Sec 4 Express Mathematics 2024 Prelims Marking Scheme

1		$a(5a-2b)(5a+2b)$		
		$=a\left((5a)^2-(2b)^2\right)$		
		$=a\left(25a^2-4b^2\right)$	**M1**	Correct expansion
		$=25a^3-4ab^2$	**A1**	

2		$3^x+3^{x+2}=90$		
		$3^x+3^x\times3^2=90$	**M1**	
		$3^x(1+9)=90$		
		$3^x=9$		
		$3^x=3^2$		
		$x=2$	**A1**	

3		$\dfrac{2x^2+4xy-3x-6y}{2x^2+xy-6y^2}$		
		$=\dfrac{2x(x+2y)-3(x+2y)}{2x^2+xy-6y^2}$	**M1**	Factorise by grouping for numerator
		$=\dfrac{(x+2y)(2x-3)}{2x^2+xy-6y^2}$		
		$=\dfrac{(x+2y)(2x-3)}{(2x-3y)(x+2y)}$	**M1**	Factorise denominator
		$=\dfrac{2x-3}{2x-3y}$	**A1**	

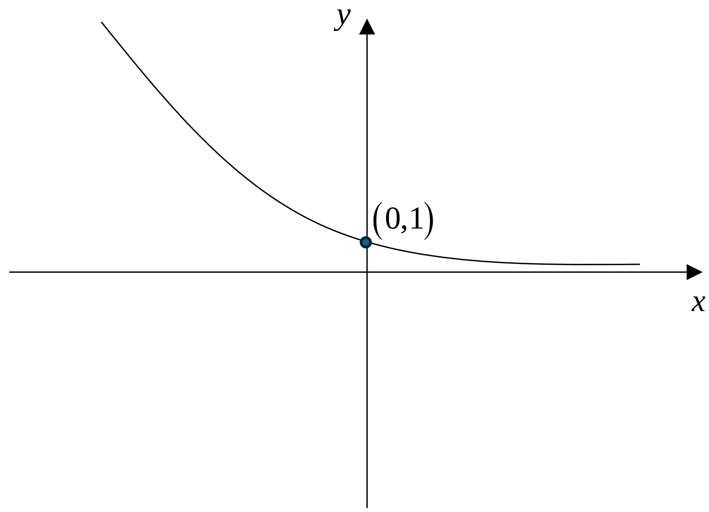
4	**(a)**	LCM of A and B is $q\times3^{m+2}\times7$ and at the same time the LCM of A and B is $3^3\times5\times7$.		
		$m=1$	**B1**	
		$q=5$	**B1**	
	(b)	$A=3^3\times7$ and $B=3\times5\times7$		
		HCF of A and B $=3\times7$ $=21$	**B1**	

5		The size/height of the bar graph.	**B1**	Stating that the graph does not start from zero is accepted too.
		It can be misleading because for example, the size/height of the graph in 2019 is twice that of the graph in 2010 but this does not represent their actual temperatures.	**B1**	A relevant example has to be given.

6	**(a)**	$\left(\dfrac{y^9}{27x^{-6}}\right)^{-\frac{2}{3}}$		
		$=\dfrac{y^{9\times\left(-\frac{2}{3}\right)}}{3^{3\times\left(-\frac{2}{3}\right)}x^{-6\times\left(-\frac{2}{3}\right)}}$ $=\dfrac{y^{-6}}{3^{-2}x^4}$	**M1**	Multiply $\left(-\dfrac{2}{3}\right)$ to each index
		$=\dfrac{3^2}{x^4y^6}$	**A1**	$\dfrac{9}{x^4y^6}$ is also accepted
	(b)	$\dfrac{25}{125^{2-x}}=5^y$		
		$\dfrac{5^2}{\left(5^3\right)^{2-x}}=5^y$ $\dfrac{5^2}{5^{6-3x}}=5^y$ $5^{2-6+3x}=5^y$	**M1**	Any other equivalent form of $5^m=5^n$ will be accepted too
		$5^{3x-4}=5^y$ Comparing index, $3x-4=y$ $x=\dfrac{y+4}{3}$	**A1**	

