Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Promotional Examination Practice Paper 2
Year 5

MATHEMATICS (Higher 2) 9758/01

3 hours

Additional Materials: List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 The selling price of a desktop monitor, keyboard and mouse is \$317.90 in total. During the Great Electronics Sale, two companies, *A* and *B*, offered the following discounts to their customers.

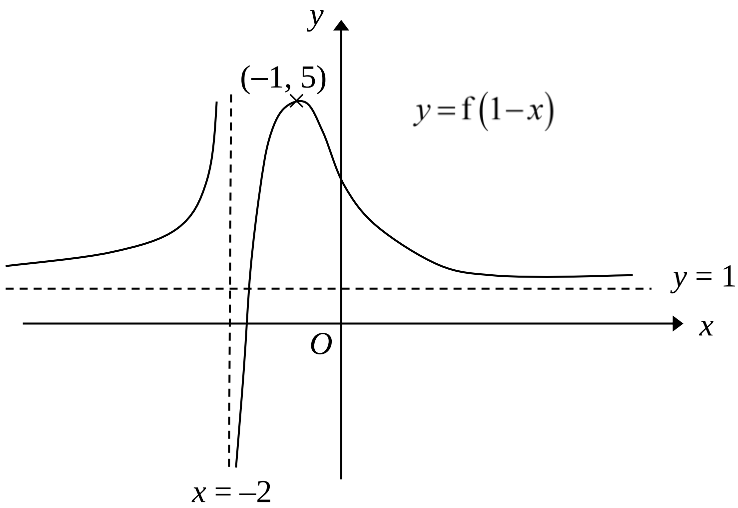
Company	Discounts given for each item			Total price after the discount
	Desktop monitor	Keyboard	Mouse	
A	10%	15%	10%	\$282.66
B	5%	25%	20%	\$283.72

(a) Find the selling prices of a desktop monitor, keyboard and mouse. [3]

(b) Kim works at Company *B* and the employees from Company *B* are offered a further 2% discount on the original price of the mouse. Explain if this additional discount will make it more attractive for Kim to purchase all the three items from her own company than from Company *A*. [1]

2 Given that $x<0$, solve the inequality $\dfrac{2}{x}<|x|-a$, where *a* is constant such that $a>3$. [4]

3 The diagram shows the curve $y=\mathrm{f}\left(1-x\right)$. The curve has a maximum point at $(-1,5)$ and two asymptotes $x=-2$ and $y=1$.



On separate diagrams, sketch the graphs of

(a) $y=\mathrm{f}(x)$, [2]

(b) $y=\mathrm{g}'(x)$, where $\mathrm{g}(x)=\mathrm{f}(1-x)$. [2]

4 A curve C has parametric equations

$$x = 3t^3, \quad y = \frac{3}{1+t^2}, \qquad t \in \mathbb{R}.$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. Explain why C has no stationary points. [2]

(b) Sketch C, indicating clearly the equation of the asymptote if any. [2]

5 It is given that $\mathrm{e}^y = 2 + \mathrm{e}^x$.

(a) Show that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \frac{\mathrm{d}y}{\mathrm{d}x}$. Hence find the Maclaurin series for y up to and including the term in x^2, giving the coefficients in exact form. [4]

(b) By using appropriate standard series expansions from the List of Formulae (MF27), verify the correctness of the series expansion found in **part (a)**. [3]

6 Jane decides to purchase an apartment. To finance her apartment, she intends to apply for a bank housing loan of \$847 500 which charges a monthly interest rate of 0.25%. Interest is charged on the outstanding loan on the first day of each month, starting from the second month. A fixed monthly repayment of $\$M$ is done on the last day of each month throughout the loan tenure, starting from the first month.

