INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION 2
in preparation for General Certificate of Education Advanced Level
Higher 2

CANDIDATE NAME	
CLASS	
INDEX NUMBER	

MATHEMATICS **9740/01**

Paper 1 **13 Sep 2012**

3 hours

Additional Materials: Answer Paper
Cover Page
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper. You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

1 A renovation company wishes to obtain sand, stone and brick for construction work. The company sends its requirements for sand, stone and brick, measured in units, to three different suppliers, AngGui, BaBao and CaiTao. Their quotations are as follows:

Price per unit (**$**)	**AngGui**	**BaBao**	**CaiTao**
Sand	15.00	11.00	12.00
Stone	10.50	17.30	13.00
Brick	8.10	7.00	10.00
Total price	205.2	229.4	208

(i) Find the number of units of sand, stone and brick required by the company. [3]

(ii) Another supplier, DaoBi, charged 10% lower per unit of sand, stone and brick than the company that charges the lowest for **each** of the materials. Find the total amount that the renovation company must pay if all the materials were purchased from DaoBi, leaving your answers to the nearest cent. [2]

2 Find the general solution of the differential equation

$$\operatorname{cosec} x \frac{d^2 y}{dx^2} = x .$$ [3]

Find the equation of the solution curve whose tangent at the origin is parallel to the line $y = 3x - 5$. [3]

3 **(i)** Prove by induction that

$$\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2 (n+1)^2 .$$ [4]

(ii) The r^{th} term of a series is given by $\ln\left(2a^{r^3}\right)$ where a is a positive constant. Show that the sum of the first n terms of the series is given by $\frac{n}{4} \ln\left(16a^{n(n+1)^2}\right)$. [3]

4 The equation of a curve is given by

$$xy - 2y^2 + 4x^2 = 66 .$$

(i) Find the exact coordinates of the points on the curve where the tangent is parallel to the y-axis. [4]

(ii) Show that every line parallel to the x-axis cuts the curve at two distinct points. [3]

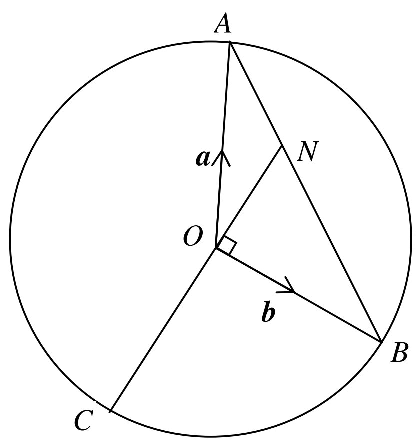
5 Sketch, on the same diagram, the graphs of $y=\ln(2x+9)$ and $y=\sqrt{10-x^2}$, including the coordinates of the points where the graphs cross the x-axis and the equations of any asymptotes. [3]

Hence solve the inequality

$$\ln(2x+9)\geq\sqrt{10-x^2}$$

and deduce the solution to the inequality $\ln(2|x|+9)\geq\sqrt{10-x^2}$. [5]

6



The above diagram shows a circle with radius r units and centre O. The points A and B on the circle are such that $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$ and $\angle AOB=120^\circ$. The point N divides AB in the ratio $\lambda:1-\lambda$ and ON is perpendicular to OB.

(i) Show that $\overrightarrow{ON}=\frac{1}{3}(2\mathbf{a}+\mathbf{b})$ and hence find the area of triangle OAN in the form of $k|\boldsymbol{a}\times\boldsymbol{b}|$, where k is a constant to be determined. [6]

(ii) It is given that the point C lies on the circle such that O, N and C are collinear. By considering the length of ON, find $\overrightarrow{OC}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. [2]

7 **(a)** If $z = 2(\cos\theta + \mathrm{i}\sin\theta)$, where $0 < \theta < \dfrac{\pi}{2}$, label the points P, Q and S representing the complex numbers z, $-\dfrac{4}{z}$ and $\dfrac{4}{z}$ respectively on an Argand diagram. [2]

It is given that PR is a diagonal of the rectangle $PQRS$.

