Name of Candidate: ______________________ (   ) Class: ______ Calculator Model: [   ]

BUKIT PANJANG GOVERNMENT HIGH SCHOOL

PRELIMINARY EXAMINATION 2023

SECONDARY 4/5

GCE 'O' LEVEL SYLLABUS

ADDITIONAL MATHEMATICS

4049 / 02

Paper 2

Date: 24 August, 2023

Duration: 2h 15 min

Time: 0750 – 1005 h

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

[Turn over]

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad .$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)......(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 (a) Solve the equation $\log_{2x+5} 4 - \log_2 (2x+5)^5 = -3$. [5]

(b) Given that $2^{x+1}+2^{x+3}=16^{x}$, find the value of x. [4]

2 The equation of a curve is $y = \ln\left(\sqrt[3]{\dfrac{2x+1}{2x-5}}\right)$.

(i) Explain why the curve does not have a stationary point. [4]

(ii) If y decreases at the rate of 0.4 units/s, find the rate of change of x when $x = 2$. [2]

3 (i) Express $\dfrac{4x^2-2x+1}{2x^3+x^2}$ in partial fractions. [5]

(ii) Hence, show that $\int_2^4 \frac{4x^2 - 2x + 1}{2x^3 + x^2}\, dx = 6\ln\frac{9}{5} - 4\ln 2 + \frac{1}{4}$. [4]

4 **Solution to this question by accurate drawing will not be accepted.**

The parallelogram *ABCD* is such that *A* is $(-2, 6)$ and *C* is $(4, 3)$. Given that point *B* lies on the *x*-axis and *BC* is perpendicular to the line $2y + 12x = 20,$ find

(a) (i) the coordinates of *B* and of *D*, [4]

(ii) the area of parallelogram *ABCD*. [2]

(b) *E* is a point on *AC* produced such that *AE* : *CE* = 4 : 1. Find the coordinates of *E*. [2]

5 A circle C with centre P and radius a cm lies on and above the x-axis at (4, 0).

(i) Find the equation of the circle in terms of a. [1]

(ii) State the equation of the tangent to the circle at the highest point in terms of a. [1]

(iii) It is given that the gradient of the tangent to the circle at K is $-\frac{4}{3}$. The line joining P and K is produced to touch the x-axis at S.
Find the length of PS in terms of a. [5]

(iv) Find the equation of another circle which is the reflection of circle C about the line $x = 1$. [1]

6 (i) Prove that $\frac{\sec^2 x}{8}(1-\cos 4x) = \sin^2 x$. [3]

(ii) Hence, find the exact values of x that satisfy the equation [5]

$$\frac{\sec^2 x}{8}\left(1-\cos 4x\right)=\cos^2 x+\sin x \text{ for } -\pi \le x \le \pi \text{ radians.}$$

7 (i) Write $5+\sqrt{7}\cos\theta-13\sin\theta$ in the form $p+R\cos(\theta+\alpha)$, where $R>0$ and $0°\le\alpha\le 90°$. [3]

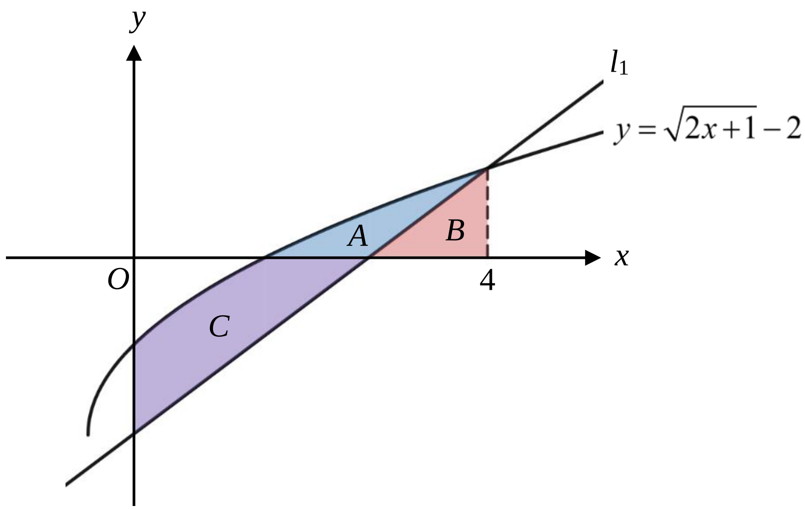
(ii) Hence find the largest value of $5+\sqrt{7}\cos\theta-13\sin\theta$ for $60°\le\theta+\alpha\le 120°$. [2]