(a) Find an expression for the loan balance at the end of the second month. Hence show that the calculated loan balance at the end of the nth loan month is given by

$$847500(1.0025)^{n-1} + 400M\left(1 - 1.0025^n\right).$$ [3]

(b) TDSR, also known as total debt servicing ratio, refers to the portion of a borrower's gross monthly income that goes towards the total monthly debt. For a bank to approve any loan, the borrower's TDSR should be less than or equal to 55%. The formula for calculating TDSR is as follows:

$$\frac{\text{Borrower's total monthly debt}}{\text{Borrower's gross monthly income}} \times 100 \cdot$$

If Jane intends to repay the loan in 25 years, determine whether the bank will approve Jane's loan given that her minimum gross monthly income is \$7 300. [3]

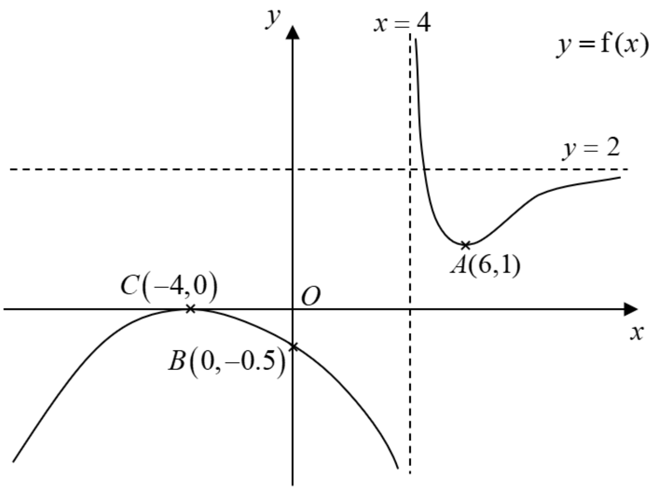
(c) State one assumption that you have used in your calculation for part **(b)**. [1]

7 **(a)** The curve C has equation $4y^2 = 1 + x^2$.

(i) Sketch C, stating the equations of any asymptotes and the coordinates of the points where C cut the axes. [3]

(ii) State a sequence of transformations that will transform C onto the curve with equation $(y-1)^2 - x^2 = 1$. [2]

(b) The diagram below shows the graph of $y = \mathrm{f}(x)$. The curve has a minimum point at $A(6,1)$, a maximum point at $C(-4, 0)$, and has asymptotes with equations $x = 4$ and $y = 2$. The curve cuts the y-axis at $B(0, -0.5)$. By first considering the graph of $y = \mathrm{f}(2x)$, or otherwise, sketch the graph of $y = \dfrac{1}{\mathrm{f}(2x)}$, labelling the corresponding coordinates of A, B, C where possible [3]



8 **(a)** It is given that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$.

(i) Find $\sum_{r=1}^{n}\left(2^{r+1} + 3r - r^2\right)$ in the form $A(2^n - 1) + \mathrm{f}(n)$, where A is a constant and $\mathrm{f}(n)$ is in fully factorised form. [3]

(ii) Using your answer in part **a(i)**, find $\sum_{r=4}^{N}\left(2^r + 3r - (r-1)^2\right)$, leaving your answer in the form $B\left(2^N\right) + C\left[N(N-1)(5-N)\right] + DN + E$ where B, C, D and E are constants to be determined. [4]

(b) A sequence is such that $v_1 = p$, where p is a constant, and

$$v_{n+1} = 3v_n - 2, \quad \text{for } n \geq 1.$$

Describe how the sequence behaves when

(i) $p = 5$, [1]

(ii) $p = 1$. [1]

9 **(a)** Find $\int \frac{5}{(2x-3)(x+1)}\,\mathrm{d}x$. [2]

(b) Using the substitution $x=\sec\theta$, where $0<\theta<\frac{\pi}{2}$, find the exact value of $\int_{\sqrt{2}}^{2} \frac{\sqrt{x^2-1}}{x}\,\mathrm{d}x$. [4]

(c) Find $\int \frac{x}{\sqrt{1-m^2x^2}}\,\mathrm{d}x$, where m is a constant. Hence find $\int \left(\sin^{-1} mx\right)\frac{x}{\sqrt{1-m^2x^2}}\,\mathrm{d}x$. [4]

10 **(a)** The functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{1}{x+a}, \quad x\in\mathbb{R}, \quad x\neq -a,$$
$$\mathrm{g}: x \mapsto x^2+2, \quad x\in\mathbb{R},$$

where a is a positive constant.

(i) Show that f^{-1} exists and define f^{-1} in a similar form. [3]

(ii) Show that the composite function gf exists and find its exact range. [3]

(b) The function h is defined by

$$\mathrm{h}: x \mapsto \begin{cases} \tan^{-1} x & \text{for } -1<x\leq 1, \\ -\frac{\pi}{4}x+\frac{\pi}{2} & \text{for } 1<x\leq 3. \end{cases}$$

It is further given that $\mathrm{h}(x+4)=\mathrm{h}(x)$ for all real values of x.

