Chapter 13 ELECTRIC FIELDS

Content

- Concept of an electric field
- Electric field strength of a point charge
- Electric force between point charges
- Electric field lines
- Electric potential energy and electric potential for point charges
- Relationships and graphs
- Equipotential lines
- Uniform electric fields
- Electric fields and conductors
- Comparison between electric fields and gravitational field

Learning Outcomes

Candidates should be able to:

(a) show an understanding of the concept of an electric field as an example of a field of force and define electric field strength at a point as the electric force exerted per unit positive charge placed at that point.

(b) represent an electric field by means of field lines.

(c) recognise the analogy between certain qualitative and quantitative aspects of electric and gravitational fields.

(d) recall and use Coulomb's law in the form $F = Q_1Q_2/4\pi\varepsilon_0 r^2$ for the electric force between two point charges in free space or air.

(e) recall and use $E = Q/4\pi\varepsilon_0 r^2$ for the electric field strength of a point charge in free space or air.

(f) calculate the electric field strength of the uniform field between charged parallel plates in terms of potential difference and plate separation.

(g) calculate the forces on charges in uniform electric fields.

(h) describe the effect of a uniform electric field on the motion of charged particles.

(i) define the electric potential at a point as the work done per unit positive charge in bringing a small test charge from infinity to that point.

(j) state that the field strength of the electric field at a point is numerically equal to the potential gradient at that point.

(k) use the equation $V = Q/4\pi\varepsilon_0 r$ for the electric potential in the field of a point charge in free space or air.

13 Introduction

Links Between Sections and Topics

There are four fundamental forces in physics: the gravitational, electromagnetic, strong and weak interactions. While the strong and weak interactions explain phenomena at the sub-atomic level, daily human experiences can be explained by gravitational and electromagnetic interactions.

Electromagnetic interactions involve particles that have a property called electric charge, an attribute that appears to be just as fundamental as mass, or even more so – charge seems to be precisely quantised, while it is not clear if mass is quantised. An object with mass experiences a force in a gravitational field, and electrically-charged objects experience forces in electric and magnetic fields. Like mass-energy, charge obeys a conservation law as well. There are important analogies and distinctions between concepts in the gravitational field and in the electrical field topics.

A charge produces an electric field in the space around it, and a second charge placed in this field experiences a force due to this field. The electric force between two isolated point charges is governed by Coulomb's law, which is mathematically similar to Newton's law of gravitation for isolated point masses. Similar to the gravitational case, work done against the electric force depends solely on the initial and final positions but is independent of the path connecting the two positions. Thus, terms like electric potential and electric potential energy can be defined and used similar to their gravitational counter parts.

Practical use of electricity often occurs in circuits rather than in free space. Circuits provide a means of conveying energy and information from one place to another. Within a circuit, the complicated effects of forces and electric fields at the microscopic level result in a macroscopic description where consideration of energy and electric potentials mostly suffices. The collective movement of charges results in electrical current, driven by potential differences (also known as voltages). Both current and potential difference can be experimentally measured. Applying the principles of charge and energy conservation provide powerful tools to analyse a variety of electrical circuits.

The mystery of magnetism was first discovered in magnetic stones by the ancients. Today, we understand magnetism as an effect inseparable from electricity, summarised by Maxwell's laws of electromagnetism. Unlike electric forces, which act on electric charges whether moving or stationary, magnetic forces act only on moving charges. Moving charges produce a magnetic field, and another moving charge or current placed in this magnetic field experiences a force. This apparent asymmetry in electromagnetic phenomena contributed to the development of the theory of relativity.

Applications and Relevance to Daily Life

Technologies harnessing electrical and magnetic properties pervade modern society. The generation of electrical energy from other forms of energy traditionally involves the induced electromotive force and current produced by a changing magnetic flux or a time-varying magnetic field. Transmitting electrical energy over long distances is made feasible by the use of alternating current and voltage transformers. Semiconductor devices in computers and smartphones are the product of our deep understanding of the physics of electricity and magnetism in solid state materials. Innovations are also pushing on the quantum frontier.

Microscopically, elastic forces in springs and contact forces between surfaces arise from electrical forces at the atomic level. In biology, electricity is also important in signalling and control. The heart rhythms are maintained by waves of electrical excitation, from nerve impulses that spread through special tissue in the heart muscles.

Links to Core Ideas

Systems and Interactions	Models and Representations	Conservation Laws
• F_E as the interaction between a charge and an external *E*-field • F_B as the interaction between a moving charge and an external *B*-field	• Microscopic model of the flow of charges • Ohm's law (for ohmic conductors) • Faraday's law • Common representations: diagrams of electric circuits, field lines and equipotential lines, magnetic flux density patterns, e.t.c. • Simplifying assumptions: e.g. point charges, negligible internal resistance, infinitely extended planes	• Conservation of charges in circuits • Conservation of energy in circuits • Lenz's law as conservation of energy

13.1 Electric Field

Electric Field

A charged particle creates a *field* of influence around itself that permeates space. Other charged particles present in this field experience a force, which is called an **electric force**.

Definition

An **electric field** is a region of space in which a charge placed in that region experiences an electric force.

13.1.1 Electric Field Strength E

13.1.1.1 General Cases

Relationship between E and F

The ratio between the electric force F experienced by a charged particle in an electric field and its charge q is termed the electric field strength E.

Definition

The **electric field strength** at a point is defined as the electric force exerted per unit positive charge placed at that point.

Formula

$$E = \frac{F}{q}$$

Electric field strength is a **vector** quantity.
S.I. unit for electric field strength is **N C^{-1}** or **V m^{-1}**.

If the electric field strength E at a point in space is known, the electric force F experienced by a charge q placed at that point is given by

Formula

$$F = qE$$

Electric force is a **vector** quantity.
S.I. unit for electric force is the **newton (N)**.

Note

- The **direction of the electric force** on a charged particle depends on whether its charge is positive or negative. A **positive charge** will experience an **electric force towards a region of lower electric potential**. A **negative charge** will experience an **electric force towards a region of higher electric potential.**
- The **direction of the electric field strength** is that of the **electric force acting on a small positive test charge** at that point.
- Some A-Level questions refer to electric field strength as just "electric field".
- The formulae above relating the electric force and electric field strength apply to any electric field. Here, we mainly discuss two types: non-uniform fields due to point charges and uniform fields produced by a pair of charged parallel plates.

Example 1 A charge of −2.0 μC experiences a force of 8.0 N when placed at a point in an electric field. Determine the electric field strength at the point.

Solution:

The magnitude of electric field strength is $E = \frac{F}{q} = \frac{8.0}{2.0 \times 10^{-6}} = 4.0 \times 10^{6}\ \text{N C}^{-1}$

13.1.1.2 Point Charges

***F* between 2 Point Charges**

The electric force between two *point charges* in free space is given by **Coulomb's law.**

Definition

Coulomb's law states that the force between two point charges is proportional to the product of the charges and inversely proportional to the square of the distance between them.

Coulomb's law in equation form is

Formula

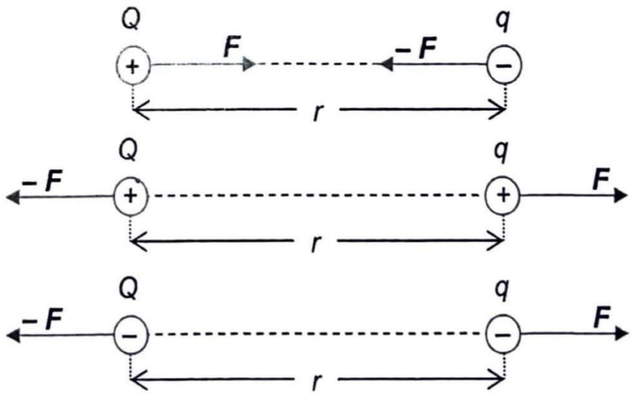
$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

where F is the magnitude of the electric force between point charges with charges Q and q (in Coulomb, C) that are separated by a distance r.

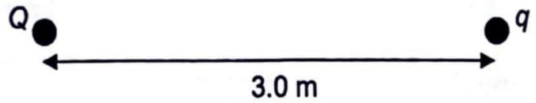
The constant of proportionality is $(4\pi\varepsilon_0)^{-1}$, where the permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{F m}^{-1}$.

Note

- The electric forces that two point charges act on each other are equal in magnitude and opposite in direction; they constitute an action and reaction pair.
- Both Coulombs' law and Newton's law of gravitation are referred to as inverse-square law, due to their r^{-2} dependence.
- The major difference between gravitational force and **electric (or electrostatic) force** is that the former is always an attractive force while the latter can be **attractive or repulsive**. This is because there are two types of charges – positive and negative. Like charges repel, while unlike charges attract.