(iii) Solve the equation $5+\sqrt{7}\cos\theta-13\sin\theta=0$ where $0° \le \theta \le 180°$. [3]

8 (i) Find $\frac{d}{dx}\left(x\tan^2 x\right)$. [2]

(ii) It is given that the curve $y = \mathrm{f}(x)$ passes through the point $\left(\dfrac{\pi}{4}, \dfrac{7}{5}\right)$ and is such that $\mathrm{f}'(x) = \dfrac{4x\tan x\sec^2 x}{5}$. Using the result of part **(i)**, find $\mathrm{f}(x)$. [6]

9



The diagram shows part of the curve $y=\sqrt{2x+1}-2$ intersecting the line l_1 at $x = 4$.
The shaded region A is bounded by the curve, the line l_1 and the x-axis.
The shaded region B is bounded by the line l_1, the x-axis and the line $x = 4$.
The shaded region C is bounded by the curve, the line l_1, the x-axis and the y-axis.

(i) Given that region A and region B have the same area, find the equation of the line l_1. [6]

Continuation of working space for Question 9(i)

(ii) Hence, find the area of shaded region C. [3]

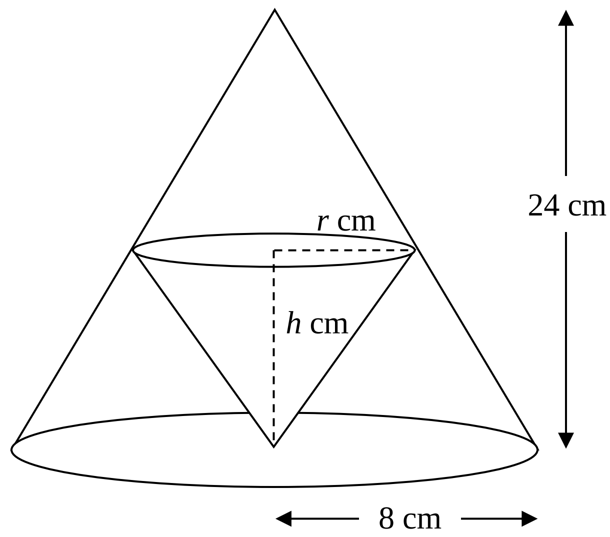
10 A particle moves in a straight line, so that, t seconds after passing O, its velocity, v m/s, is given by $v = 10e^{-0.4t} - 4$.

(i) Find the initial acceleration of the particle. [2]

(ii) Find the value of t when the particle is at instantaneous rest. [3]

(iii) Find the total distance travelled by the particle in the third second. [4]

11 The diagram shows an inverted smaller cone of radius r cm and height h cm, where r and h can vary, inside a bigger upright cone of base radius 8 cm and height 24 cm. The vertex of the smaller cone touches the centre of the base of the bigger cone.



(i) Show that the volume, V cm^3, of the smaller cone is given by

$$V = \frac{\pi}{27}\left(576h - 48h^2 + h^3\right).$$

[3]

(ii) Hence, find the greatest volume of the smaller cone. [5]

