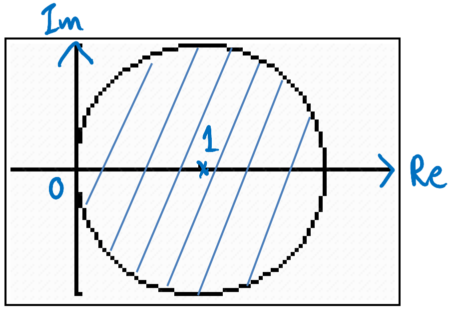
CATHOLIC JUNIOR COLLEGE
H2 MATHEMATICS 9740
JC2 PRELIM PAPER 1 SOLUTIONS 2012

1	INEQUALITIES + COMPLEX NUMBERS		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
(i)	- solve an equality involving modulus functions either algebraically or graphically - deduce the solution of an inequality of the form $\frac{f(x)}{g(x)} \geq 0$ where f(x) and g(x) are linear expressions, based on the solution obtained from solving the equality	Suppose $w \in \Re$, $\lvert 2w-1 \rvert \leq \lvert w+1 \rvert$ $(2w-1)^2 \leq (w+1)^2$ $4w^2 - 4w + 1 \leq w^2 + 2w + 1$ $3w^2 - 6w \leq 0$ $3w(w-2) \leq 0$ $0 \leq w \leq 2$ $\left\lvert \frac{2w-1}{w+1} \right\rvert \geq 1$ $\lvert 2w-1 \rvert \geq \lvert w+1 \rvert$ $3w(w-2) \geq 0$ $w \leq 0,\ w \geq 2, \quad w \neq -1$	
(ii)	- express a complex number in its algebraic form - represent complex numbers expressed in cartesian form by points in the Argand diagram	Suppose $w \in C$, let $w = x + iy$, where $x, y \in \mathbb{R}$. Since $\lvert 2w-1 \rvert \leq \lvert w+1 \rvert$, $\lvert 2(x+iy) - 1 \rvert \leq \lvert x + iy + 1 \rvert$, $\lvert (2x-1) + 2yi \rvert \leq \lvert (x+1) + iy \rvert$, $\lvert (2x-1) + 2yi \rvert^2 \leq \lvert (x+1) + iy \rvert^2$, $(2x-1)^2 + 4y^2 \leq (x+1)^2 + y^2$, note that $\lvert a+bi \rvert^2 \equiv a^2 + b^2$, $4x^2 - 4x + 1 + 4y^2 \leq x^2 + 2x + 1 + y^2$, $3x^2 - 6x + 3y^2 \leq 0$, $\quad x^2 - 2x + y^2 \leq 0$.	

CATHOLIC JUNIOR COLLEGE
H2 MATHEMATICS 9740
JC2 PRELIM PAPER 1 SOLUTIONS 2012

1	**INEQUALITIES + COMPLEX NUMBERS**		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
		Then, via completing the square, $(x-1)^2 + y^2 \le 1$, which represents a circle of radius 1 and centred about (1, 0), together with its interior.  	

2	AP & GP		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
(a)		Let *n* be the no. of months. Then $\frac{n}{2}(2a+(n-1)d)\geq 2000$ $\Rightarrow \frac{n}{2}(100+(n-1)10)\geq 2000$ $\Rightarrow \frac{n}{2}(90+10n)\geq 2000$ $\Rightarrow n^2+9n-400\geq 0$ Solving, $n\leq -25, n\geq 16$. 16 complete months to fully repay the debt.	
(b)(i)	Find the general of a GP and use it to find a particular value	Balance at end of 1 month $= 1.02(2000-50)$ Balance at end of 2 months $= 1.02[1.02(2000-50)-50]$ $= 1.02^2(2000)-50(1.02^2+1.02)$ Balance at end of *n* months $= 1.02^n(2000)-50(1.02^n+\cdots+1.02)$ $= 1.02^n(2000)-50\left(\frac{1.02(1.02^n-1)}{1.02-1}\right)$ $= 2550-1.02^n(550)$ Outstanding amount at 1 January 2013 $= 2550-1.02^{12}(550) = \$1852.47$	
(ii)	Solve inequality using general term.	loan repaid $\Rightarrow$ balance $\leq 0 \Rightarrow 2550-1.02^n(550)\leq 0 \Rightarrow n\geq$ 77.46 so he takes 78 months.	

CATHOLIC JUNIOR COLLEGE
H2 MATHEMATICS 9740
JC2 PRELIM PAPER 1 SOLUTIONS 2012

3	**SIGMA NOTATION/METHOD OF DIFF/MI**		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
(i)	Able to prove a statement using the method of mathematical induction involving a recurrence relation.	Let P_n be the statement $u_n = \dfrac{n-1}{(n+1)^2}$ for all $n \geq 1$. When $n=1$, LHS $= u_1 = 0$ RHS $= \dfrac{1-1}{(1+1)^2} = 0$ Since LHS = RHS, P_1 is true and forms the basis for induction. Assume P_k to be true, i.e. $u_k = \dfrac{k-1}{(k+1)^2}$. RTP: $u_{k+1} = \dfrac{k}{(k+2)^2}$. $\text{LHS} = u_{k+1} = u_k + \dfrac{4+k-k^2}{[(k+1)(k+2)]^2}$ $= \dfrac{k-1}{(k+1)^2} + \dfrac{4+k-k^2}{[(k+1)(k+2)]^2}$ $= \dfrac{(k-1)(k+2)^2 + 4 + k - k^2}{(k+1)^2(k+2)^2}$ $= \dfrac{k^3 + 3k^2 - 4 + 4 + k - k^2}{(k+1)^2(k+2)^2}$ $= \dfrac{k^3 + 2k^2 + k}{(k+1)^2(k+2)^2} = \dfrac{k(k^2+2k+1)}{(k+1)^2(k+2)^2}$ $= \dfrac{k}{(k+2)^2} = \text{RHS}$ Therefore, $P_k \Rightarrow P_{k+1}$. Hence by MI, P_n is true for all $n \geq 1$.	