Example 2 Two charges Q (+2.0 μC) and q (+1.0 μC) are separated by a distance of 3.0 m.



(a) Determine the magnitude and direction of the force acting on q by Q.
(b) If q is negatively charged instead, but with the same magnitude, determine the magnitude and direction of the force on q by Q.

Solution:

(a) Magnitude of force,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{(2.0\times10^{-6})(1.0\times10^{-6})}{4\pi(8.85x10^{-12})(3.0)^2} = 1.998\times10^{-3} = 2.00\times10^{-3}\ \text{N}$$

Since both charges are positive, the force on q due to Q is directed away from Q the right.

(Like charges repel.)

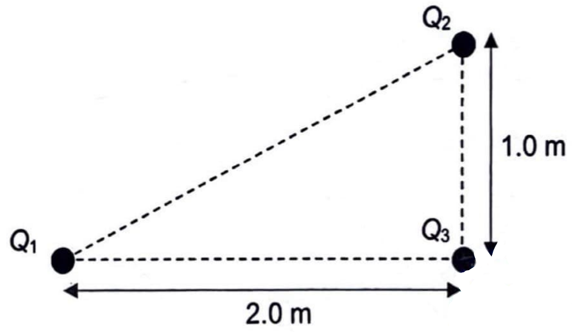
(b) Magnitude of force,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = 2.00\times10^{-3}\ \text{N}$$

Since both charges are oppositely charged, the force acting on q due to Q is directed towards Q or to the left.

(Unlike charges attract.)

Example 3 Three charges Q_1 = +2.0 μC, Q_2 = +1.0 μC and Q_3 = +3.0 μC are placed at the corners of a right-angle triangle with dimensions as shown.



Determine the magnitude and direction of the resultant force acting on Q_3.

Solution:

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_3}{r_1^2} == \frac{1}{4\pi\varepsilon_0}\frac{(2.0\times10^{-6})(3.0\times10^{-6})}{(2.0)^2} = 0.013488\ \text{N}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2Q_3}{r_2^2} = \frac{1}{4\pi\varepsilon_0}\frac{(1.0\times10^{-6})(3.0\times10^{-6})}{(1.0)^2} = 0.026975\ \text{N}$$

$$|F| = \sqrt{F_1^2 + F_2^2} = \sqrt{(0.013488)^2 + (0.026975)^2} = 0.03016 = 0.0302\ \text{N}$$

direction of the resultant force is at an angle θ below the horizontal

$$\tan\theta = \frac{F_2}{F_1}$$

$$\theta = \tan^{-1}\left(\frac{F_2}{F_1}\right) = \tan^{-1}\left(\frac{0.026975}{0.013488}\right) = 63.43 = 63.4°$$

E due to a Point Charge

The electric force acting on point charge q, which is a distance r away from the source of the field (Q), is given by $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$, according to Coulomb's law.

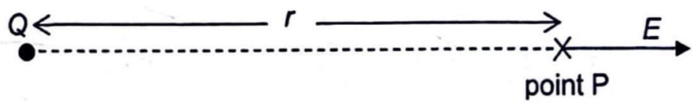
This formula can also be written as follows:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = q\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right) = qE$$

Hence, the electric field strength due to **a point charge** Q at a distance r away is

Formula

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$



Note

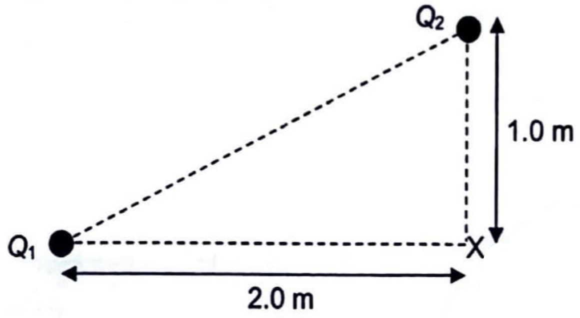
- When considering the electric force on charge q, only the field strength due to the other charge (Q) is considered. In other words, the electric field of q does not act on q itself. Similarly, if the electric force on Q is considered, only the field strength due to q should be considered:

  $$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = Q\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)$$

- The direction of the field strength due to a charged particle depends on the sign of its charge: it points **radially away from a positive point charge**, and **radially towards a negative point charge**. These directions, by definition, are the **directions of the electric forces acting on a small positive test charge** placed at that point.

- The **resultant field strength** at a point due to more than one charge can be found from the **vector sum** of the individual electric field strength due to each charge at that point.

Example 4 Two charges Q_1 = +2.0 μC and Q_2 = +1.0 μC are placed at the corners of a right-angle triangle with dimensions as shown.



(a) Determine the electric field strength at the third corner X.
(b) If a charge Q_3 of +3.0 μC is placed at corner X, determine the force it experiences.

Solution:

a

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1^2} = \frac{1}{4\pi\varepsilon_0}\frac{(2.0\times10^{-6})}{(2.0)^2} = 4.4959\times10^3 \text{ N C}^{-1}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2^2} = \frac{1}{4\pi\varepsilon_0}\frac{(1.0\times10^{-6})}{(1.0)^2} = 8.9918\times10^3 \text{ N C}^{-1}$$

$$|E| = \sqrt{E_1^2 + E_2^2} = \sqrt{(4.4959\times10^3)^2 + (8.9918\times10^3)^2} = 10053 = 10050 \text{ N C}^{-1}$$

direction of the resultant field strength is at an angle θ below the horizontal

$$\tan\theta = \frac{E_2}{E_1}$$

$$\theta = \tan^{-1}\left(\frac{E_2}{E_1}\right) = \tan^{-1}\left(\frac{8.9918\times10^3}{4.4959\times10^3}\right) = 63.43 = 63.4°$$

b.

$$|E| = 10053 = 10050 \text{ N C}^{-1}$$

$$|F| = q|E| = (3.0\times10^{-6})(10053) = 0.03016 = 0.0302 \text{ N}$$

Since charge Q_3 is a positive charge, the force acting on it is in the direction of the electric field strength at the point.

13.1.2 Electric Field Lines

13.1.2.1 General Cases

Drawing Field Lines

Drawing of field lines is a tool that helps to visualise fields (gravitational, electric or magnetic). Each field is a 3-D pattern, but on paper it is represented by a 2-D slice of the pattern.

The field lines we draw around the sources of the fields (masses, charges, magnets) are used to denote the direction of the forces experienced by test bodies placed within the fields.

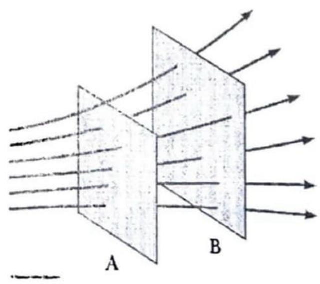
5 General guidelines for drawing field lines

1. Direction of the field line indicates the direction of the resultant force acting on a small positive test charge.

2. Field lines point from higher to lower potential.
   - Since the direction of the field line is determined by the resultant force acting on a small positive test charge, the field line will always point from a region of higher electric potential to a region of lower electric potential.

3. Field lines never cross one another.
   - The direction of the electric field strength at any point is along the tangent to the electric field line at that point and is in the same direction as the field line.
   - If the lines cross one another, it means that there are two tangents at that point. This is not possible as the electric field strength cannot point towards two different directions at the same point.

4. The density of the field lines (number of lines per unit area) represents field strength.
   - The spacing between the field lines indicates the relative magnitude of the field strength.



   - Since the spacing of the field lines at region A is closer than at region B, the electric field strength at A has a larger magnitude than that at B.

5. Field lines are perpendicular to the surface of the source.
   - As the charges on the surface of the source will redistribute until there is no potential difference between the charges, the direction of the resultant force acting on a small positive test charge any point near the surface of the source will always be perpendicular to its surface.