(i) State, in terms of z, the complex number represented by the point R. [1]

(ii) Find, in terms of θ, the area of the rectangle $PQRS$, leaving your answer as a single trigonometric function. [2]

(b) Show that $3 + 3\mathrm{e}^{\mathrm{i}\theta} = 6\mathrm{e}^{\mathrm{i}\frac{\theta}{2}}\cos\dfrac{\theta}{2}$, where $0 < \theta < \pi$. [2]

Hence find the exact values of a and θ if $3 + 3\mathrm{e}^{\mathrm{i}\theta} = \dfrac{a}{\sqrt{2}}(1+\mathrm{i})$. [3]

8 It is given that $y = \mathrm{e}^{-x}\ln(1-2x)$.

(i) Show that $(1-2x)\dfrac{\mathrm{d}y}{\mathrm{d}x} = -2\mathrm{e}^{-x} - y(1-2x)$ and hence show that

$$(1-2x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\frac{\mathrm{d}y}{\mathrm{d}x} + 4\mathrm{e}^{-x} + (3-2x)y.$$ [3]

(ii) Hence find the Maclaurin series for y up to and including the term in x^3. [4]

(iii) Verify that the same result is obtained if the standard series expansions for e^x and $\ln(1+x)$ are used. [2]

9 A curve C has parametric equations

$$x = 2\cos t + 1, \quad y = \sin t, \quad 0 \le t < 2\pi.$$

(i) Sketch C, indicating the coordinates of the intersections with the x-axis. [2]

(ii) Find the numerical value of the volume of revolution obtained when C is rotated 180° about the x-axis. [3]

(iii) Find a cartesian equation of C. [1]

(iv) Describe a sequence of transformations that will transform the curve C into a circle with radius 2 centred at the origin. [2]

10 **(a)** The functions h and g are defined as follows:

$$\text{g} : x \mapsto \sqrt{x-a}\,, \qquad x \geq a\,,$$
$$\text{h} : x \mapsto x^2 + a\,, \qquad x < 0\,.$$

(i) Show that the composite function gh exists. [2]

(ii) Define gh in a similar form. [3]

(b) The function f is defined by

$$\text{f} : x \mapsto \frac{1}{1+2\sin x}, \qquad \frac{\pi}{2} < x < \pi.$$

(i) By using differentiation, show that $\text{f}\left(x\right)$ increases as x increases. [2]

(ii) Find $\text{f}^{-1}\left(x\right)$ stating the domain of f^{-1}. [3]

11 **(a)** The positive multiples of 5 are grouped into sets $M_1 = \{5\}$, $M_2 = \{10, 15\}$, $M_3 = \{20, 25, 30\}$, …, where the set M_n has n elements.

(i) Find the total number of elements in the first n sets and show that the last element of M_n is $\frac{5}{2}n(n+1)$. [3]

(ii) Hence find the sum of all the elements in M_{n+1} in terms of n. [2]

(b) David, a JC student is training for the 2.4 km run of his NAPFA fitness test on the jogging track, where he has to run 6 rounds. The time taken for every round is $\frac{23}{20}$ of the time taken of his previous round as he gets more tired.

(i) If he takes 2 min 30 seconds to run the fourth round, find the time taken for him to run 2.4km, leaving your answer to the nearest second. [3]

(ii) Tommy starts the 2.4 km run one minute after David started to run. For Tommy, the time taken for every round is $\frac{11}{10}$ of the time taken of his previous round. If Tommy runs his first round in 1 min 50 seconds, will he be able to complete the 2.4km rum before David? Give a reason for your answer. [2]

12 **(i)** On a single Argand diagram sketch the loci given by

(a) $|z-12-12\mathrm{i}|=12$,

(b) $|\mathrm{i}z-1-12\mathrm{i}|=5$. [4]

(ii) Hence, or otherwise, find the complex numbers z_1 and z_2 that satisfy both **(a)** and **(b)**. [4]

(iii) Given that z_1 and z_2 also satisfy $|z-12-12\mathrm{i}|=|z-a-b\mathrm{i}|$. Find the real values of a and b. [2]

(iv) Find the least possible value of $\arg(w-30-12\mathrm{i})$ given that w satisfy $|w-12-12\mathrm{i}|=12$ and $\mathrm{Re}(w)\geq 12$.

[2]