7	**(a)**	$9x^2 + 24xy + 16y^2$		
		$= (3x + 4y)^2$	**B1**	
	(b)	Let $x = a^4$, $144a^8 - (9a^8 + 24a^4 y + 16y^2)$ $= 144x^2 - (9x^2 + 24xy + 16y^2)$ $= (12x)^2 - (3x + 4y)^2$	**M1**	Attempt to factorise an expression in the form $a^2 - b^2$, after making use of part **(a)** answer
		$= (12x + 3x + 4y)(12x - 3x - 4y)$ $= (15x + 4y)(9x - 4y)$ $= (15a^4 + 4y)(9a^4 - 4y)$	**A1**	

8	**(a)**	$q=\dfrac{k}{r^2}$, where k is the proportionality constant. q and r are the initial intensity and distance respectively.		
		When the distance is reduced by 40%, the new distance is $\dfrac{3r}{5}$. New intensity $=\dfrac{k}{\left(\dfrac{3r}{5}\right)^2}$	**M1**	
		$=\dfrac{k}{\dfrac{9r^2}{25}}$ $=\dfrac{25k}{9r^2}$ $=\dfrac{25}{9}q$	**A1**	
	(b)	Percentage difference $=\dfrac{\text{New Intensity}-\text{Initial Intensity}}{\text{Initial Intensity}}\times 100\%$ $=\dfrac{\dfrac{25}{9}q-q}{q}\times 100\%$ $=\dfrac{16}{9}\times 100\%$ $=178\%\ \text{(3 s.f.)}$	**B1**	$177\dfrac{7}{9}\%$ will be accepted too.

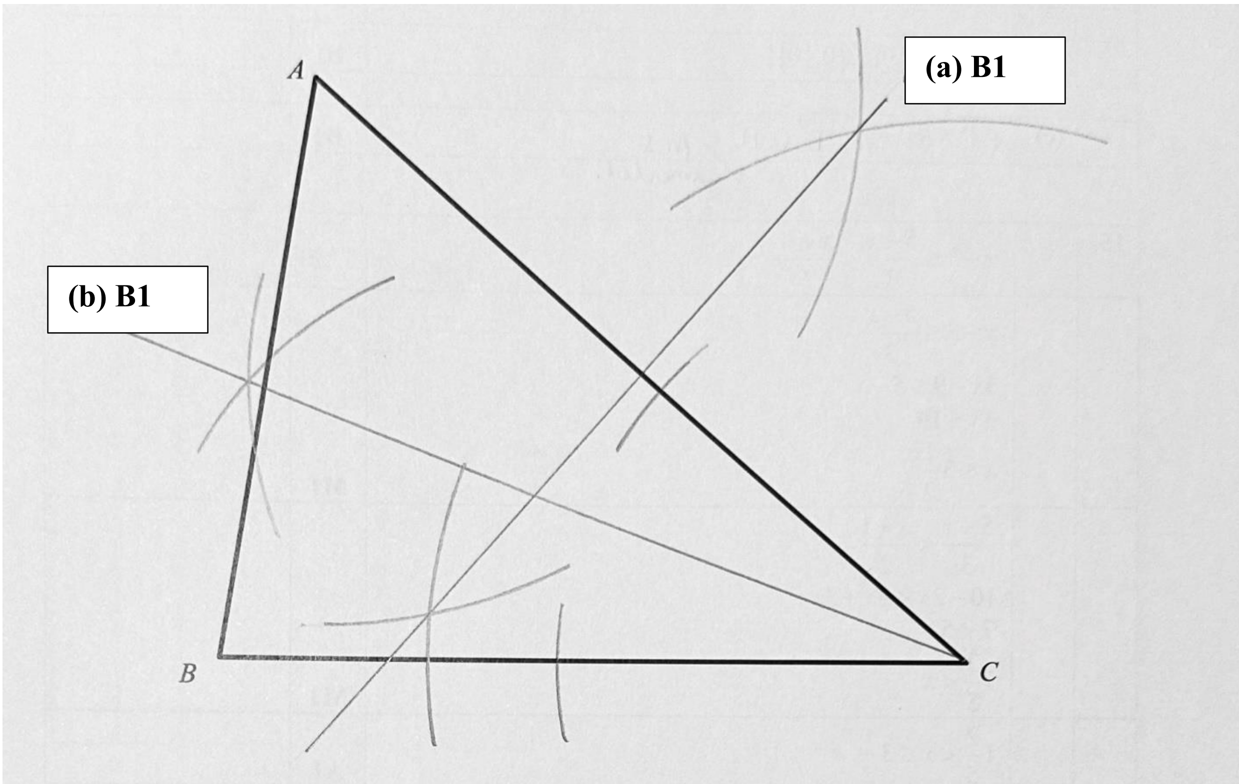
9			**B1**	Both shape and y-intercept needs to be drawn and indicated correctly