END OF PAPER

2023 BPGHS AMath Prelim P2 solution

1 (a) Solve the equation $\log_{2x+5} 4 - \log_2 (2x+5)^5 = -3$. [5]

$$\frac{\log_2 4}{\log_2 (2x+5)} - \log_2 (2x+5)^5 = -3$$

$$\frac{2}{\log_2 (2x+5)} - 5\log_2 (2x+5) = -3$$

Let $\log_2 (2x+5) = a$,

$$\frac{2}{a} - 5a = -3$$

$$2 - 5a^2 = -3a$$

$$5a^2 - 3a - 2 = 0$$

$$(5a+2)(a-1) = 0$$

$a = -\frac{2}{5}$ or $a = 1$

$\log_2 (2x+5) = -\frac{2}{5}$ or $\log_2 (2x+5) = 1$

$2^{-\frac{2}{5}} = 2x+5$ or $2^1 = 2x+5$

$x = -2.12$ (to 3s.f.) // or $x = -\frac{3}{2}$ //

(b) Given that $2^{x+1}+2^{x+3}=16^x$, find the value of x. [4]

$$2^x 2^1 + 2^x 2^3 = 16^x$$

$$2^x(2+2^3) = 16^x$$

$$10 = \frac{16^x}{2^x}$$

$$8^x = 10$$

$$\lg 8^x = \lg 10$$

$$x \lg 8 = 1$$

$$x = \frac{1}{\lg 8}$$

$$= 1.11 \text{ (to 3s.f.)} \,//$$

2 The equation of a curve is $y=\ln\left(\sqrt[3]{\frac{2x+1}{2x-5}}\right)$.

(i) Explain why the curve does not have a stationary point. [4]

$$y = \ln\left(\frac{2x+1}{2x-5}\right)^{\frac{1}{3}}$$

$$= \frac{1}{3}\left[\ln(2x+1) - \ln(2x-5)\right]$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{2}{2x+1} - \frac{2}{2x-5}\right)$$

$$= \frac{1}{3}\left[\frac{2(2x-5)-2(2x+1)}{(2x+1)(2x-5)}\right]$$

$$= \frac{1}{3}\left[\frac{-12}{(2x+1)(2x-5)}\right]$$

$$= \frac{-4}{(2x+1)(2x-5)}$$

Since numerator $\neq 0$ and denominator $\neq 0$, $\frac{dy}{dx} \neq 0$.

Since $\frac{dy}{dx} \neq 0$, the curve does not have a stationary point.

(ii) If y decreases at the rate of 0.4 units/s, find the rate of change of x when $x = 2$. [2]

$$\frac{dy}{dt} = -0.4$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$-0.4 = \frac{-4}{[2(2)+1][2(2)-5]} \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = -0.4 \div \frac{4}{5}$$

$$= -0.5 \text{ units/s}$$

3 (i) Express $\frac{4x^2-2x+1}{2x^3+x^2}$ in partial fractions. [5]

$$\frac{4x^2-2x+1}{x^2(2x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x+1}$$

$$4x^2-2x+1 = A(x)(2x+1) + B(2x+1) + Cx^2$$

Sub $x=0$,

$1 = B$

Sub $x = -\frac{1}{2}$,

$$4\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 1 = C\left(-\frac{1}{2}\right)^2$$

$$\frac{1}{4}C = 3$$

$$C = 12$$

Sub $x=1$,

$$4-2+1 = A(3) + 1(3) + 12$$

$$3A = -12$$

$$A = -4$$

$$\therefore \quad \frac{-4}{x} + \frac{1}{x^2} + \frac{12}{2x+1}$$

(ii) Hence, show that $\int_2^4 \frac{4x^2-2x+1}{2x^3+x^2}\,dx = 6\ln\frac{9}{5} - 4\ln 2 + \frac{1}{4}$. [4]

$$\int_2^4 \frac{-4}{x} + \frac{1}{x^2} + \frac{12}{2x+1}\,dx$$

$$= \int_2^4 -4x^{-1} + x^{-2} + 12(2x+1)^{-1}\,dx$$

$$= \left[-4\ln x + \frac{x^{-1}}{-1} + \frac{12\ln(2x+1)}{2}\right]_2^4$$

$$= \left[-4\ln 4 + \frac{4^{-1}}{-1} + \frac{12\ln 9}{2}\right] - \left[-4\ln 2 + \frac{2^{-1}}{-1} + \frac{12\ln 5}{2}\right]$$