CATHOLIC JUNIOR COLLEGE
H2 MATHEMATICS 9740
JC2 PRELIM PAPER 1 SOLUTIONS 2012

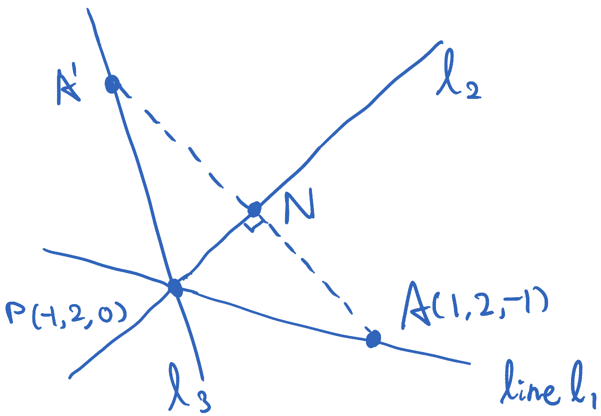
3	SIGMA NOTATION/METHOD OF DIFF/MI		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
(ii)	Able to apply the method of difference to find the sum of a series. Able to evaluate sum to infinity of a convergent series.	$\sum_{n=2}^{N}\frac{4+n-n^2}{[(n+1)(n+2)]^2}=\sum_{n=2}^{N}u_{n+1}-u_n$ $=u_{N+1}-u_2$ $=\frac{N+1-1}{(N+1+1)^2}-\frac{2-1}{(2+1)^2}$ $=\frac{N}{(N+2)^2}-\frac{1}{9}$ Sum to infinity $=-\frac{1}{9}$	
(iii) HOT	Able to identify a change of index for a summation in order to evaluate it using a previously known summation.	$\sum_{n=2}^{\infty}\frac{2+3n-n^2}{[n(n+1)]^2}=\sum_{n=2}^{\infty}\frac{4+(n-1)-(n-1)^2}{[(n-1+1)(n-1+2)]^2}$ $=\sum_{r=1}^{\infty}\frac{4+r-r^2}{[(r+1)(r+2)]^2}$ $=\frac{4+1-1^2}{[(1+1)(1+2)]^2}-\frac{1}{9}$ $=0$	

4	VECTORS		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**

CATHOLIC JUNIOR COLLEGE
H2 MATHEMATICS 9740
JC2 PRELIM PAPER 1 SOLUTIONS 2012

4	**VECTORS**		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
(i)	Determine if 2 lines intersect and find intersection point.	$\frac{x-1}{2}=-z-1, y=2 \Rightarrow \mathbf{r}=\begin{pmatrix}1\\2\\-1\end{pmatrix}+\lambda\begin{pmatrix}2\\0\\-1\end{pmatrix}, \lambda \in \Re$, and $x+3=4-y=z+2 \Rightarrow \mathbf{r}=\begin{pmatrix}-3\\4\\-2\end{pmatrix}+\mu\begin{pmatrix}1\\-1\\1\end{pmatrix}, \mu \in \Re$. At the intersection point (denoted by *P*), $\overrightarrow{OP}=\begin{pmatrix}1+2\lambda\\2\\-1-\lambda\end{pmatrix}=\begin{pmatrix}-3+\mu\\4-\mu\\-2+\mu\end{pmatrix}$, for some scalars λ, μ. Solving, $\lambda=-1, \mu=2$. (via our GC.) Hence, $\overrightarrow{OP}=\left.\begin{pmatrix}1+2\lambda\\2\\-1-\lambda\end{pmatrix}\right\|_{\lambda=-1}=\begin{pmatrix}-1\\2\\0\end{pmatrix}$. As such, l_1 and l_2 intersect at the point $P \equiv (-1, 2, 0)$. <u>Alternative solution</u> : Let $P(x, y, z)$ denote the point of intersection between lines l_1 and l_2, if it exists. Since *P* lies on l_1, $\frac{x-1}{2}=-z-1$ and $y=2$.	