10		$SC = RD$ (given)		No marks for this step
		$\angle SCD = \angle RDA$ (alternate angles)	**M1**	
		$CD = DA$ (sides of a rhombus are equal)	**M1**	
		By SAS congruency test, triangle SCD is congruent to triangle RDA.	**A1**	

11		$x^2 - 10x - 2$ $= (x-5)^2 - 25 - 2$ $= (x-5)^2 - 27$	**M1**	
		Minimum point is $(5, -27)$	**A1**	

12	**(a)**	$\mathrm{n}(\xi)$ $=3\times 3$ $=9$	**B1**	
	(b)	P $=\{(-1,0),(0,-1),(0,0),(0,1)\}$	**B1**	
	(c)	Q $=\{(-2,1),(-1,1)\}$	**B1**	

13 The diagram below shows a triangle ABC.



Note that marks will not be awarded if relevant arcs are not drawn for parts **(a)** and **(b)** respectively.

14	**(a)**	$\mathrm{n}(W)=2$	**B1**	
	(b)	$\varnothing, \{0\}, \{\{0\}\}, \{0, \{0\}\}$	**B1**	
	(c)	$A \cup B'$ / $(A' \cap B)'$/ $(A \cup B)' \cup A$/ $(A \cap B) \cup B'$	**B1**	

15		$x-3 \le \frac{5-x}{3} < \frac{x+1}{2}$		
		$x-3 \le \frac{5-x}{3}$ $3x-9 \le 5-x$ $4x \le 14$ $x \le 3\frac{1}{2}$	**M1**	
		$\frac{5-x}{3} < \frac{x+1}{2}$ $10-2x < 3x+3$ $7 < 5x$ $1\frac{2}{5} < x$	**M1**	
		$1\frac{2}{5} < x \le 3\frac{1}{2}$	**A1**	

16		$\angle BAC = 90^\circ$ (right angle in a semicircle)	**M1**	
		$\cos \angle ABC = \frac{AB}{BC}$ $\frac{8}{17} = \frac{AB}{34}$	**M1**	Only awarded with the relevant values substituted in this step.
		$AB = 16$ cm $AC = \sqrt{34^2 - 16^2}$ $= 30$ $\cos \angle ACD = -\cos \angle ACB$	**M1**	
		$= -\frac{AC}{BC}$ $= -\frac{30}{34}$ $= -\frac{15}{17}$	**A1**	

17	**(a)**	$T_7 = 8^2 - 7$ $= 57$	**B1**	
	(b)	$T_n = (n+1)^2 - n$	**B1**	
	(c)	$T_{n+1} - T_n$ $= (n+2)^2 - (n+1) - \left[(n+1)^2 - n\right]$	**M1**	
		$= n^2 + 4n + 4 - n - 1 - \left[n^2 + 2n + 1 - n\right]$ $= 2n + 2$ $= 2(n+1)$	**M1**	
		<u>I agree</u>. Since the difference $T_{n+1} - T_n$ is <u>a multiple of 2</u>, hence the difference will always be an even integer.	**A1**	

