

JURONG JUNIOR COLLEGE

J2 Preliminary Examination

## MATHEMATICS Higher 2

9740/02

Paper 2

24 August 2009

3 hours

Additional materials:

Answer Paper List of Formulae (MF15) Cover Page

## READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in. Write in dark blue or black pen on both sides of the paper. You may use a soft pencil for any diagrams or graphs. Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands. You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question. At the end of the examination, fasten all your work securely together, with the cover page in front.

This document consists of 7 printed pages and 1 blank page.

[Turn over

## Section A: Pure Mathematics [40 marks]

1 Given that lines 
$$l_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 8 \\ 1 \end{pmatrix}$$
 and  $l_2 : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \alpha \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$  where  $\alpha, \lambda \in \Box$ , meet at point A.

B is the point (6, 8, 4) and lies on  $l_1$  while C is a point on  $l_2$  with non-negative coordinates.

- (i) Given that AB and AC are of equal lengths, show that  $\overrightarrow{OC} = \begin{pmatrix} 10 \\ 3 \\ 3 \end{pmatrix}$ . [3]
- (ii) A point *D* is such that *ABDC* is a parallelogram. Find the position vector of *D*. [2]
- (iii) Find the sine of the angle BAC exactly and hence find the exact area of the parallelogram ABDC.[3]



answer in exact form.

2

The graph of  $y = x^2 e^x$ , for  $0 \le x \le 1$  is shown in the diagram. Rectangles, each of width  $\frac{1}{n}$  are drawn under the curve as shown.

(a) Show that A, the total area of the rectangles may be expressed as  $\frac{1}{n^3} \sum_{r=1}^{n-1} f(r)$ , where f(r) is to be found. [2] Find, using integration by parts, the limiting value of A when  $n \to \infty$ , giving your

[4]

(b) A region *R* in the first quadrant is bounded completely by the curve  $y = x^2 e^x$ , the line y = e and the *y*-axis. Find the volume of revolution formed when *R* is rotated completely about the *x*-axis, giving your answer correct to 2 decimal places. [2]

3 (a) Expand  $\left(\frac{1-3x}{2+x}\right)^n$  where  $n \in \square^+$ , in ascending powers of x, up to and including the term in  $x^2$ . State the range of values of x for which the expansion is valid. [4]

(**b**) (**i**) Given that 
$$\ln y = \frac{\tan^{-1} x}{2}$$
, show that  $2(1+x^2)\frac{d^2 y}{dx^2} = (1-4x)\frac{dy}{dx}$ . [2]

(ii) Find the first four terms of the Maclaurin's series of y in ascending powers of x.[3]

- (iii) Given that x is sufficiently small for  $x^3$  and higher powers of x to be neglected, deduce an expansion for  $e^{\left(\frac{\tan^{-1} x}{2}\right) - \ln(\sec x)}$  in ascending powers of x. [2]
- 4 The curve *C* has equation  $y = \frac{2x^2 + kx 1}{x + 1}$ , where *k* is a constant.
  - (i) Show that *C* has no stationary point if k > 1. [2]
  - (ii) Given that y = 2x + 1 is an asymptote of *C*, show that k = 3. [2]

(iii) State a sequence of transformations which transform the graph of  $y = x - \frac{1}{x}$  to the

graph of 
$$y = \frac{2x^2 + 3x - 1}{x + 1}$$
. [3]

(iv) Sketch on separate diagrams, the graphs of  $y = \frac{2x^2 + 3x - 1}{x + 1}$  and  $y^2 = \frac{2x^2 + 3x - 1}{x + 1}$ , showing clearly the coordinates of the points where the graphs cross the axes and the equations of any asymptotes. [4] Hence deduce the number of real roots of the equation  $2x^4 + 3x^3 - x^2 - x = 1$ . [2]

## Section B: Statistics [60 marks]

5 A company has 2000 employees belonging to the following groups:

Production	1200
Marketing	600
Management	100
Others	100

The company's president wants to obtain an estimate of the views of all employees about a pending executive decision.

A sample of 100 employees is to be chosen to take part in a survey.