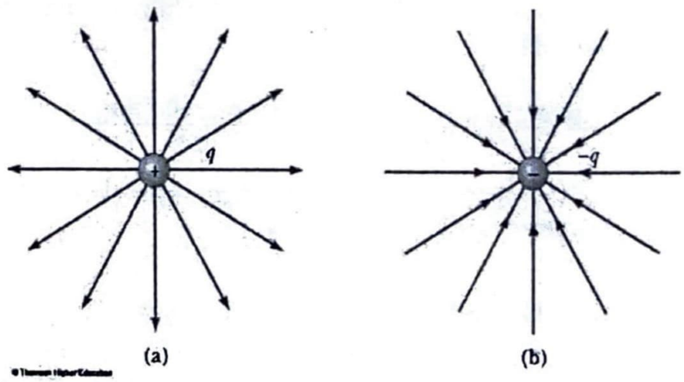
13.1.2.2 Point Charges

Field Lines for Point Charges

The field lines for point charges follow the general guidelines.
Specifically, electric field lines **point outwards from positive charges and point inwards towards negative charges.**

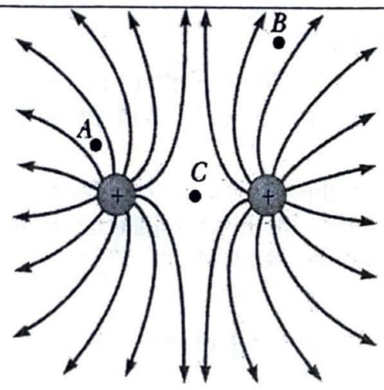
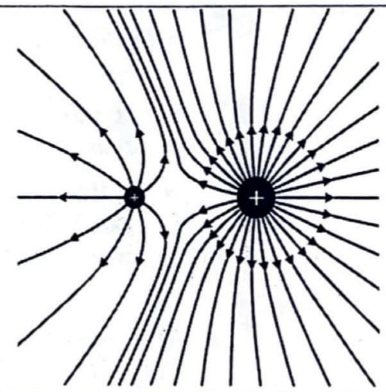
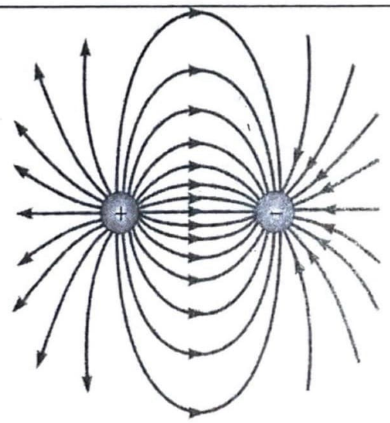
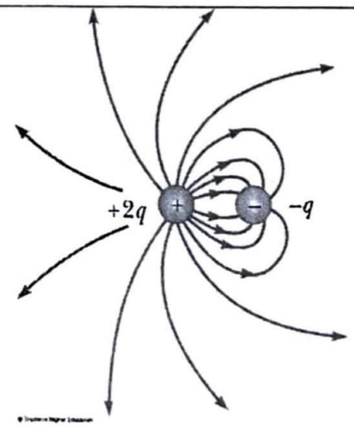
Single Point Charges

These are the field line patterns for
(a) an isolated positive charge,
(b) an isolated negative charge.



Two Point Charges

Here are the field line patterns for two charges.

	
(a) Positive charges of the same magnitude	**(b) Positive charges of different magnitudes**
	
(c) Opposite charges of the same magnitude	**(d) Opposite charges of different magnitudes**

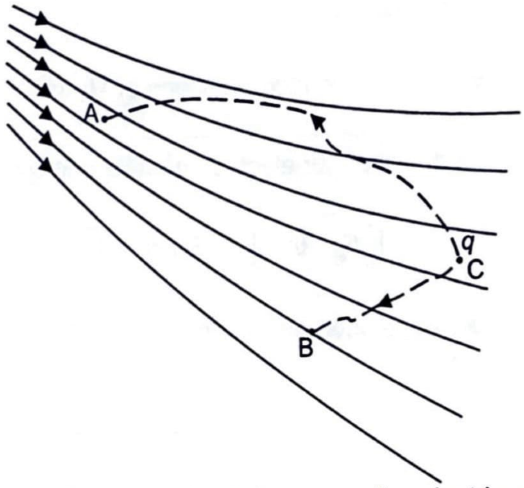
13.2 Electric Potential Energy *U* Electric Potential *V*

13.2.1 General Cases

Electric Potential Energy *U*

The electric force has a special property: the work done against it is independent of the path taken, but only depends on the initial and final positions.

For this reason, we can define an energy function associated with the positions of the charges within an electric field, called the electric potential energy. This will enable us to simplify the calculation for work done against the electric force (an integration along the path) to the difference between the potential energy values at the final and initial positions.



Choosing a fixed (but arbitrary) point C in space, the electric potential energy of a charged particle *q* at any other point is defined as the work done by an external force against the electric force in moving *q* from C to that point.

The point of reference C is conventionally chosen to be at infinity.

Definition

The **electric potential energy** of a charge at a point in an electric field is defined as the work done by an external force in bringing the charge from infinity to that point.

In equation form, the above definition says, the electric potential energy for arbitrary points A and B is

$$U_A = \int_{\infty}^{r_A} F_{ext}\ dr = \int_{\infty}^{r_A} (-F_E)\ dr$$

$$U_B = \int_{\infty}^{r_B} F_{ext}\ dr = \int_{\infty}^{r_B} (-F_E)\ dr$$

where the integrations are along arbitrary paths from infinity to points A and B.

Electric potential energy is a **scalar** quantity.
S.I. unit for electric potential energy is the **joule (J)**.

Note

- It must be emphasised that, in the above definition, the **work done is by an external force** (F_{ext}). It is taken to be equal but opposite to the electric force (F_E), because we are calculating the **work done against the electric force** and, for simplicity, the kinetic energy of the charged particle is assumed to be constant along the way (hence the net force must be zero).

- One way to look at this definition is that the **work done by the external force changes the electric potential energy** of the system. Similar considerations go into the definition of the gravitational potential energy.

The **work done by an external force** against the electric force to move q from A to B (an integration along any path between A and B) can be shown to be **equal to the difference between the potential energies**, U_A and U_B, at A and B respectively.

To demonstrate this, we can choose the path A → C → B.

work done against the electric force to move q from A to B

$$= \int_{r_A}^{r_B} (-F_E)\ dr = \int_{r_A}^{r_C} (-F_E)\ dr + \int_{r_C}^{r_B} (-F_E)\ dr = -\int_{r_C}^{r_A} (-F_E)\ dr + \int_{r_C}^{r_B} (-F_E)\ dr$$
$$= -U_A + U_B = U_B - U_A$$

where we have used the definitions of U_A and U_B.

Electric Potential V

From the definition for electric potential energy of a charged particle:

$$U = \int_{\infty}^{r} F_{ext}\ dr = \int_{\infty}^{r} (-F_E)\ dr = \int_{\infty}^{r} (-qE)\ dr = q \times \int_{\infty}^{r} (-E)\ dr = q \times V$$

where we have used electric force $F_E = qE$, and defined

$$V = \frac{U}{q} = \int_{\infty}^{r} (-E)\ dr.$$

V is called the electric potential.

Definition

The **electric potential** at a point in an electric field is defined as the work done per unit positive charge by an external force in bringing a small test charge from infinity to that point.

In equation form, the above definition can be written as

Formula

$$V = \frac{U}{q}$$

Electric potential is a **scalar** quantity.
S.I. unit for electric potential is the **volt (V)** or **J C^{-1}**.

Given the electric potential V at a point in space, the electric potential energy U of a point charge q placed at that point is

Formula

$$U = qV$$

With the reference point chosen at infinity, both the **electric potential energy and electric potential are zero at infinity**.

For a charge being moved from one point to another in an electric field, the change in electric potential ΔV, also known as the **potential difference** between the two points, is calculated as

Formula

$$\Delta V = V_{final} - V_{initial}$$

The corresponding change in potential energy ΔU, which is equal to the work done against the electric force between the two points, is given by

Formula

$$\Delta U = q\Delta V$$

The **values of *U*, *V* and *q* are either positive or negative** and their values must be substituted into the above equations with their signs.

The electron-volt (eV)

In many situations, the energy gained by a charged particle is small and hence it is convenient to introduce a new unit of energy called the electron-volt (eV).

Definition

The electron-volt is defined as the energy gained by an electron, which carries a charge of 1.60 ×10^{-19} C, when it is accelerated through a potential difference of 1 V (using $\Delta U = q\Delta V$):

$$1\text{ eV} = 1.60\times10^{-19}\text{ C} \times 1\text{ V} = 1.60\times10^{-19}\text{ J}$$

13.2.2 Point Charges

***U* due to 2 Point Charges**

Consider moving a point charge *q* from infinity to a point in the electric field produced by another point charge Q. This point is at a distance *r* away from Q.

The electric force on *q* at this point is $F_E = \frac{Qq}{4\pi\varepsilon_0 r^2}$ and *r* is the distance between *q* and Q.

