: Dog is not yellow, the other item is a yellow Horse/Rider/Bird
Probability = $\frac{10}{56}\times\frac{7}{55}\times2$
Required probability = $\frac{7}{56}\times\frac{32}{55}\times2+\frac{10}{56}\times\frac{7}{55}\times2=\frac{21}{110}$</td><td></td></tr>
<tr><td>(iii)</td><td>$\frac{a}{56}\times\frac{b}{55}\times2=\frac{1}{77}$
where a,b refer to the number of his first and second favourites
$\therefore ab=20=1\times20$ or 2×10 or 4×5
Reference to the table, only 4,5 is possible
Thus, possible combinations are as follow
<table>
<tr><td>4</td><td>5</td></tr>
<tr><td>White Horse</td><td>White Rider</td></tr>
<tr><td>White Horse</td><td>Yellow Bird</td></tr>
<tr><td>Orange Bird</td><td>White Rider</td></tr>
<tr><td>Orange Bird</td><td>Yellow Bird</td></tr>
</table>
</td><td></td></tr>
</table>

Question 9

No.	Suggested Solution	Remarks for Student
(i)	Since the manager wants to check "whether the mean resistance is in fact 750 ohms", significant variation of both more or less than 750 is not acceptable. Thus, he should carry out a 2-tail test to see if the mean resistance differs from 750 ohms. Null hypothesis, $\mathrm{H}_0 : \mu = 750$ Alternative hypothesis, $\mathrm{H}_1 : \mu \neq 750$ where μ is the population mean resistance of resistors rated at 750 ohms.	
(ii)	Let X denote the resistance (in ohms) of a resistor rated at 750 ohms. Using GC, $\bar{x} = 756$ Perform an 2-tailed test at 5% significance level. Under H0, $\bar{X} \sim \mathrm{N}\left(750, \frac{100}{8}\right)$ $p\text{-value} = 2\mathrm{P}\left(\bar{X} > 756\right) = 0.0897 > 0.05$, hence we do not reject $\mathrm{H}_0 : \mu = 750$. The manager does not have **sufficient evidence at 5%** level of significance to claim that the mean resistance is not 750 ohms.	Since X follows a Normal distribution, then so does $\bar{X}$. The population variance is given in the question so we use it. Answer in context with reference to the alternative hypothesis.
(iii)	**How** Take a large random sample, say 50 instead of 8 in the previous test, of resistors rated at 1250 ohms, and take down the resistances. Calculate the sample mean and unbiased estimate of population variance using this new sample. Use Central Limit Theorem to approximate the distribution of $\bar{X}$, to determine the p-value, and carry out the test. **Why** Distribution of the restistance of the population of resistors is unknown. (so we need a ***large sample*** to approximate the probability distribution of $\bar{X}$) The population variance is also unknown. (so we need to estimate it by s^2)	

Question 10

No.	Suggested Solution	Remarks for Student
(i) **(a)**	Do on graph paper.	For $y = 35 - \frac{1}{3}x$, it would be easier to plot 2 points and join them. Example join (42, 21) and (54, 17) to get the line $y = 35 - \frac{1}{3}x$
(b)	On same graph paper	
(c)	$\frac{4}{3}$	Using GC
(d)	Residual = $b - \mathrm{f}(a)$ could be either positive and negative. Measuring vertical distance require the use of modulus, that is $\lvert \mathrm{f}(a) - b \rvert$ and minimizing the sum of (absolute) residuals is difficult. Squaring, that is $\left(\mathrm{f}(a) - b\right)^2$, is easier to investigate when there is a need to find the minimum value as we could perform differentiation. Squaring also makes the larger values even larger, so more important. Thus, in general, the sum of the squares of the residuals rather than the sum of the residuals is used.	
(ii)	Since $1 < \frac{4}{3}$, Bhani's model is a better fit.	
(iii)	(50, 18.6)	Using GC
(iv)	$y = 33.6 - 0.3x$ $r = -0.985$	Using GC
(v)	24.6 This value may not be reliable as 30 is not in the given range of speed, that is, 40 to 60 km/h.	Using GC
(vi)	Sum of residuals = 0 implies that ALL data points lie on the regression line, which is impossible in any experiment. Deduction: Data points in Cerie's experiment are possibly falsified.	

Question 11

No.	Suggested Solution	Remarks for Student
(i)	Let W denote mass of a white ball in grams. Carbon Let B denote mass of a black ball in grams. Hydrogen $W \sim \mathrm{N}\left(110, 4^2\right),\ B \sim \mathrm{N}\left(55, 2^2\right)$ $W_1 + W_2 + W_3 + W_4 \sim \mathrm{N}\left(440, 64\right)$ $\mathrm{P}\left(W_1 + W_2 + W_3 + W_4 > 425\right) = 0.96960 \approx 0.970$	
(ii)	$W + B \sim \mathrm{N}\left(165, 20\right)$ $\mathrm{P}\left(161 < W + B < 175\right) = 0.80178 \approx 0.802$	
(iii)	$W_1 + W_2 + B_1 + B_2 + B_3 \sim \mathrm{N}\left(385, 44\right)$ $\mathrm{P}\left(W_1 + W_2 + B_1 + B_2 + B_3 < M\right) = 0.271$ $\therefore M = 380.955 \approx 381$	
(iv)	Let W' denote mass of a drilled white ball in grams. Let B' denote mass of a drilled black ball in grams. Let R denote mass of a rod in grams. $W' \sim \mathrm{N}\left(77, 7.84\right)$ $B' \sim \mathrm{N}\left(49.5, 3.24\right)$ $R \sim \mathrm{N}\left(20, 0.81\right)$ Let X denote mass of the model in grams $X = W' + B_1' + B_2' + B_3' + B_4' + R_1 + R_2 + R_3 + R_4$ $X \sim \mathrm{N}\left(355, 24.04\right)$ $\mathrm{P}\left(X > 350\right) = 0.84608 \approx 0.846$	