(i) Sketch the graph of $y=\mathrm{h}(x)$ for $-2\leq x\leq 6$. [3]

(ii) Write down $\int_{-2}^{6} \mathrm{h}(x)\,\mathrm{d}x$. [1]

11 The curves C_1 and C_2 have equations $y=\sqrt{15x+4}$ and $y=x^2-2$ respectively. The region R is bounded by the y-axis, C_1 and C_2.

(a) Show algebraically that the point with coordinates (3,7) is the only point of intersection between C_1 and C_2. [3]

(b) Without the use of a graphing calculator, find the exact area of R. [4]

(c) Find the volume of revolution when R is rotated 2π radians about the y-axis. [3]

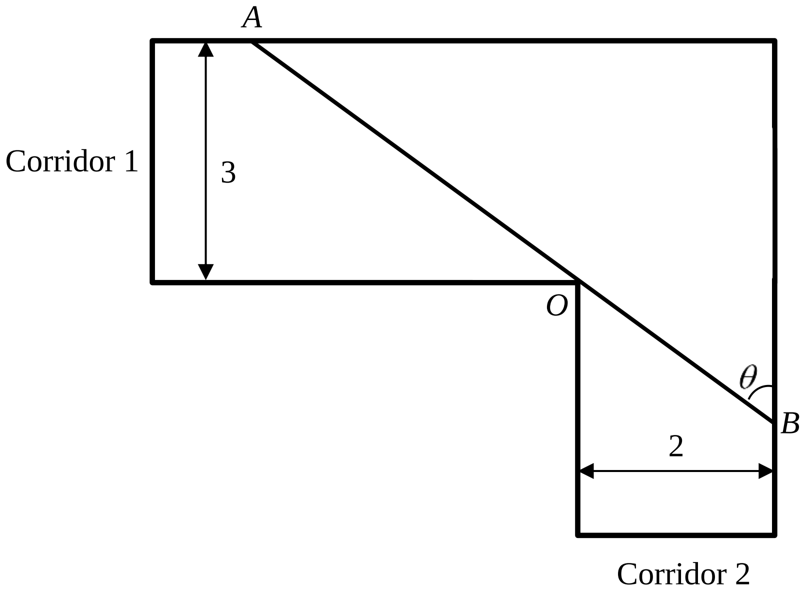
12 **(a)** The sum of the first n terms of a series is given by $S_n = 3n(n+2)$. Show that this series follows an arithmetic progression. [3]

(b) The second, seventh and mth term of the series in part **(a)** are the first three consecutive terms of a geometric series, such that its nth term is given by v_n. Find the value of m and explain whether the sum to infinity of the geometric series exists. [3]

(c) The nth term of another geometric series is given by $w_n = \mathrm{e}^{5+nx(x+1)}$, where x is a constant. Find the range of values of x such that this geometric series converges. [3]

(d) Using $x = -0.5$ in part **(c)** and given that the sum of the first n terms of the geometric series in part **(b)** first exceeds the nth term of the geometric series in part **(c)**, i.e., $\left(\sum_{r=1}^{n} v_r\right) > w_n$, determine the least value of n. [2]

13 In a particular building, there is an L-shaped corridor (see diagram below). Corridor 1 has a fixed width of 3 m and Corridor 2, which is perpendicular to the first corridor, has a fixed width of 2 m. The points A and B are on one side of the wall along Corridor 1 and Corridor 2 respectively, such that line AB touches the corner O and makes an angle of θ radians with one of the walls as shown where $0 \le \theta \le \frac{\pi}{2}$.



(a) Show that $AB = \dfrac{\alpha}{\cos\theta} + \dfrac{\beta}{\sin\theta}$, where α and β are constants to be determined. [1]

(b) If θ is decreasing at 0.1 radians per second, find the rate at which AB is increasing at the instant when $OB = 4$ m. [4]

(c) A pole of length p m is carried through this L-shaped corridor parallel to the ground. By using differentiation, show that the maximum possible value of p is $\left[(2)^k + (3)^k\right]^{\frac{1}{k}}$, where k is a constant to be determined exactly. [7]