4	VECTORS		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
		Since P lies on l_2, $x+3=4-y=z+2$. As $y=2$ and $x+3=4-y$ in particular, $x=-1$. As $y=2$ and $4-y=z+2$ in particular, $z=0$. Check that $P(x,y,z)=(-1,2,0)$ satisfies the remaining equation $\frac{x-1}{2}=-z-1$ --- (eq. 1), so as to determine whether $P(x,y,z)=(-1,2,0)$ is indeed a point of intersection btw. the lines l_1 and l_2, i.e. satisfies all equations of line l_1 as well as all equations of line l_2. LHS of (eq. 1) = $\frac{-1-1}{2}=-1$. RHS of (eq. 1) = $-0-1=-1$. As such the lines l_1 and l_2 intersect at $(-1,2,0)$.	
(ii)	Find angle between 2 lines	The acute angle between lines l_1 and l_2 is $=\cos^{-1}\left(\frac{\left\lvert\begin{pmatrix}2\\0\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\1\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}2\\0\\-1\end{pmatrix}\right\rvert\left\lvert\begin{pmatrix}1\\-1\\1\end{pmatrix}\right\rvert}\right)=\cos^{-1}\left(\frac{\lvert 1\rvert}{\sqrt{5}\sqrt{3}}\right),$. $=75.03678\ldots^\circ$ or $1.309638\ldots$,	

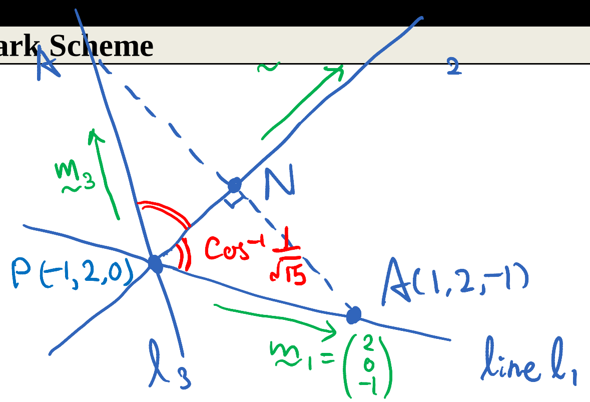
4	VECTORS		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
		$\approx 75.0°$ (1 d.p.) or 1.31 (3 s.f.). (Accept answer for angle in radians, which is dimensionless.)	
(iii)	Find foot of perpendicular from point to line Use Midpoint/Ratio Theorem to find the reflection point Form equation of line joining 2 points given their position vectors	Let *N* be foot of perpendicular from *A*(1, 2, -1) to l_2, and *A'* be the reflection of *A* in l_2. (Note that the point (-1, 2, 0) lies on line l_3.)  Since $N \in l_2$ $\overrightarrow{ON} = \begin{pmatrix} -3+\mu \\ 4-\mu \\ -2+\mu \end{pmatrix}$ for some $\mu \in \Re$. Since $\overrightarrow{AN} \perp l_2$,	

CATHOLIC JUNIOR COLLEGE
H2 MATHEMATICS 9740
JC2 PRELIM PAPER 1 SOLUTIONS 2012

4	VECTORS		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
		so $\overrightarrow{AN}\cdot\underbrace{\begin{pmatrix}1\\-1\\1\end{pmatrix}}_{\substack{\text{a direction}\\ \text{vector for } l_2}}=0$, i.e. $\begin{pmatrix}-4+\mu\\2-\mu\\-1+\mu\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\1\end{pmatrix}=0$. $\Rightarrow \mu=\frac{7}{3}$. As such, $\overrightarrow{ON}=\left.\begin{pmatrix}-3+\mu\\4-\mu\\-2+\mu\end{pmatrix}\right\|_{\mu=\frac{7}{3}}=\frac{1}{3}\begin{pmatrix}-2\\5\\1\end{pmatrix}$. $\overrightarrow{ON}=\frac{1}{2}(\overrightarrow{OA}+\overrightarrow{OA'}) \quad \Rightarrow \overrightarrow{OA'}=2\overrightarrow{ON}-\overrightarrow{OA}=\frac{1}{3}\begin{pmatrix}-7\\4\\5\end{pmatrix}$ A direction vector of l_3 is $\begin{pmatrix}-1\\2\\0\end{pmatrix}-\frac{1}{3}\begin{pmatrix}-7\\4\\5\end{pmatrix}=\frac{1}{3}\begin{pmatrix}4\\2\\-5\end{pmatrix}$. So an equation of l_3 is $\mathbf{r}=\begin{pmatrix}-1\\2\\0\end{pmatrix}+\nu\begin{pmatrix}4\\2\\-5\end{pmatrix}, \nu\in\Re$. --- <u>Alternative Solution 1</u> : Let $\mathbf{m}_3$ be a direction vector for line l_3. $m_2=\begin{pmatrix}1\\-1\\1\end{pmatrix}$ $\quad l$	

4 VECTORS

Part	Assessment Objectives	Mark Scheme	Feedback



Since the acute angle between l_2 and l_3 is the same as the acute angle between l_1 and l_2, which is $\cos^{-1}(\frac{1}{\sqrt{15}})$ so

$$\frac{1}{\sqrt{15}} = \frac{\mathbf{m}_2 \cdot \mathbf{m}_3}{|\mathbf{m}_2||\mathbf{m}_3|} \qquad \text{(eq. 1)}$$