Using the definition for electric potential energy, we have

$$U = \int_\infty^r F_{ext}\ dr = \int_\infty^r (-F_E)\ dr = \int_\infty^r \left(-\frac{Qq}{4\pi\varepsilon_0 r^2}\right)dr = \left[\frac{Qq}{4\pi\varepsilon_0 r}\right]_\infty^r = \frac{Qq}{4\pi\varepsilon_0 r}$$

Thus, the potential energy of the system of two point charges is

Formula

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

Note

- The **sign of the potential energy *U*** in the above formula is determined by the **signs of *q* and *Q*** (include the signs of *q* and *Q* when substituting into the formula). It is positive between two charges of the same sign (due to the repulsion between the charges), and negative between two charges of opposite signs (due to the attraction between them). This is different from the gravitational potential energy between two point masses ($U_{gravitational} = -\frac{Gm_1m_2}{r^2}$), which carries an explicit negative sign because the gravitational force is always attractive and the masses are always positive.

- One advantage of the choosing the reference point at infinity is the simple form of the above expression. If the reference point is at any finite distance, there would be an additional constant term ($-\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_C}$, which goes to 0 if $r_C \rightarrow \infty$) from the integration.

- Another advantage of choosing the reference point at infinity concerns bound systems, e.g. a positive charge and a negative charge orbiting each other, or a planet-satellite system bound by gravitational force. The potential energy (hence the total energy) of such bound systems, if the reference point for potential energy is chosen at infinity, is always negative.

V due to a Point Charge

Combining the definition of the electric potential and the expression for the electric potential energy for point charges, the electric potential due to **a point charge Q** at a distance r away from Q is

Formula

$$V = \frac{U}{q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Note

- The **sign of the potential V** in the above formula is determined by the **sign of Q** (include the sign of Q when substituting into the formula). It is positive when Q is a positive charge and negative when Q is a negative charge.

- If more than one charge is present, the **total electric potential** at a point is the **algebraic sum** of the individual electric potential due to each charge at that point, since electric potential is a scalar quantity.

13.2.3 Important Relationships

Relationship between F & U

The definition of potential energy

$$U = \int_{\infty}^{r} F_{ext} dr = \int_{\infty}^{r} (-F_E)\, dr$$

implies that the electric force on a charge at a point is equal to the negative of the derivative of electric potential energy U with respect to r at that point:

$$F_E = -\frac{dU}{dr}$$

Note

- From a U–r graph, the **negative of the gradient** of the tangent at a point on the graph gives the **electric force** at that point.

- The negative sign in the formula means that the **electric force points in the direction of decreasing electric potential energy**.

Relationship between *E* & *V*

Similarly, the definition of the electric potential

$$V = \int_{\infty}^{r} (-E)\, dr$$

implies that the electric field strength at a point is equal to the negative of the derivative of electric potential *V* with respect to *r* at that point:

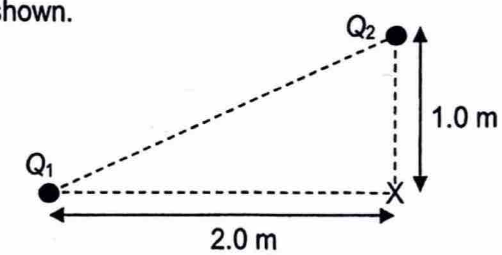
$$\boxed{E = -\frac{dV}{dr}}$$

Note

- This equation can also be obtained by dividing both sides of the equation $F_E = -\frac{dU}{dr}$ by charge *q*, since $F_E = qE$ and $U = qV$.
- From a *V–r* graph, the **negative of the gradient** of the tangent at a point on the graph gives the **electric field strength** at that point.
- The magnitude of the **electric field strength at a point is numerically equal to the gradient of the potential at that point**. The negative sign means that the **electric field strength always points in the direction of decreasing potential,** that is **from higher potential to lower potential.**

Example 5

Two charges Q_1 = +2.0 μC and Q_2 = –2.0 μC are placed at the corners of a triangle with dimensions as shown.



(a) Determine the electric potential at the third corner X.
(b) If a charge Q_3 of +1.0 μC is brought to corner X, determine the change in electric potential energy of the system.

Solution:

(a) $V_1 \text{ (due to } Q_1) = \frac{Q_1}{4\pi\varepsilon_0 r_1} = \frac{2.0\times10^{-6}}{4\pi\varepsilon_0(2.0)} = 8992 \text{ V}$

$V_2 \text{ (due to } Q_2) = \frac{Q_2}{4\pi\varepsilon_0 r_2} = \frac{-2.0\times10^{-6}}{4\pi\varepsilon_0(1.0)} = -17984 \text{ V}$

The total electric potential
$V = V_1 + V_2 = 8992 + (-17984) = -8992 = -8.99 \text{ kV}$

(b) $\Delta U = q\Delta V$

$= Q_3(V_X - V_i)$

$= (+1.0\times10^{-6})(-8992 - 0)$

$= -0.008992$

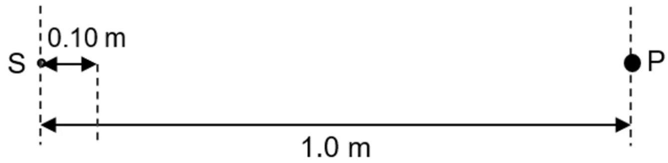
$= -8.99\times10^{-3} \text{ J}$

Example 6 A fixed, positive point charge S carries a charge of 1.0×10^{-4} C. A particle P of mass 2.0×10^{-5} kg and charge -1.5×10^{-10} C is released from rest at a distance of 1.0 m from S.

Calculate the speed of particle P when it is at a distance of 0.10 m from S. Neglect any gravitational effect.

Solution:

Because the particle moves from one point to another,
- we will first calculate the change in potential,
- then the change in potential energy
- and finally use the principle of conservation of energy to determine its speed.



Electric potential at the initial position of P due to S:

$$V_i = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_i} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1.0\times10^{-4}}{1.0}\right) = 8.9918\times10^{5}\ \text{V}$$

Electric potential at the final position of P due to S:

$$V_f = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_f} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1.0\times10^{-4}}{0.10}\right) = 8.9918\times10^{6}\ \text{V}$$

$$\begin{aligned}\Delta U_E &= q\Delta V = q\left(V_f - V_i\right)\\ &= \left(-1.5\times10^{-10}\right)\left(8.9918\times10^{6} - 8.9918\times10^{5}\right)\\ &= -1.2139\times10^{-3}\ \text{J}\end{aligned}$$

By the principle of conservation of energy,

increase in KE = decrease in EPE

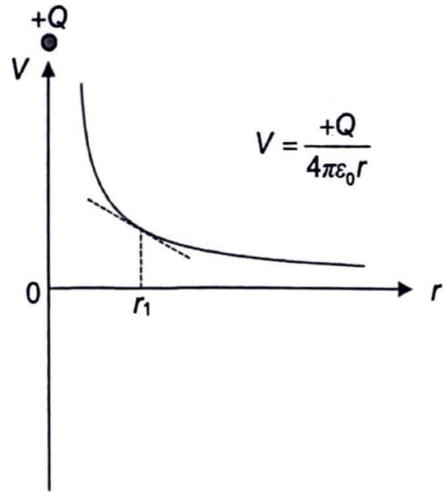
$$\frac{1}{2}mv^2 - 0 = 1.2139\times10^{-3}$$

$$v = \sqrt{\frac{2\times1.2139\times10^{-3}}{2.0\times10^{-5}}} = 11.018 = 11.0\ \text{m s}^{-1}$$

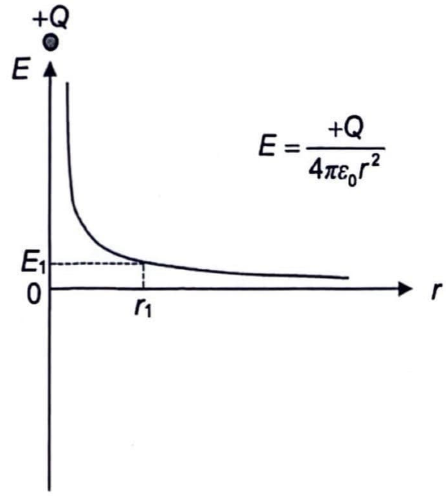
13.3 V–r and E–r Graphs

13.3.1 Single Point Charges

Positive Charge



- V is a scalar
- V is positive at all values of r as charge is positive



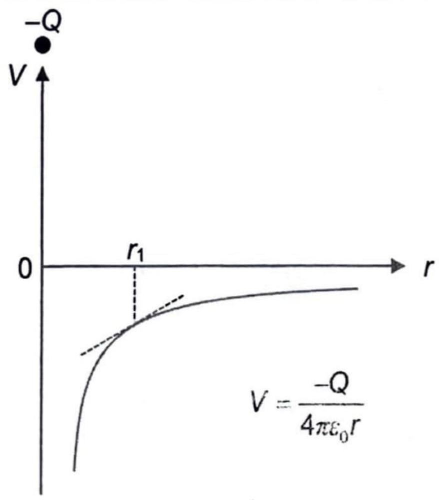
- E is a vector
- E is positive at all values of r as it points outwards from +Q