$$= -4\ln 4 - \frac{1}{4} + 6\ln 9 + 4\ln 2 + \frac{1}{2} + 6\ln 5$$

$$= 6(\ln 9 - \ln 5) - 4(\ln 4 - \ln 2) + \frac{1}{4}$$

$$= 6\ln\frac{9}{5} - 4\ln 2 + \frac{1}{4} \quad \text{(shown)}$$

//

4 **Solution to this question by accurate drawing will not be accepted.**
The parallelogram *ABCD* is such that *A* is $(-2, 6)$ and *C* is $(4, 3)$. Given that point *B* lies on the x-axis and *BC* is perpendicular to the line $2y+12x=20$, find

(i) the coordinates of *B* and of *D*, [4]

$$2y+12x=20$$
$$y+6x=10$$
$$y=-6x+10$$
gradient of BC $= -1 \div -6$
$$= \frac{1}{6}$$

eqⁿ BC : $y=\frac{1}{6}x+c$

sub $(4,3)$,

$3=\frac{1}{6}(4)+c$

$c=\frac{7}{3}$

$y=\frac{1}{6}x+\frac{7}{3}$

sub $y=0$,

$\frac{1}{6}x+\frac{7}{3}=0$

$x=-14$

$\therefore$ B$(-14,0)$ //

$\frac{3-0}{4-x}=\frac{1}{6}$

$4-x=18$

$x=-14$

$\therefore$ B$(-14,0)$

midpt AC = midpt BD

$\left(\frac{-2+4}{2}, \frac{6+3}{2}\right)=\left(\frac{-14+x}{2}, \frac{0+y}{2}\right)$

$\frac{-14+x}{2}=1$, $\frac{y}{2}=\frac{9}{2}$

$x=16$ $\qquad y=9$

$\therefore$ D$(16,9)$ //

(ii) the area of parallelogram *ABCD*. [2]

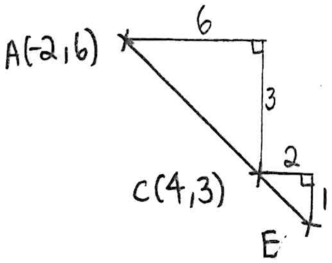
$$\text{Area} = \frac{1}{2}\begin{vmatrix} -2 & -14 & 4 & 16 & -2 \\ 6 & 0 & 3 & 9 & 6 \end{vmatrix}$$

$$= \frac{1}{2}\left| 0-42+36+96-(-18)-48-0-(-84) \right|$$

$$= \frac{1}{2}\left| 144 \right|$$

$$= 72 \text{ units}^2$$

(iii) *E* is a point on *AC* produced such that *AE* : *CE* = 4 : 1. Find the coordinates of *E*. [2]



From A to C :

Difference in $x = 6$

Difference in $y = 3$

From C to E :

Diff. in $x = 2$

Diff. in $y = 1$

$\therefore E(4+2, 3-1)$

$= (6, 2)$

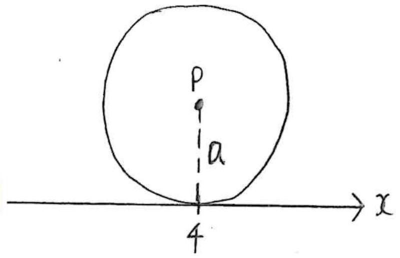
and above ·

5 A circle C_1 with centre P and radius a cm lies on the x-axis at (4, 0).

(i) Find the equation of the circle in terms of a. [1]

centre $(4, a)$

$(x-4)^2 + (y-a)^2 = a^2$

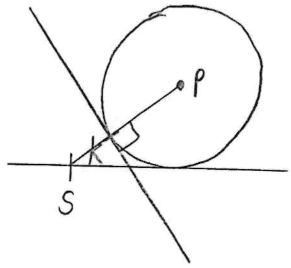


(ii) State the equation of the tangent to the circle at the highest point in terms of a. [1]