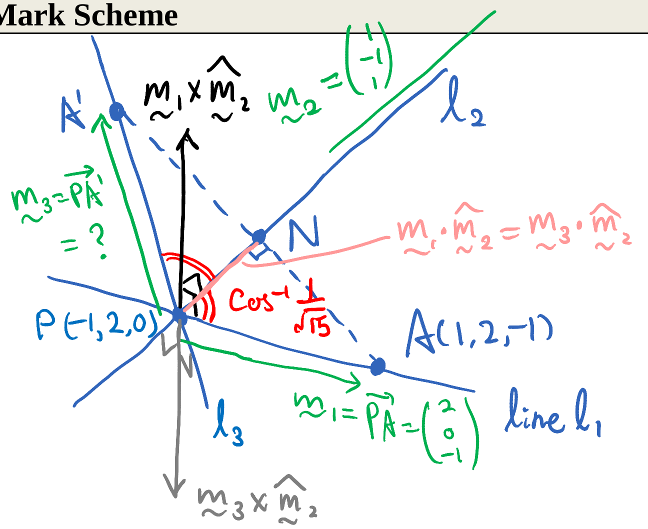
Since the (obtuse) angle between l_1 and l_3 is twice the acute angle between l_1 and l_2, which is $\cos^{-1}(\frac{1}{\sqrt{15}})$ so

$$\underbrace{\cos\left(2\cos^{-1}\left(\frac{1}{\sqrt{15}}\right)\right)}_{=2\left[\cos\left(\cos^{-1}\left(\frac{1}{\sqrt{15}}\right)\right)\right]^2-1=\frac{2}{15}-1=-\frac{13}{15}.} = \frac{\mathbf{m}_1 \cdot \mathbf{m}_3}{|\mathbf{m}_1||\mathbf{m}_3|} \qquad \text{(eq. 2)}$$

Choose $\mathbf{m}_3$ to be a unit vector, i.e. $|\mathbf{m}_3| = 1$ (eq. 3)

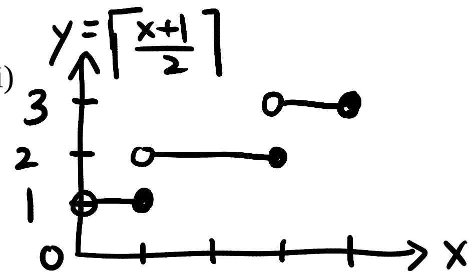
CATHOLIC JUNIOR COLLEGE
H2 MATHEMATICS 9740
JC2 PRELIM PAPER 1 SOLUTIONS 2012

4	VECTORS		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
		With $\mathbf{m}_1 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$, $\mathbf{m}_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\mathbf{m}_3 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ for some *x*, *y* & *z*, (eq. 1) yields $x - y + z = \frac{\sqrt{5}}{5}$, (eq. 2) yields $2x - z = -\frac{13}{15}\sqrt{5}$, and (eq. 3) yields $x^2 + y^2 + z^2 = 1$. Solving this set of equations gives $x = -\frac{4}{15}\sqrt{5}$, $y = -\frac{2}{15}\sqrt{5}$ and $z = \frac{1}{3}\sqrt{5}$. As such, $\mathbf{m}_3 = -\frac{\sqrt{5}}{15}\begin{pmatrix} 4 \\ 2 \\ -5 \end{pmatrix}$, So an equation of l_3 is $\mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + v\begin{pmatrix} 4 \\ 2 \\ -5 \end{pmatrix}, v \in \Re$. --- <u>Alternative Solution 2</u> : Let *A′* be the reflection image of point *A* (on l_1) in l_2. Let $\mathbf{m}_3 = \overrightarrow{PA'}$ and $\mathbf{m}_1 = \overrightarrow{PA}$.	

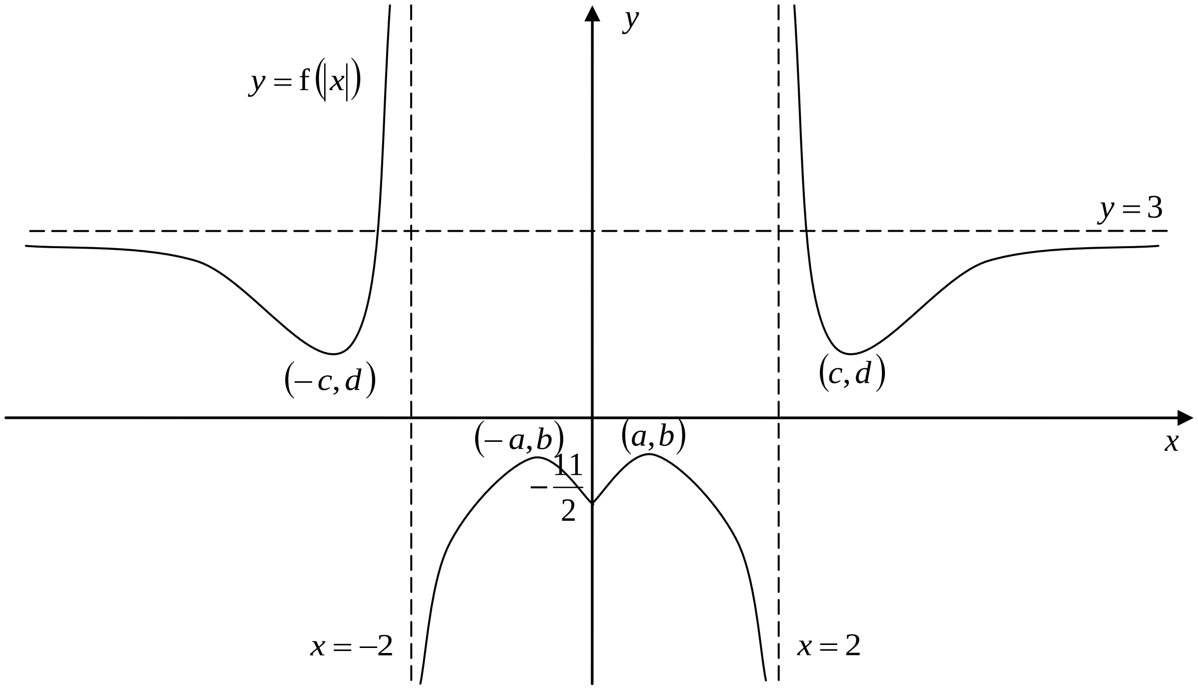
4	VECTORS		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
		 Observe that $\mathbf{m}_1 \cdot \hat{\mathbf{m}}_2 = \mathbf{m}_3 \cdot \hat{\mathbf{m}}_2$, (eq. 1) and that $\mathbf{m}_1 \times \hat{\mathbf{m}}_2 = -\mathbf{m}_3 \times \hat{\mathbf{m}}_2$. (eq. 2) (Note that $\mathbf{m}_1 \cdot \hat{\mathbf{m}}_2 = \mathbf{m}_3 \cdot \hat{\mathbf{m}}_2$ = *PN*, while $\mathbf{m}_1 \times \hat{\mathbf{m}}_2$ and $\mathbf{m}_3 \times \hat{\mathbf{m}}_2$ are vectors perpendicular to the plane containing the lines l_1, l_2, l_3, with opposite direction to each other, and with length *AN*.) Let $\mathbf{m}_3 = \begin{pmatrix}x\\y\\z\end{pmatrix}$. Since $\mathbf{m}_1 = \begin{pmatrix}2\\0\\-1\end{pmatrix}$, & $\hat{\mathbf{m}}_2 = \frac{1}{3}\begin{pmatrix}1\\-1\\1\end{pmatrix}$, hence (eq. 1) yields $\frac{1}{3} = \frac{x-y+z}{3}$, and	