18	**(a)**	$15 \div 3 = 5$ $Q = (-2 + 5 \times 2, 1)$ $= (8,1)$	**B1**	
	(b)	Gradient of *PR* $= \dfrac{4-1}{13-(-2)}$ $= \dfrac{1}{5}$	**M1**	
		Let the equation of the line *PR* be $y = mx + c$ $4 = \dfrac{1}{5} \times 13 + c$ $c = \dfrac{7}{5}$ Equation of line *PR*: $y = \dfrac{1}{5}x + \dfrac{7}{5}$	**A1**	
	(c)	$W = (13, 0)$	**B1**	
	(d)	Length of line segment *QR* $= \sqrt{(13-8)^2 + (4-1)^2}$ $= \sqrt{34}$	**M1**	
		Perimeter of *PQRS* $= 2 \times \sqrt{34} + 2 \times 10$	**M1**	
		$= 31.7$ units (3 s.f.)	**A1**	

19		$20.3 \times 10^3 \times 365$ $= 7409500$ $= 7.41 \times 10^6$ (3 s.f.)	**B1**	

20	**(a) (i)**	$\mathbf{P}=\begin{pmatrix}3\\4.50\end{pmatrix}$	**B1**	
	(a) (ii)	$\mathbf{R}=\begin{pmatrix}36 & 40\\48 & 39\\45 & x\end{pmatrix}\begin{pmatrix}3\\4.50\end{pmatrix}$ $=\begin{pmatrix}288\\319.5\\135+4.5x\end{pmatrix}$	**B1**	
	(b)	The elements in matrix **R** represents the total production cost for the muffins sold at Outlets *A*, *B* and *C* respectively.	**B1**	
	(c)	$1.7\mathbf{R}$ $=1.7\begin{pmatrix}288\\319.5\\135+4.5x\end{pmatrix}$	**M1**	
		$=\begin{pmatrix}489.6\\543.15\\229.5+7.65x\end{pmatrix}$		
		$\begin{pmatrix}\frac{5}{6} & \frac{4}{5} & 1\end{pmatrix}\begin{pmatrix}489.6\\543.15\\229.5+7.65x\end{pmatrix}$	**M1**	
		$=(1072.02+7.65x)$ Total amount collected from sale $=\$(1072.02+7.65x)$	**A1**	
	(d)	$1072.02+7.65x=1454.52$ $x=50$	**B1**	

21	**(a)**	Number of students who are in a sports team $=\frac{1}{3}\times 36$ $=12$ Number of leaders in a sports team $=12-4$ $=8$	**B1**	
	(b)	Let the number of students who are members in a performing arts club be *n*. $\mathrm{P}\left(\text{both students selected are members in a performing arts club}\right)$ $=\frac{n}{36}\times\frac{n-1}{35}$ Hence, $\frac{n\left(n-1\right)}{1260}=\frac{1}{42}$	**M1**	
		$n\left(n-1\right)=30$ $n^2-n-30=0$ $\left(n-6\right)\left(n+5\right)=0$	**M1**	Completing the square or quadratic formula is accepted too.
		$n=6$ or $n=-5$ (rej $\because n$ cannot be negative)	**A1**	
	(c)	I disagree with Derrick's claim because; a student <u>can be both a leader</u> and <u>in a performing arts club.</u> Or <u>being a leader and being in a performing arts club</u> are <u>not mutually exclusive</u>.	**B1**	

22		$8\text{ cm}^2 : 2048\text{ m}^2$ $=1\text{ cm}^2 : 256\text{ m}^2$ $=1\text{ cm}^2 : 2560000\text{ cm}^2$	**M1**	
		$=1\text{ cm} : 1600\text{ cm}$ $n=1600$	**A1**	

23		4 technicians can repair 416 computers in 16 days 4 technicians can repair 26 computers in 1 day 4 technicians can repair 260 computers in 10 days	**M1**	
		4 technicians can repair 156 computers in 6 days 1 technician can repair 156 computers in 24 days 3 technicians can repair 156 computers in 8 days	**M1**	
		Total number of days taken $=10+8$ $=18$	**A1**	