Note

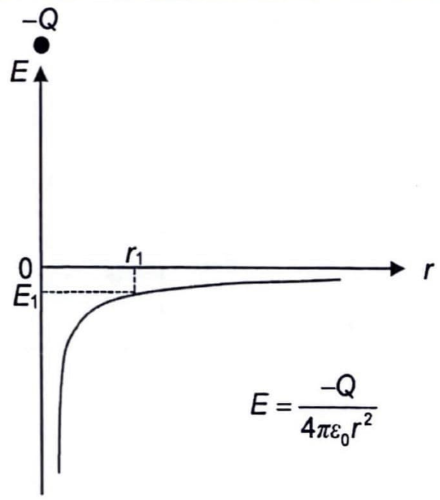
Recall: General relationship between V and E is $E = -\frac{dV}{dr}$

At r_1, since gradient of the tangent i.e. $\frac{dV}{dr}$ is negative, $E_1 = -\frac{dV}{dr}$ is positive.

Negative Charge



- V is a scalar
- V is negative at all values of r as charge is negative



- E is a vector
- E is negative at all values of r as it points inwards towards –Q

Note

At r_1, since gradient of the tangent i.e. $\frac{dV}{dr}$ is positive, $E_1 = -\frac{dV}{dr}$ is negative.

13.3.2 Two Point Charges

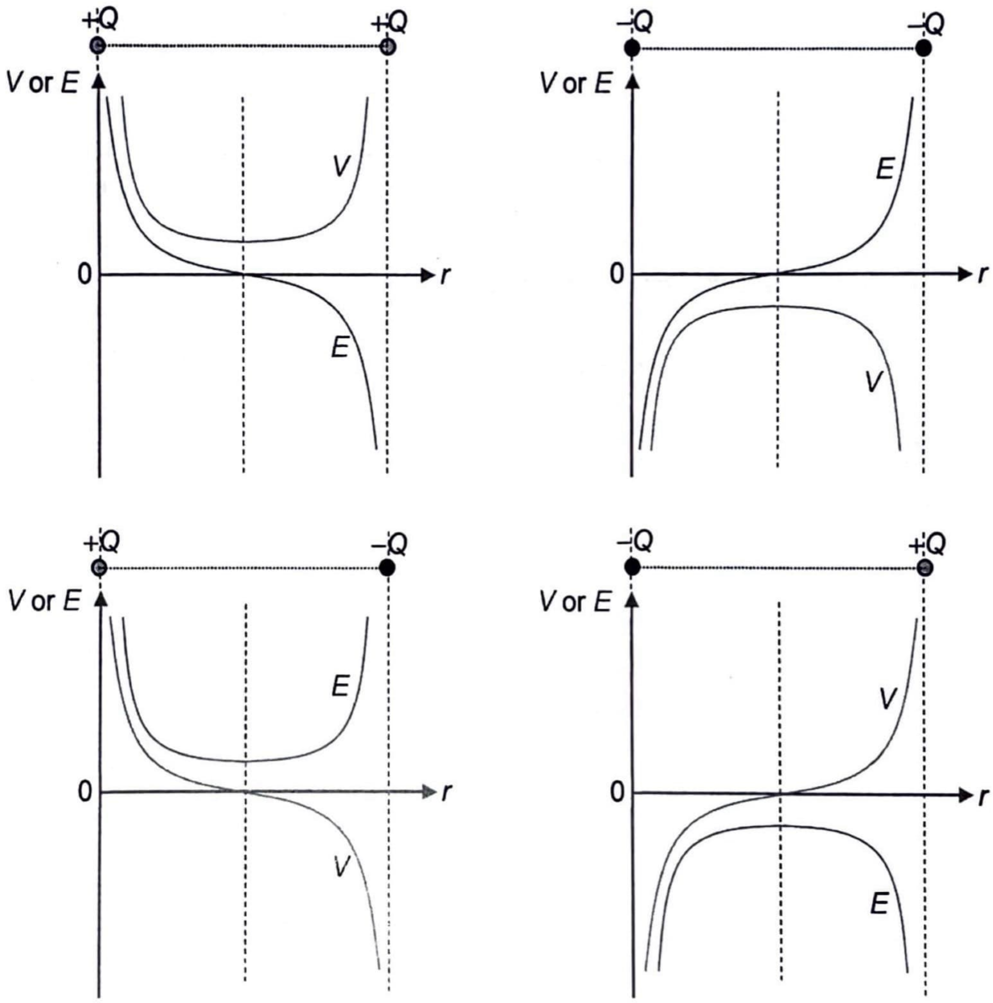
Total V and Resultant E

Electric potential is a **scalar** quantity. The **total potential** at a point between two point charges is an **algebraic sum** of the potentials due to each point charge at that point.

Electric field strength is a **vector** quantity. The **resultant field strength** at a point between two point charges is a **vector sum** of the field strengths due to each point charge at that point. The **sign** of the resultant field strength indicates its **direction**. Conventionally, positive indicates direction to the right and negative indicates direction to the left.

The relationship $E = -\frac{dV}{dr}$ still applies to the graphs for the total potential and resultant field strength between two point charges.

Possible Scenarios



13.4 Equipotential Lines and Surfaces

13.4.1 General Cases

Work Done and Equipotential Lines

The work done by a force F over a displacement s is given by

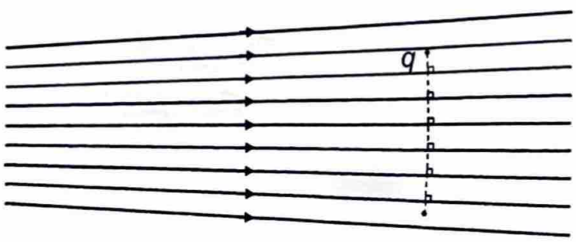
$$\text{work done} = Fs\cos\theta,$$

where θ is the angle between the vectors F and s. If F and s are perpendicular to each other, i.e. $\theta = 90°$, the work done is zero.

Consider a charged particle q being moved by an external force within an electric field as shown in the diagram. The electric force is parallel or antiparallel to the electric field lines.

If the trajectory (dotted line) of the charged particle q is perpendicular to the electric field lines at every point, the trajectory will also be perpendicular to the electric force.

Hence the work done by the external force against the electric force is zero. This, in turn, implies that the electric potential change along this trajectory is zero, i.e., the whole trajectory is at the same potential.



Such a trajectory traces out what we call an **equipotential line**, where all points on this line are at the same potential. In 3D space, equipotential lines form equipotential surfaces where all points on the same surface have the same potential.

As discussed above, **equipotential lines are perpendicular to electric field lines at every point. Movement along an equipotential line or surface requires no work done against the electric force**, since such a path is always perpendicular to the electric field lines.

Equipotential lines are like contour lines on a map which trace lines of equal altitude. In this case, the "altitude" is the electric potential. **Dashed lines without arrowheads** (potential is a scalar) are used to represent **equipotential lines**, while solid lines with arrowheads (field strength is a vector) are electric field lines. When a set of equipotential lines are drawn, **the potential difference between consecutive equipotential lines is usually a fixed value.**

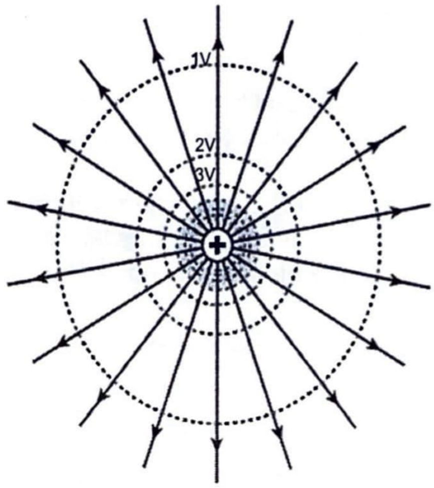
13.4.2 Point Charges

Non-uniform Field

In 3D, spherical equipotential surfaces surround a point charge and are centred on the charge.

In 2D, the equipotential lines around a point charge are concentric circles centred on the charge (a slice of the 3D spherical equipotential surfaces).