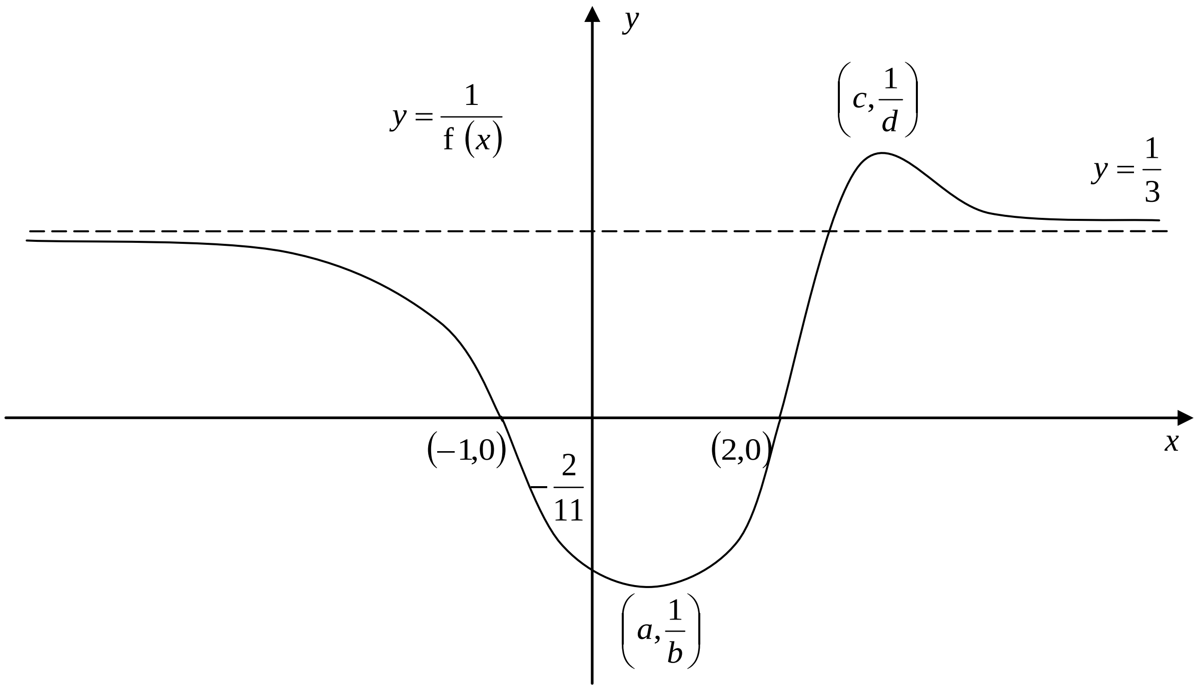
4	VECTORS		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
		(eq. 2) yields $\frac{1}{3}\begin{pmatrix}-1\\-3\\-2\end{pmatrix}=-\frac{1}{3}\begin{pmatrix}y+z\\z-x\\-x-y\end{pmatrix}$. Consolidating the comparison of the *x* and *y*-component, as well as using the former equation yields the system of equations $\left\{\begin{matrix}x-y+z=1\\y+z=1\\-x+z=3\end{matrix}\right\}$ whose solution is $x=-\frac{4}{3}, y=-\frac{2}{3}, z=\frac{5}{3}$. As such, $\mathbf{m}_3=-\frac{1}{3}\begin{pmatrix}4\\2\\-5\end{pmatrix}$, and A vector equation for l_3 is $\mathbf{r}=\begin{pmatrix}-1\\2\\0\end{pmatrix}+v\begin{pmatrix}4\\2\\-5\end{pmatrix}, v\in\Re$.	