- (i) Describe one disadvantage of obtaining the sample using quota sampling. [1]
- (ii) Name and briefly describe another method of sampling, in which each group is represented proportionately. [3]
- 6 Past records have shown that the mean reading speed per minute of Primary 2 school children is 163 words. The numbers of words, *w*, per minute read by a random sample of 80 Primary 2 school children are taken and summarised by

$$\sum w = 13560 \qquad \qquad \sum w^2 = 2388670.$$

- (i) Test, at the 5% level of significance, whether the mean reading speed of Primary 2 school children has increased.
  [5]
- (ii) Explain what do you understand by the phrase "at the 5% level of significance" in the context of this question. [1]
- (iii) Explain whether the test is still valid if the 80 children were all chosen from the same school.

7 (a) In an organisation of 10 workers, how many ways can a leader and 3 other committee members be appointed? [1]

After appointing the leader and the 3 committee members, in how many ways can these 10 people sit at a round table, if the leader must have any two of his committee members seated beside him, when

- (i) the seats are not numbered? [2]
- (ii) the seats are numbered? [1]
- (b) Five cards each have a single digit written on them. The digits are 1, 2, 2, 4, 6 respectively. How many even numbers greater than 4000 can be formed by placing some or all of these cards side by side? [3]
- 8 A deck of 36 cards has four colours, red, blue, green and yellow. Each colour set consists of 9 cards numbered 1, 2, 3, ..., 9.
  - (a) One card is taken at random from the deck. Events *A* and *B* are defined as follows:*A*: The card taken is blue.
    - *B*: The card taken is a 1.

(i)	Determine whether A and B are independent events.	[2]
(ii)	Describe in words what the event $A \cup B$ represents and find the probability	/ of

- this event. [2]
- (iii) Find P(A'|B'). [2]
- (b) Three cards are taken from the deck, at random and without replacement. Find the probability that all three cards are of different colours with the same number. [2]
- (c) James takes 20 cards from this deck of 36 cards at random and without replacement.
  Find the probability that none of these cards are numbered '1'. [2]

- 9 In a large canteen, a quarter of the customers drink coffee.
  - (a) Find the probability that at least 3 out of a random sample of 7 customers drink coffee.[2]
  - (b) If the probability that fewer than k out of a random sample of 500 customers drink coffee is at least 0.99, using a suitable approximation, find the least value of k. [5]
  - (c) Assuming that 99.9% of the customers do not make a complaint and that complaints occur independently, using a suitable approximation, find the probability that more than 498 out of a random sample of 500 customers will not make a complaint. [3]
- 10 Eric loves to solve online Sudoku puzzles. The puzzles are classified as "Beginner", "Intermediate" and "Expert". The times, in minutes, that Eric takes to solve the puzzles are independent and normally distributed with the means and standard deviations shown in the following table:

Туре	Mean (min.)	Standard Deviation (min.)		
Beginner	3.2	0.5		
Intermediate	μ	2.1		
Expert	12.3	σ		

- (a) If the probability that the total time taken by Eric to solve a "Beginner" and an "Intermediate" puzzle in more than 9.5 minutes is 0.8, find the value of μ, correct to 1 decimal place.
  [3]
- (b) If the probability that Eric takes less than 10 minutes to solve an "Expert" puzzle is at most 0.25, find the maximum value of  $\sigma$ , correct to 1 decimal place. [2]

Suppose  $\mu = 7.8$  and  $\sigma = 3.2$ . Find the probability that

- (c) the total time taken by Eric to solve an "Intermediate" and an "Expert" puzzle is more than four times the time he spends on a "Beginner" puzzle. [3]
- (d) the average time for Eric to solve three "Beginner", two "Intermediate" and one "Expert" puzzles is not more than 7 minutes.[3]

11 It is believed that the pressure P of a gas is related to its volume V by the relation  $PV^c = k$ , where c and k are positive constants.

The table below gives observed values of V for 7 different values of P.

Р	1	2	3	4	7	10	14
V	7.7	5.8	4.5	3.4	2.3	1.9	1.4

The variables x and y are defined as  $x = \ln P$  and  $y = \ln V$ .

- (i) Show that *x* and *y* have a linear relationship. [1]
- (ii) Find the linear product moment correlation coefficient between *x* and *y*. [2]
- (iii) Find the equation of the estimated regression line of y on x and use it to predict the value of V when P = 8. Comment on the reliability of your prediction. [3]
- (iv) Deduce the estimated values of c and k, giving your answers to 3 significant figures. [2]
- (v) For each of the seven values of x, Y' is given by Y' = a + bx, where a and b are any constants. Calculate the minimum value of  $\sum (y Y')^2$ . [3]