The dashed lines represent potential at equal increments – the equipotential lines or surfaces **get further apart with increasing distance** from the charge, because the **field gets weaker and the field strength decreases.**



13.5 Uniform Electric Fields

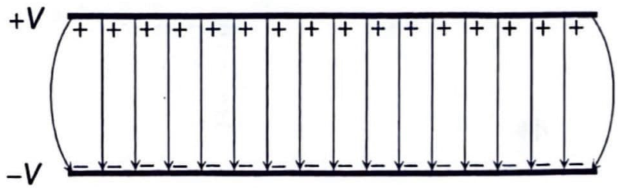
Constant E, F and a

Two parallel plates of infinite dimensions, each having a different electric potential, produce a uniform electric field between them.

Just like the uniform gravitational field close the to the surface of the Earth, the electric field lines of a uniform field are:

- **parallel** and **evenly spaced**
- and represent field strengths of the **same direction** and **same magnitude** at all points.

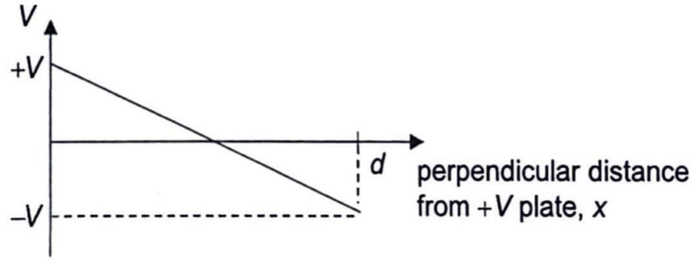
The diagram below shows the electric field lines between two equally but oppositely charged parallel plates. As real-life plates have finite dimensions, the field at the ends of the plates are not uniform due to the fringing effects. The electric field lines point **from** the plate with **higher potential** plate **to** the plate with **lower potential**.



Note

Instead of labelling the plates in terms of charges, we often label them in terms of potential instead. For e.g. $+V$ and $-V$.

Since E of a uniform field is constant and $E=-\frac{dV}{dr}$, the potential gradient $\frac{dV}{dr}$ is also constant. This implies that electric potential V is a linear function of the distance x between the two plates as shown below.



The magnitude of the field strength E can be found by calculating the potential gradient i.e.

Formula

$$|E|=\left|-\frac{dV}{dx}\right|=\frac{|\Delta V|}{d}$$

where ΔV is the potential difference between the two plates and d is the perpendicular distance between the plates.

Charged Particle in Uniform Electric Field

When a particle of charge q of mass m is placed in a uniform electric field of field strength E, a constant electric force F acts on the charge where,

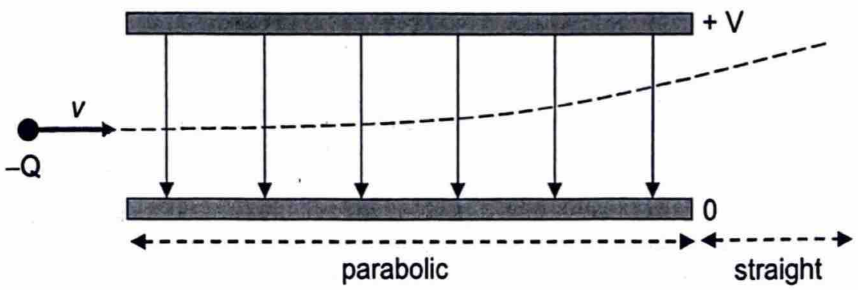
$$F = qE$$

Formulae

If the constant electric force F is the only force acting on the particle, the acceleration a of the particle is also constant. Thus, the kinematics equations of motion can be applied.

$$a = \frac{F}{m} = \frac{qE}{m}$$

Since the acceleration of the particle is constant, if the particle is projected with a component of speed in the direction parallel to the plates, it will move in a parabolic path as shown below. This is similar to a mass moving in a parabolic path in a uniform gravitational field.



Note

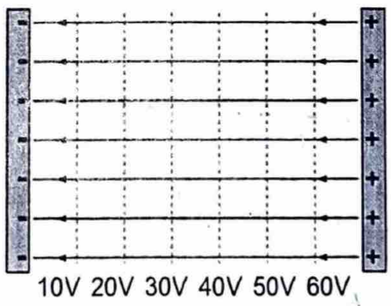
1. One common mistake is to apply formulae such as $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ and $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ to uniform fields. These formulae only apply to non-uniform field of point charges.

2. As the electric force acting on charged particles such as protons and electrons are much larger than the gravitational force acting on them due to their very small masses, i.e. $qE >> mg$, we usually do not consider the weight of these sub-atomic particles when calculating the acceleration or net force acting on them.

Equipotential Lines

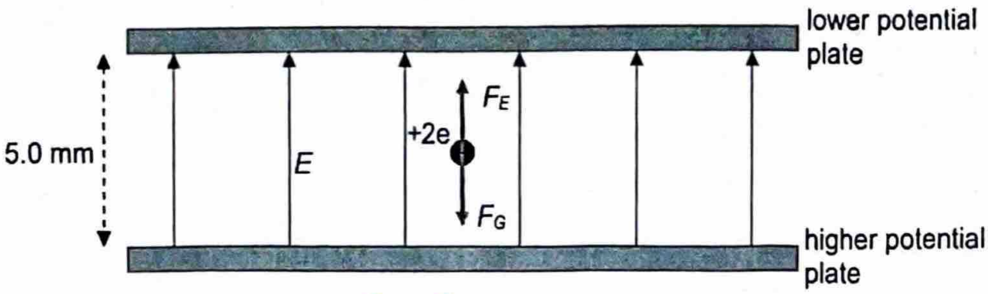
For a uniform electric field between charged parallel plates, the equipotential lines are perpendicular to the field lines and hence parallel to the plates.

Equipotential lines of **equal interval** are **equally spaced** since the **field strength is the same everywhere**.



Example 7 A particle of mass 2.0×10^{-15} kg and charge $+2e$ remains stationary within two parallel plates maintained at a potential difference V and separated by a vertical distance of 5.0 mm. By considering the forces acting on the particle, determine V.

(elementary charge $e = 1.60 \times 10^{-19}$ C)



Solution:

$$F_E = F_G$$

$$qE = mg$$

$$q\frac{V}{d} = mg$$

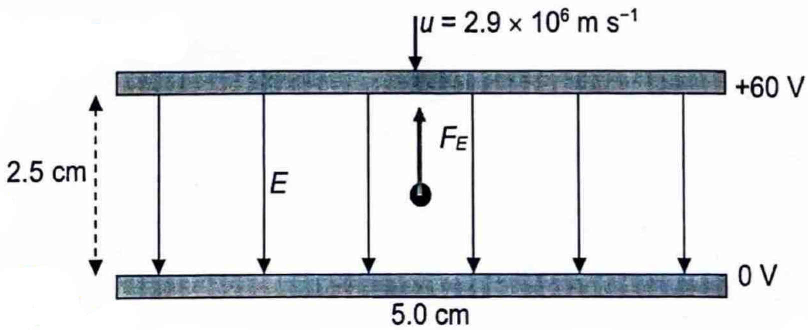
$$V = \frac{mgd}{q} = \frac{(2.0 \times 10^{-15})(9.81)(5.0 \times 10^{-3})}{2 \times 1.60 \times 10^{-19}} = 306.56 = 307 \text{ V}$$

Example 8 Two horizontal plates of length 5.0 cm are placed 2.5 cm apart in a vacuum. The upper plate is maintained at a potential of +60 V relative to the lower plate.

An electron is injected vertically through a hole in the upper plate with a speed of 2.9×10^6 m s^{-1} towards the lower plate.

Determine the distance s from the upper plate where the electron is momentarily at rest.

(rest mass of electron $m_e = 9.11 \times 10^{-31}$ kg, elementary charge $e = 1.60 \times 10^{-19}$ C)



Solution:

In the uniform field, the electron experiences a constant upward electric force F_E:

$$F_E = qE = q\frac{\Delta V}{d} = (1.60 \times 10^{-19})\left(\frac{60}{2.5 \times 10^{-2}}\right) = 3.84 \times 10^{-16} \text{ N}$$

As the electron moves away from the upper plate, its kinetic energy decreases until it is momentarily at rest.