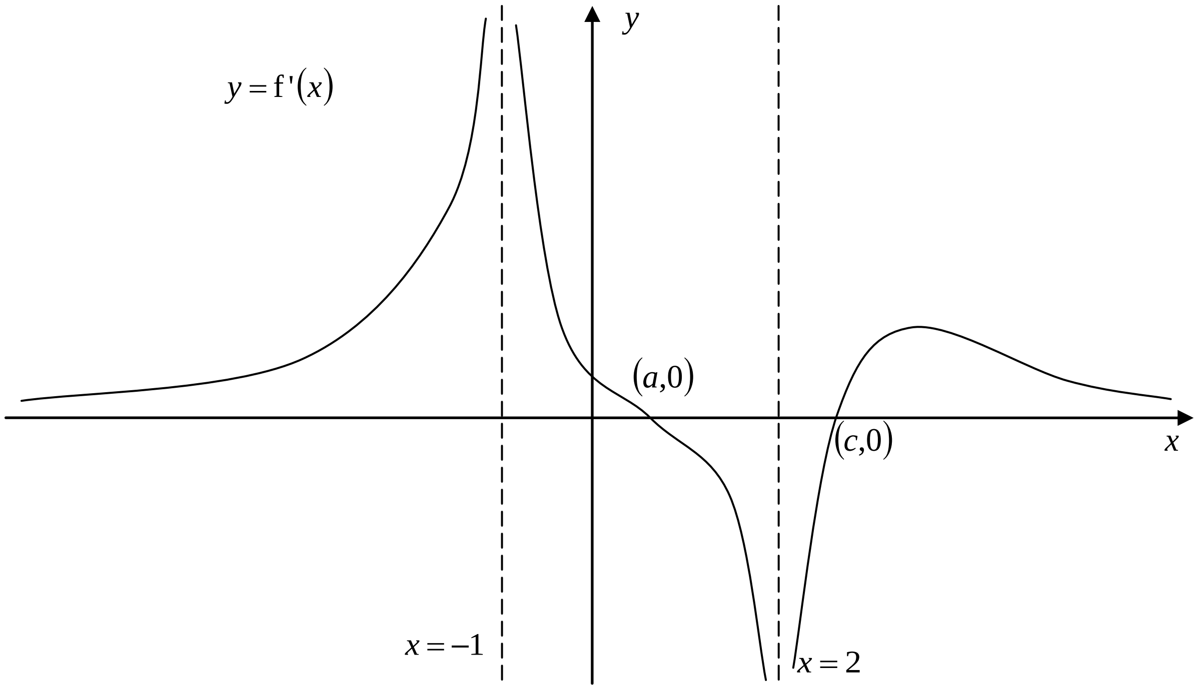
5	FUNCTIONS / TRANSFORMATIONS		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
(a)	- Using of function notation to represent the various transformations and subsequently, the combination of these transformations	After Step 1, $y = f(x) -> y = f(x+1)$ After Step 2, $y = f(x+1) -> y = f(x+1)+5$ After Step 3, $y = f(x+1)+5 -> y = f(2x+1)+5$ Thus, $h(x) = f(2x+1)+5$ $h(x)-5 = f(2x+1)$ $h(\frac{x-1}{2})-5 = f(x)$ Thus, $f(x) = 4\left(\frac{x-1}{2}\right)^2 + 4\left(\frac{x-1}{2}\right)+9-5 = x^2+3$	1.
(b)	- understanding the rule of a given function to obtain output with the given input - determining the range of a function from its graph	$2\lceil 2.5 \rceil + \lceil 3.09 \rceil = 2(3)+4 = 10$ (i)	

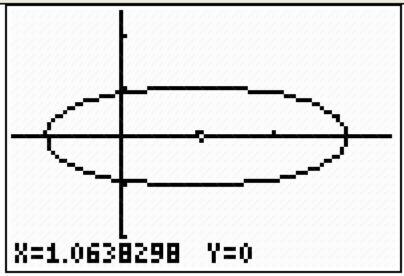
5	FUNCTIONS / TRANSFORMATIONS		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
		$g(0.12)+g(3)$ $=\left\lceil \frac{0.12+1}{2} \right\rceil + \left\lceil \frac{3+1}{2} \right\rceil$ $=\lceil 0.56 \rceil + \lceil 2 \rceil$ $=1+2$ $=3$ (ii)  (iii) Range of $g = \mathbb{Z}^+$	

CATHOLIC JUNIOR COLLEGE
H2 MATHEMATICS 9740
JC2 PRELIM PAPER 1 SOLUTIONS 2012

<table>
<tr><th>6</th><th colspan="3">TRANSFORMATIONS</th></tr>
<tr><th>Part</th><th>Assessment Objectives</th><th>Mark Scheme</th><th>Feedback</th></tr>
<tr><td></td><td>Relating the graph of $y=\mathrm{f}\left(|x|\right)$ to the graph of $y=\mathrm{f}(x)$.</td><td>(i)

</td><td></td></tr>
</table>

6	**TRANSFORMATIONS**		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
	Relating the graph of $y=\dfrac{1}{\mathrm{f}(x)}$ to the graph of $y=\mathrm{f}(x)$.		