Highest point $= (4, 2a)$

$\therefore \; y = 2a$ //

(iii) It is given that the gradient of the tangent to the circle at K is $-\frac{4}{3}$. The line joining P and K is produced to touch the x-axis at S.
Find the length of PS in terms of a. [5]



Gradient of PS $= -1 \div -\frac{4}{3}$

$= \frac{3}{4}$

$P(4,a)$, $S(x,0)$

$\frac{a-0}{4-x} = \frac{3}{4}$

$4a = 12 - 3x$

$3x = 12 - 4a$

$x = 4 - \frac{4}{3}a$

$S(4-\frac{4}{3}a, 0)$

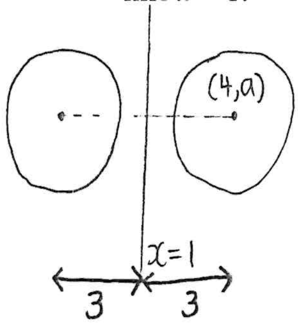
$PS = \sqrt{(4-\frac{4}{3}a-4)^2+(0-a)^2}$

$= \sqrt{\frac{16}{9}a^2+a^2}$

$= \sqrt{\frac{25}{9}a^2}$

$= (\frac{5}{3}a)$ units //

(iv) Find the equation of another circle which is the reflection of circle C_1 about the line $x = 1$. [1]



new centre $= (-2, a)$

radius $= a$

$(x+2)^2 + (y-a)^2 = a^2$ //

6 (i) Prove that $\frac{\sec^2 x}{8}(1-\cos 4x) = \sin^2 x$. [3]

$$\text{LHS} = \frac{\sec^2 x}{8}(1-\cos 4x)$$

$$= \frac{\sec^2 x}{8}\left[1-(1-2\sin^2 2x)\right]$$

$$= \frac{\sec^2 x}{8}(2\sin^2 2x)$$

$$= \frac{1}{8\cos^2 x}\left[2(2\sin x\cos x)^2\right]$$

$$= \frac{1}{8\cos^2 x}(8\sin^2 x\cos^2 x)$$

$$= \sin^2 x$$

$$= \text{RHS (proven)}$$

(ii) Hence, find the exact values of x that satisfy the equation $\frac{\sec^2 x}{8}(1-\cos 4x)=\cos^2 x+\sin x$ for $-\pi \le x \le \pi$ radians. [5]

$$\sin^2 x = \cos^2 x + \sin x$$

$$\sin^2 x = 1 - \sin^2 x + \sin x$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$\sin x = -\frac{1}{2}$ or $\sin x = 1$

basic $\angle = \sin^{-1}\frac{1}{2}$ $\quad$ $x = \frac{\pi}{2}$

$= \frac{\pi}{6}$

Q3, Q4

$x = \pi + \frac{\pi}{6}$ or $2\pi - \frac{\pi}{6}$

$= \frac{7\pi}{6}$ (rej.) $\quad = \frac{11\pi}{6}$ (rej.)

$x = \frac{7\pi}{6} - 2\pi$ or $\frac{11\pi}{6} - 2\pi$

$= -\frac{5\pi}{6}$ $\quad = -\frac{\pi}{6}$

$\therefore x = -\frac{5\pi}{6}$ or $-\frac{\pi}{6}$ or $\frac{\pi}{2}$ //

7 (i) Write $5+\sqrt{7}\cos\theta-13\sin\theta$ in the form $p+R\cos(\theta+\alpha)$, where $R>0$ and $0°\leq\alpha\leq 90°$. [3]

$R=\sqrt{(\sqrt{7})^2+(13)^2}$

$=\sqrt{176}$

$\alpha=\tan^{-1}\left(\frac{13}{\sqrt{7}}\right)$

$=78.49631204$

$\therefore\ 5+\sqrt{176}\cos(\theta+78.5°)$ (to 1d.p.)