By the principle of conservation of energy,
decrease in KE = increase in EPE = work done against electric field

$$\frac{1}{2}mu^2 - 0 = F_E \times s$$

$$s = \frac{mu^2}{2F_E} = \frac{(9.11 \times 10^{-31})(2.9 \times 10^6)^2}{2(3.84 \times 10^{-16})} = 0.0099759 = 9.98 \times 10^{-3} \text{ m}$$

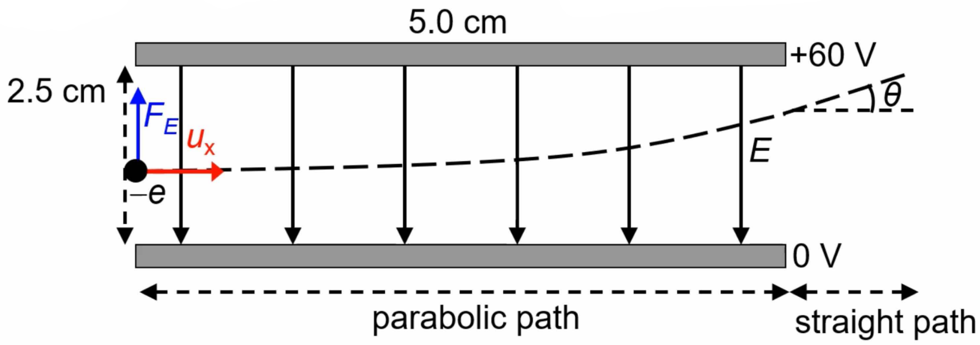
Example 9 An electron is projected horizontally between the plates in Example 8 with an initial speed of 7.5×10^6 m s^{-1}. Neglecting any gravitational and edge effects,

(a) describe and explain the path taken by the electron while moving between the plates and after leaving the plates,

(b) determine the angle the electron makes with the horizontal upon leaving the plates.

(rest mass of electron $m_e = 9.11 \times 10^{-31}$ kg, elementary charge $e = 1.60 \times 10^{-19}$ C)

Solution:



Since the plates are parallel, the electric field between them is uniform.

The electron will experience a constant acceleration (due to the constant electric force) vertically upwards towards the upper positive plate, while its horizontal velocity remains constant.

Hence, the electron will move in a parabolic path towards the positive plate.

Beyond the parallel plates, the electron moves along a straight path.

Horizontally, taking direction to the right as positive,

time taken to travel through the plates

$$t = \frac{s_x}{u_x} = \frac{5.0 \times 10^{-2}}{7.5 \times 10^6} = 6.667 \times 10^{-9} \text{ s}$$

Vertically, taking direction upwards as positive,

$$F_E = ma \quad \Rightarrow \quad qE = m_e a_y$$

$$a_y = \frac{q(\Delta V/d)}{m_e} = \frac{1.60 \times 10^{-19}\left(60/(2.5 \times 10^{-2})\right)}{9.11 \times 10^{-31}} = 4.215 \times 10^{14} \text{ m s}^{-2}$$

$$v_y = u_y + a_y t = 0 + (4.215 \times 10^{14})(6.667 \times 10^{-9}) = 2.810 \times 10^6 \text{ m s}^{-1}$$

$$\text{Hence, } \tan\theta = \frac{v_y}{v_x} = \frac{2.810 \times 10^6}{7.5 \times 10^6} \quad \Rightarrow \quad \theta = 20.539 = 20.5°$$

13.6 Electric Fields and Conductors

13.6.1 Electric Field Inside a Charged Conductor

Zero E and constant V

Any excess charges added to a conductor will redistribute themselves **uniformly** onto the **surface** of the conductor until **no excess charge is present inside** the conductor. This is because the repulsion between charges of the same sign pushes the charges as far away as possible; eventually (but quickly), they all end up on the surface. It can also be understood from the energy perspective. The potential energy of two point charges q_1 and q_2 a distance r from each other is $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$, which is positive if the two charges are of the same sign. Thus, getting as far away from each other as possible (which maximises r) helps to reduce the energy of the system.

Inside the conductor, the **electric field strength is zero**, and the **entire conductor is at the same potential**, when electrostatic equilibrium is reached (i.e. when all excess charges finally settle down on the surface). This is because as long as the electric field inside the conductor is non-zero (which is the case when the excess charges are initially added), the electric force will drive the charges towards a distribution which weakens the electric field inside the conductor. This process lasts until the electric field inside the conductor is zero, when the collective movement of the electrons inside ceases.

The above mechanism can also be understood from the energy point of view. When the excess charge is just added to the conductor, the entire conductor is not at the same potential. The non-zero potential gradient (which implies the non-zero electric field, according to $E = -dV/dr$) inside the sphere drives free electrons from regions of high potential energy to regions of low potential energy, a process that reduces the potential gradient (electrons move from low potential to high potential). This collective movement thus decreases the energy of the system. It ceases when **the entire conductor is at the same potential**, at which point the potential energy of the entire conductor is at a minimum.

Note that the two characteristics of a charged conductor in electrostatic equilibrium, namely "zero field inside" and "entire conductor being at the same potential" are equivalent to each other. This is because $E = -dV / dr$.

13.6.2 Electric Field of a Charged Conducting Sphere

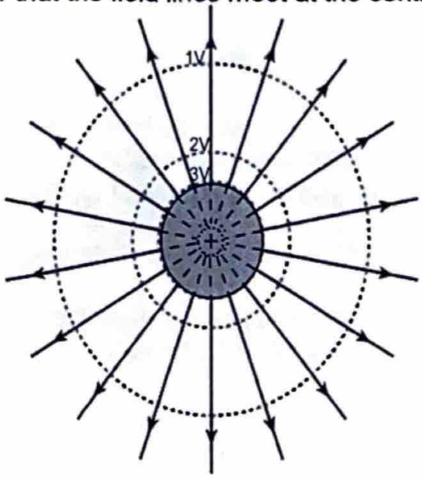
Variation of E and V with r

Outside of a **charged solid** or **hollow conducting sphere**, due to its spherical symmetry, it can be shown that the electric field behaves as though **all its charges are concentrated at its centre**. Therefore, we can use the equation for point charges to determine the electric field strength outside the sphere.

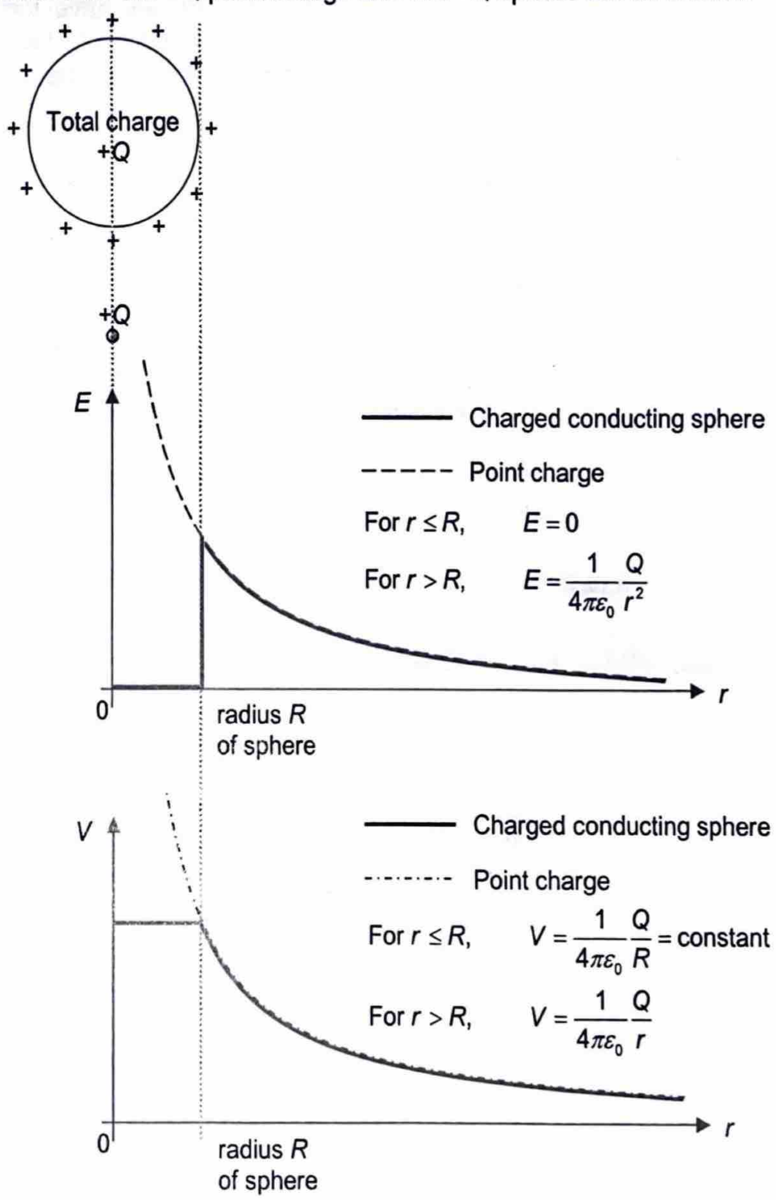
Consider a point charge $+Q$ and a sphere with charge $+Q$.