6	TRANSFORMATIONS		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
			

7	**PARAMETRIC EQUATIONS/DEFINITE INTEGRALS/ TANGENT/NORMAL**		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
(i)	- obtain the graph of a pair of parametric equations using a graphic calculator	 Intercepts at (0, $\frac{\sqrt{3}}{2}$), (0, $-\frac{\sqrt{3}}{2}$), (-1, 0) and (3, 0)	
(ii)	- find the area under a curve given its parametric equations	$Area = 2\int_{-1}^{3} y\,dx$ $= 2\int_{\pi}^{0} \sin\theta(-2\sin\theta)\,d\theta$ $= 2\int_{\pi}^{0} \cos 2\theta - 1\,d\theta$ $= 2\left[\frac{\sin 2\theta}{2} - \theta\right]_{\pi}^{0}$ $= 2\pi$	
(iii)	- find and use the equation of normal to the curve, defined parametrically	$\frac{dx}{d\theta} = -2\sin\theta \qquad \frac{dy}{d\theta} = \cos\theta$ $\frac{dy}{dx} = \cos\theta\left(\frac{-1}{2\sin\theta}\right) = \frac{-1}{2}\cot\theta$ Gradient of tangent = $\frac{-1}{2}\cot\theta$	

7	PARAMETRIC EQUATIONS/DEFINITE INTEGRALS/ TANGENT/NORMAL		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
		Gradient of normal = $2\tan\theta$ Equation of normal at the point $(2\cos\theta+1,\ \sin\theta)$: $y-\sin\theta=2\tan\theta(x-2\cos\theta-1)$ When x = 0, $y-\sin\theta=2\tan\theta(-2\cos\theta-1)$ $y=-4\sin\theta-2\tan\theta+\sin\theta$ $\quad=-3\sin\theta-2\tan\theta$ Coordinates of R = $(0,\ -3\sin\theta-2\tan\theta)$ When y = 0, $-\sin\theta=2\tan\theta(x-2\cos\theta-1)$ $-\sin\theta=2x\tan\theta-4\sin\theta-2\tan\theta$ $2x\tan\theta=3\sin\theta+2\tan\theta$ $x=\frac{3}{2}\cos\theta+1$ Coordinates of Q = $\left(\frac{3}{2}\cos\theta+1,\ 0\right)$ Area of triangle OPQ $=\frac{1}{2}\left(\frac{3}{2}\cos\theta+1\right)\left\|-3\sin\theta-2\tan\theta\right\|=\frac{1}{2}\left(\frac{3}{2}\cos\theta+1\right)(3\sin\theta+2\tan\theta)$	

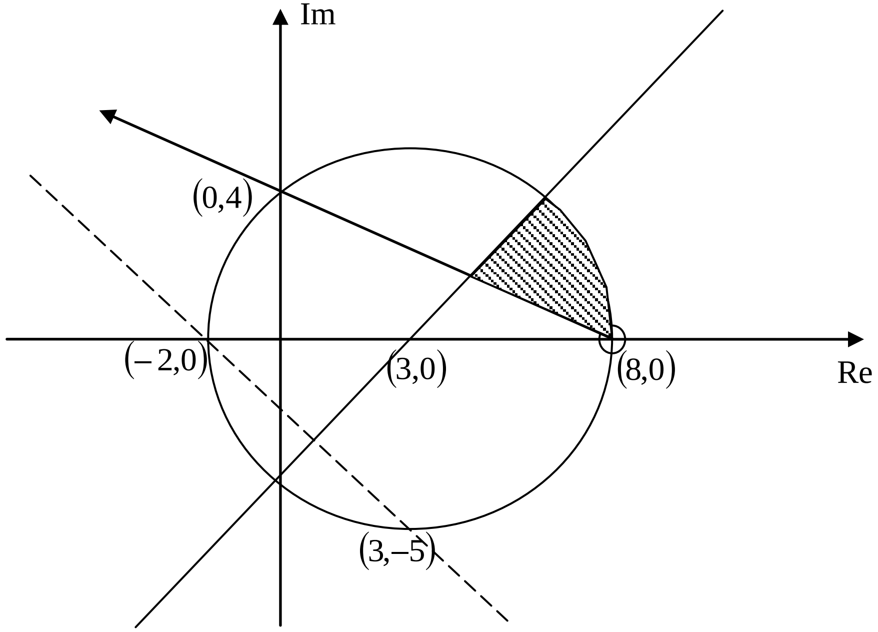
8	APPLICATIONS OF DIFFERENTIATION		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
(i)	- to find the area of a regular hexagon	Base Area of Packaging $=6\left(\frac{1}{2}x^2\sin 60^0\right)=\frac{3\sqrt{3}x^2}{2}cm^2$.	
(ii)	- to solve a minima problem involving area and volume of a hexagonal prism	Volume $=\frac{3\sqrt{3}x^2}{2}h=972$ $h=\frac{1944}{3\sqrt{3}x^2}$ Area of the material, $A=2\left(\frac{3\sqrt{3}x^2}{2}\right)+6xh$ $=3\sqrt{3}x^2+\frac{6x(1944)}{3\sqrt{3}x^2}$ $=3\sqrt{3}x^2+\frac{3888}{\sqrt{3}x}$ $\frac{dA}{dx}=6\sqrt{3}x-\frac{3888}{\sqrt{3}x^2}=0$ $18x^3-3888=0$ $x^3=216$ $x=6$ $\frac{d^2A}{dx^2}=6\sqrt{3}+\frac{7776}{\sqrt{3}x^3}>0$ Area is minimum when x = 6cm Minimum Area	