(ii) Hence find the largest value of $5+\sqrt{7}\cos\theta-13\sin\theta$ for $60°\leq\theta+\alpha\leq 120°$. [2]

when $\theta+\alpha=60°$,

$5+\sqrt{176}\cos(60°)$

$=5+\sqrt{176}\left(\frac{1}{2}\right)$

$=11.6$ (to 3s.f.)

(iii) Solve the equation $5+\sqrt{7}\cos\theta-13\sin\theta=0$ where $0°\leq\theta\leq180°$. [3]

$$5+\sqrt{176}\cos(\theta+78.49631204)=0$$

$$\cos(\theta+78.49631204)=\frac{-5}{\sqrt{176}}$$

$$\text{basic } \measuredangle = \cos^{-1}\left(\frac{5}{\sqrt{176}}\right)$$

$$=67.858876$$

Q2, Q3

$$\theta+78.49631204 = 180-67.858876 \text{ or } 180+67.858876$$

$$\theta = 33.6° \text{ or } 169.4° \text{ (to 1d.p.)}$$

8 (i) Find $\frac{d}{dx}\left(x\tan^2 x\right)$. [2]

$$\frac{d}{dx}\left(x\tan^2 x\right) = \tan^2 x + x(2\tan x)\sec^2 x$$

$$= \tan^2 x + 2x\tan x\sec^2 x.$$

(ii) It is given that the curve $y = \mathrm{f}(x)$ passes through the point $\left(\frac{\pi}{4}, \frac{7}{5}\right)$ and is such that $\mathrm{f}'(x) = \frac{4x\tan x\sec^2 x}{5}$. Using the result of part **(i)**, find $\mathrm{f}(x)$. [6]

$$\int \tan^2 x + 2x\tan x\sec^2 x \, dx = x\tan^2 x + C_1$$

$$\int \tan^2 x \, dx + \int 2x\tan x\sec^2 x \, dx = x\tan^2 x + C_1$$

$$\int 2x\tan x\sec^2 x \, dx = x\tan^2 x - \int \sec^2 x - 1 \, dx + C_1$$

$$= x\tan^2 x - (\tan x - x) + C_2$$

$$= x\tan^2 x - \tan x + x + C_2$$

$$\int \frac{4x\tan x\sec^2 x}{5} \, dx = \frac{2}{5}(x\tan^2 x - \tan x + x) + C_3$$

when $x = \frac{\pi}{4}$, $y = \frac{7}{5}$

$$\frac{2}{5}\left[\frac{\pi}{4}\tan^2\left(\frac{\pi}{4}\right) - \tan\frac{\pi}{4} + \frac{\pi}{4}\right] + C_3 = \frac{7}{5}$$

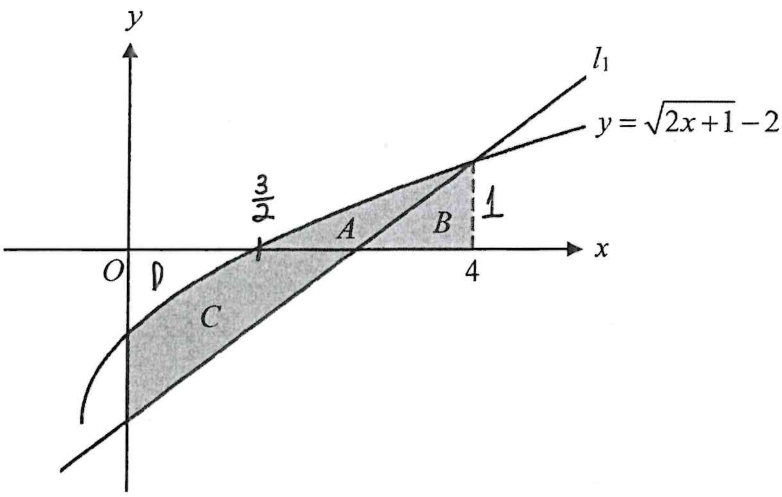
$$\frac{2}{5}\left(\frac{\pi}{4} - 1 + \frac{\pi}{4}\right) + C_3 = \frac{7}{5}$$

$$C_3 = \frac{7}{5} + \frac{2}{5} - \frac{2}{5}\left(\frac{\pi}{2}\right)$$

$$= \frac{9}{5} - \frac{\pi}{5} \quad \text{or } 1.17 \text{ (to 3s.f.)}$$

$$\therefore \mathrm{f}(x) = \frac{2}{5}(x\tan^2 x - \tan x + x) + \frac{9}{5} - \frac{\pi}{5}$$