<table>
<tr><td>24</td><td>(a)</td><td>No, you cannot because the cumulative frequency diagram <u>only allows you to find probability of less than (less than or equals to) 5 weekly exercise hours or probability of at least (more than) 5 weekly exercise hours.</u></td><td>B1</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(b)</td><td>$(100-60)\%\times 120=48$
$k=3$</td><td>B1</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(c)</td><td><table><tr><td></td><td>Lower Quartile</td><td>Median</td><td>Upper Quartile</td><td>Interquartile Range</td></tr><tr><td>Town B</td><td>1.6</td><td>2.8</td><td>5.8</td><td>4.2</td></tr></table></td><td>B1
B1</td><td>1 mark correct median and 1 mark for correct interquartile range.</td></tr>
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<tr><td></td><td></td><td><table><tr><td></td><td>Lower Quartile</td><td>Median</td><td>Upper Quartile</td><td>Interquartile Range</td></tr><tr><td>Town A</td><td>1.9</td><td>3.5</td><td>5</td><td>3.1</td></tr></table></td><td></td><td></td></tr>
<tr><td></td><td>(d)</td><td>On average, the people in Town A have a higher weekly exercise hours than Town B since its median is higher.</td><td>B1</td><td></td></tr>
<tr><td></td><td></td><td>There is a larger spread in weekly exercise hours in Town B since its interquartile range is higher as compared to that in Town A.</td><td>B1</td><td></td></tr>
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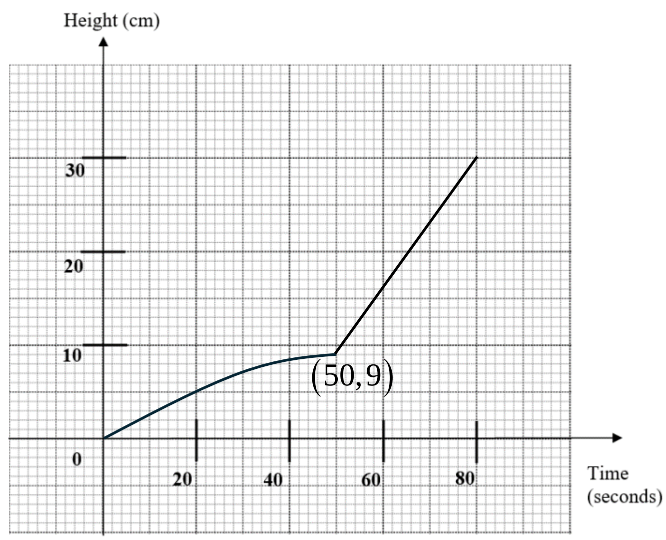
25	**(a)**	$\angle FGD = 180^\circ - \angle FHD$ $= 180^\circ - 40^\circ$ (angles in opposite segments) $= 140^\circ$	**B1**	
	(b)	$\angle CDO = 90^\circ$ (tangent $\perp$ radius)	**B1**	
	(c)	$\angle ODH = 40^\circ$ (base $\angle$ of isosceles triangle)	**M1**	
		$\angle FOD = 40^\circ + 40^\circ$ (external angle of a triangle) $= 80^\circ$		
		$\angle BOF = 360^\circ - 90^\circ - 90^\circ - 62^\circ - 80^\circ$ ($\angle$ sum of quadrilateral $BODC$) $= 38^\circ$	**A1**	
	(d)	$\angle BDF = \frac{38^\circ}{2}$ (angle at centre $= 2 \times$ angle at circumference) $= 19^\circ$	**M1**	
		$\angle OFD = \frac{180^\circ - 80^\circ}{2}$ (base $\angle$ of isosceles triangle) $= 50^\circ$ $\angle FJD = 180^\circ - 50^\circ - 19^\circ$ $= 111^\circ$	**A1**	

26	**(a)**	Let the height of the cone in Figure 2 be h. Since triangle QWS is similar to triangle VWT, $\frac{h}{h-9}=\frac{12}{8}$	**M1**	
		$12h-108=8h$ $h=27$		
		Volume of A $=\frac{1}{3}\pi\left(6^2\right)h-\frac{1}{3}\pi\left(4^2\right)(h-9)$	**M1**	
		$=\frac{1}{3}\pi(36)(27)-\frac{1}{3}\pi(16)(18)$ $=\frac{1}{3}\pi[684]$ $=228\pi$ $=716\ \text{cm}^3$ (3 s.f.)	**A1**	
	(b)		**B1** **B1**	1 mark is for correctly plotted point $(50,9)$. 1 mark is for the correct shapes for both graphs time from 0 to 50 and time 50 to 80 seconds.