8	**APPLICATIONS OF DIFFERENTIATION**		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
		$=3\sqrt{3}\left(6^2\right)+\dfrac{3888}{\sqrt{3}(6)}$ $=561.1844...$ $\approx 561.18\ \ (2\,\text{d.p.})$	
(iii)		Minimum cost of packaging $=\dfrac{\$0.05}{100\text{ cm}^2}\times\left(561.1844...\text{ cm}^2\right)=\$0.28$ (nearest cent)	

9	MACLAURIN'S SERIES		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
(i)	- to carry out implicit differentiation repeatedly	$\ln(y+1)=1+\tan^{-1}x$ $\frac{1}{y+1}\frac{dy}{dx}=\frac{1}{1+x^2}$ $\frac{dy}{dx}=\frac{y+1}{1+x^2}$ $\frac{d^2y}{dx^2}=\frac{(1+x^2)\frac{dy}{dx}-(y+1)(2x)}{(1+x^2)^2}$ $(1+x^2)^2\frac{d^2y}{dx^2}=(1+x^2)\frac{dy}{dx}-(y+1)(2x)$ $(1+x^2)\frac{d^2y}{dx^2}=\frac{dy}{dx}(1-2x)$	
(ii)	- to find the Maclaurin's series for a function	$(1+x^2)\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}(2x)=\frac{dy}{dx}(-2)+(1-2x)\frac{d^2y}{dx^2}$ When x = 0, $y=e-1$ $\frac{dy}{dx}=e$ $\frac{d^2y}{dx^2}=e$ $\frac{d^3y}{dx^3}=-e$ $y=(e-1)+ex+\frac{ex^2}{2}-\frac{ex^3}{6}+\text{.......}$	

9	MACLAURIN'S SERIES		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
(iii)	-to use the binomial expansion of $(1+x)^n$ to obtain the unknowns a and b	$(a+bx)^{-1}=\frac{1}{a}\left(1+\frac{b}{a}x\right)^{-1}=\frac{1}{a}\left(1-\frac{b}{a}x+......\right)$ $\frac{1}{a}=e-1 \quad \Rightarrow a=\frac{1}{e-1}$ $-\frac{b}{a^2}=e \quad \Rightarrow b=-\frac{e}{(e-1)^2}$	

10	COMPLEX NUMBERS		
Part	**Assessment Objectives**	**Mark Scheme**	**Feedback**
	Find the nth roots of a complex number	(a) $z^3 = 2-2\sqrt{3}i$ $z^3 = 4e^{i\left(-\frac{\pi}{3}\right)}$ $z = 4^{\frac{1}{3}}e^{\frac{1}{3}i\pi\left(-\frac{1}{3}+2k\right)}, k = 0, \pm 1$ $z = 4^{\frac{1}{3}}e^{i\pi\left(-\frac{1}{9}\right)}, 4^{\frac{1}{3}}e^{i\pi\left(\frac{5}{9}\right)}, 4^{\frac{1}{3}}e^{i\pi\left(-\frac{7}{9}\right)}$	
	Solve simple equations involving a complex variable by equating real parts and imaginary parts.	(b)(i) Since $z=3i$ is a root of $az^4+z^3+11z^2+bz+18=0$ $a(3i)^4+(3i)^3+11(3i)^2+b(3i)+18=0$ $81a-27i-99+3bi+18=0$ Compare real coefficient, $a=1$ Compare imaginary coefficient, $b=9$	
	Solve equations with real coefficients when a pair of complex roots is given.	(b)(ii) $z^4+z^3+11z^2+9z+18=0$ Since all coefficients are real, $z=-3i$ and $z=3i$ are conjugate roots $z^4+z^3+11z^2+9z+18=(z+3i)(z-3i)(z^2+pz+q)$ By comparing coefficients, $q=2,\ \ p=1$ $z^2+z+2=0$ $z=\dfrac{-1\pm i\sqrt{7}}{2}, 3i, -3i$	

11	COMPLEX NUMBERS			
Part	Assessment Objectives	Mark Scheme		Feedback
(c)	Sketch the loci of simple equations in the Argand diagram. These loci includes circle, half line and perpendicular bisector			
		draw a circle with correct center $\lvert z-3\rvert=5$	[B1]	
		draw a circle with correct radius	[B1]	
	Able to employ geometry in Argand diagram to locate the point of intersection between two loci.	Draw a half line, with the correct argument. $\pi-\tan^{-1}\left(\frac{1}{2}\right)$	[B1]	
		Draw a half line, excluding the starting point at (8, 0)	[B1]	
		Indicate the 2 reference points correctly for perpendicular bisector	[B1]	
		Draw a perpendicular bisector correctly. $\lvert z+2\rvert=\lvert z-3+5i\rvert$	[B1]	
	Able to shade the correct region based on the given inequalities.	$p=0+4i$	[B1]	
		Shade the correct region	[B1]	