9



The diagram shows part of the curve $y=\sqrt{2x+1}-2$ intersecting the line l_1 at $x=4$.
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The shaded region B is bounded by the line l_1, the x-axis and the line $x=4$.
The shaded region C is bounded by the curve, the line l_1, the x-axis and the y-axis.

(i) Given that region A and region B have the same area, find the equation of the line l_1. [6]

when $y=0$,

$\sqrt{2x+1}-2=0$

$\sqrt{2x+1}=2$

$2x+1=4$

$x=\frac{3}{2}$

$A+B=\int_{\frac{3}{2}}^{4}(2x+1)^{\frac{1}{2}}-2\,dx$

$=\left[\frac{(2x+1)^{\frac{3}{2}}}{(\frac{3}{2})(2)}-2x\right]_{\frac{3}{2}}^{4}$

$=\left[\frac{9^{\frac{3}{2}}}{3}-8\right]-\left[\frac{4^{\frac{3}{2}}}{3}-2\left(\frac{3}{2}\right)\right]$

$=\frac{4}{3}$

when $x=4$,

$y=\sqrt{9}-2$

$=1$

Area of $B=\frac{2}{3}$

$\frac{1}{2}\times b\times 1=\frac{2}{3}$

$b=\frac{4}{3}$

$(4-\frac{4}{3},0)$

$=(\frac{8}{3},0)$

Continuation of working space for question 9(i)

$(4,1)$, $(\frac{8}{3},0)$

$\text{Gradient} = \frac{1-0}{4-\frac{8}{3}}$

$= \frac{3}{4}$

$y = \frac{3}{4}x + c$

$1 = \frac{3}{4}(4) + c$

$c = -2$

$\therefore y = \frac{3}{4}x - 2$ //

(ii) Hence, find the area of shaded region C. [3]

$\text{Area of region } D = -\int_0^{\frac{3}{2}} (2x+1)^{\frac{1}{2}} - 2 \, dx$

$= -\left[\frac{(2x+1)^{\frac{3}{2}}}{3} - 2x \right]_0^{\frac{3}{2}}$

$= -\left\{ \left[-\frac{1}{3}\right] - \left[\frac{1}{3}\right] \right\}$

$= \frac{2}{3}$

when $x=0$,
$y = \frac{3}{4}(0) - 2$
$= -2$

$\text{Area of region } C = \left(\frac{1}{2} \times \frac{8}{3} \times 2\right) - \frac{2}{3}$

$= 2 \text{ units}^2$ //

10 A particle moves in a straight line, so that, t seconds after passing O, its velocity, v m/s, is given by $v = 10e^{-0.4t} - 4$.

(i) Find the initial acceleration of the particle. [2]

$$a = \frac{dv}{dt}$$

$$= 10e^{-0.4t}(-0.4)$$

$$= -4e^{-0.4t}$$

$$= -4e^{0}$$

$$= -4 \text{ m/s}^2$$

(ii) Find the value of t when the particle is at instantaneous rest. [3]

when $v = 0$,

$$10e^{-0.4t} - 4 = 0$$

$$e^{-0.4t} = \frac{4}{10}$$

$$\ln e^{-0.4t} = \ln \frac{4}{10}$$

$$-0.4t = \ln \frac{4}{10}$$

$$t = \ln\left(\frac{4}{10}\right) \div -0.4$$

$$= 2.29072683$$

$$= 2.29 \text{ s (to 3s.f.)}$$

(iii) Find the total distance travelled by the particle in the third second. [4]

$$s = \int 10e^{-0.4t} - 4 \, dt$$

$$= \frac{10e^{-0.4t}}{-0.4} - 4t + C$$

$$= -25e^{-0.4t} - 4t + C$$

when $t=0$, $s=0$,

$$0 = -25e^{0} + C$$

$$C = 25$$

$$s = -25e^{-0.4t} - 4t + 25$$

when $t = 2$,

$$s = -25e^{-0.4(2)} - 4(2) + 25$$

$$= 5.76678$$

when $t = 2.29072683$,

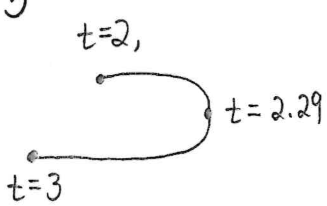
$$s = -25e^{-0.4(2.29072683)} - 4(2.29072683) + 25$$

$$= 5.83709$$

when $t = 3$,

$$s = -25e^{-0.4(3)} - 4(3) + 25$$

$$= 5.470145$$

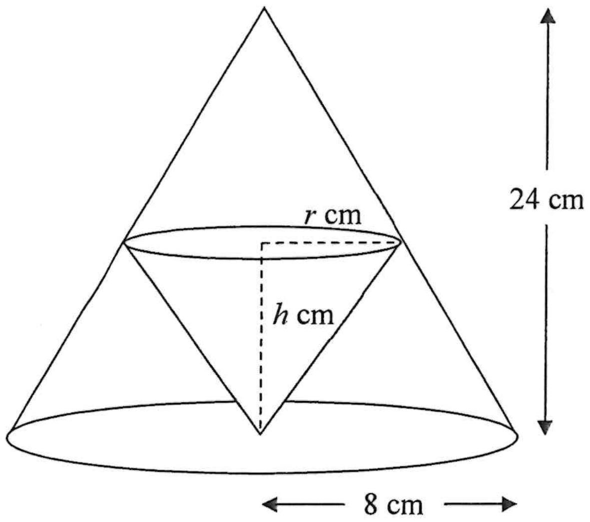


Total distance

$$= (5.83709 - 5.76678) + (5.83709 - 5.470145)$$

$$= 0.437 \text{ m} \quad (\text{to } 3s.f.)$$

11 The diagram shows an inverted smaller cone of radius r cm and height h cm, where r and h can vary, inside a bigger upright cone of base radius 8 cm and height 24 cm. The vertex of the smaller cone touches the centre of the base of the bigger cone.



(i) Show that the volume, V cm^3, of the smaller cone is given by

$$V = \frac{\pi}{27}\left(576h - 48h^2 + h^3\right).$$

[3]

$$\frac{r}{8} = \frac{24-h}{24}$$

$$r = \frac{24-h}{3}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{24-h}{3}\right)^2 h$$

$$= \frac{1}{3}\pi\left(\frac{576-48h+h^2}{9}\right)h$$

$$= \frac{\pi}{27}\left(576h-48h^2+h^3\right) \quad \text{(shown)}$$

//

(ii) Hence, find the greatest volume of the smaller cone. [5]

$$\frac{dV}{dh} = \frac{\pi}{27}(576 - 96h + 3h^2)$$

$$\frac{dV}{dh} = 0$$

$$\frac{\pi}{27}(576 - 96h + 3h^2) = 0$$

$$h^2 - 32h + 192 = 0$$

$$(h - 24)(h - 8) = 0$$

$h = 24$ (rej.) or $h = 8$

$$V = \frac{\pi}{27}\left[576(8) - 48(8)^2 + (8)^3\right]$$

$$= 238 \text{ cm}^3 \text{ (to 3s.f.)}$$

$$\frac{d^2V}{dh^2} = \frac{\pi}{27}(-96 + 6h)$$

$$= \frac{\pi}{27}[-96 + 6(8)]$$

$$= -5.59 < 0$$

Since $\frac{d^2V}{dh^2} < 0$, the volume is the greatest when $h = 8$.

END OF PAPER