Since the electric field inside the sphere is zero, there are no electric field lines within the sphere. The electric field lines outside the sphere are similar to that of a point charge of equal charge in the centre of the sphere.

The following diagram shows the +Q point charge in the centre of the +Q sphere. Field lines outside the sphere seem to originate from the centre of the sphere where the point charge is. There are no field lines within the sphere, the dotted lines within the sphere are to show that the field lines meet at the centre of the sphere.



The variation of the electric field strength E, and that of the electric potential V, with distance r for the +Q point charge and the +Q sphere are as shown.



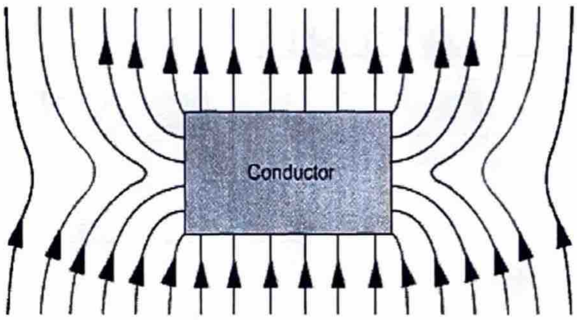
13.6.3 Conductor in an External Electric Field

Effect on Electric Field Lines

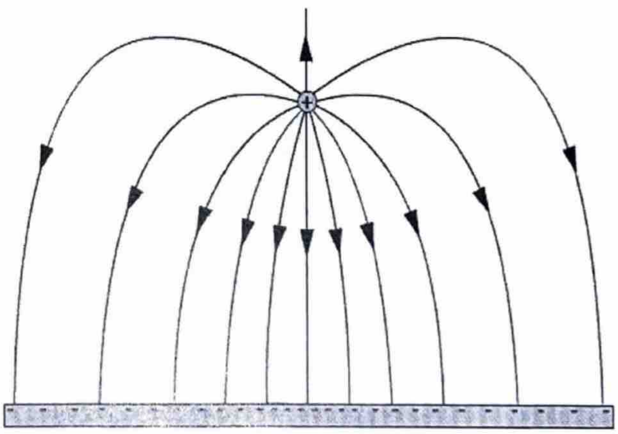
For an isolated (i.e. not connected in a closed electrical circuit) **solid or hollow conductor** in an external electric field, the free electrons within it will redistribute themselves onto the surface quickly, until the **electric field within the conductor is zero** and the **entire conductor is at the same potential**, at which point the collective movement of electrons will cease (electrostatic equilibrium is achieved). This is due to the same mechanism that resulted in zero field inside a charged conductor, which we described earlier.

Therefore,

- there are no electric field lines within the conductor
- electric field lines must start or end at the surface of the conductor
- electric field lines are perpendicular to the surface of the conductor since the surface is an equipotential surface – otherwise, there will be a component of the field strength that is tangential to the surface which will cause the charges to move and redistribute until the net force on them is zero.



Electric field lines near the surface of a neutral conductor placed within a uniform electric field



Electric field lines near the surface of an earthed conductor placed in the non-uniform electric field of a positive point charge

13.7 Electric Fields & Gravitational Field: Comparison & Summary

GENERAL RELATIONSHIPS

	Electric field	Gravitational field
Strength and Force	$E = \frac{F_E}{q}$	$g = \frac{F_G}{m}$
Force and Potential Energy	$F_E = -\frac{dU_E}{dr}$	$F_G = -\frac{dU_G}{dr}$
Strength and Potential	$E = -\frac{dV}{dr}$	$g = -\frac{d\phi}{dr}$
Potential and Potential energy	$V = \frac{U_E}{q}$	$\phi = \frac{U_G}{m}$

POINT CHARGES / POINT MASSES

	Electric field	Gravitational field
Force (vector)	$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$	$F = -G\frac{Mm}{r^2}$
	• can be attractive or repulsive depending on the charges	• M and m are positive values • always attractive (implied by the negative sign)
Field Strength (vector)	$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$	$g = -G\frac{M}{r^2}$
	• points away from positive charge • points towards negative charge	• always points towards the mass
Potential Energy (scalar)	$U = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$	$U = -G\frac{Mm}{r}$
	• signs of Q and q must be included in the expression	• M and m are positive values • always negative
Potential (scalar)	$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$	$\phi = -G\frac{M}{r}$
	• sign of Q must be included in the expression	• always negative

UNIFORM FIELDS

	Electric field between parallel plates	Gravitational field near Earth surface
Force	$F = qE$	$F = mg$
magnitude of Strength	$E = \frac{\Delta V}{d}$	$g = \frac{F}{m}$
	• points from higher potential plate to lower potential plate • *E* is constant in uniform field i.e. *V* changes linearly with distance	• always point towards the mass • *g* is constant in uniform field
Acceleration	$a = \frac{F}{m}$	$a_{freefall} = g$ *If effect of circular motion is considered,* $a_{freefall} = g - a_{centripetal}$
	• *a* due to electric force is constant • can apply kinematics equations of motion	• *a* due to gravitational force is constant • can apply kinematics equations of motion
Path of charge / mass	• +ve charge will accelerate towards lower potential plate • –ve charge will accelerate towards higher potential plate • if motion of charge has a perpendicular component to the field, it will move in a **parabolic** path	• mass will accelerate towards Earth's surface • if motion of mass has a perpendicular component to the field, it will move in a **parabolic** path
change in Potential Energy	$U = q\Delta V$	$U = mg\Delta h$
	• sign of *q* must be included in the expression • $\Delta V = V_{final} - V_{initial}$ • If *U* is +ve, there is a gain in electric potential energy • if *U* is –ve, there is a loss in electric potential energy	• $\Delta h = h_{final} - h_{initial}$ • if *U* is +ve, there is a gain in gravitational potential energy • if *U* is –ve, there is a loss in gravitational potential energy

Appendix A

Millikan's Oil-Drop Experiment

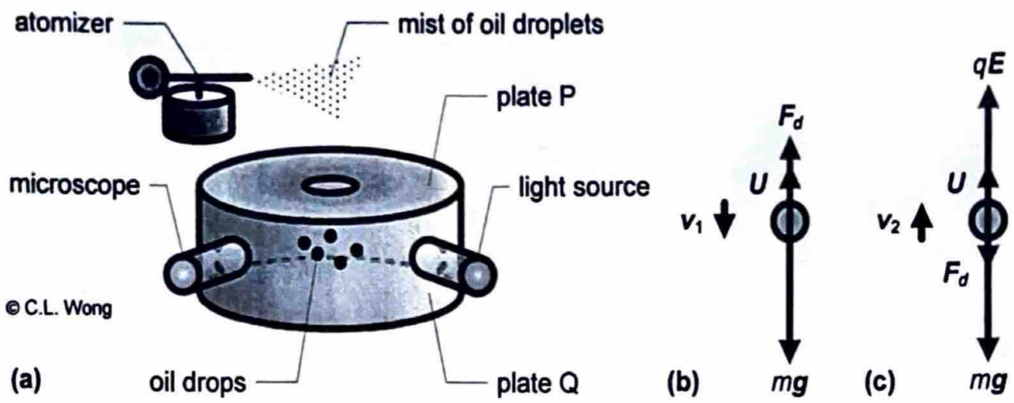


Fig. (a) shows a **simplified scheme** of the Millikan's oil-drop experiment, first used by Robert Millikan and Harvey Fletcher to determine the electric charge of the electron in 1909. By measuring the terminal speeds of the oil droplet between the plates, its electric charge can be measured. Robert Millikan was awarded the Nobel Prize in physics in 1923.

Fig. (b): **Without an electric field,** the oil droplet falls with terminal speed v_1.
An atomizer is used to produce tiny oil droplets which fall through a hole in the upper plate P. Frictional effects in the atomizer caused the oil droplets to be charged; some positively and others negatively. The droplets are illuminated by a light source and observed using a microscope.
First, we can determine the radius r and hence the weight W of an observed oil droplet by allowing it to fall with terminal speed. The drag force F_d acting on the oil droplet is given by Stokes' law: $F_d = 6\pi\eta r v$, where v is the terminal speed and η is the viscosity of air.

When the oil droplet is falling with terminal speed v_1, $\frac{4}{3}\pi r^3 \rho_{oil} g = \frac{4}{3}\pi r^3 \rho_{air} g + 6\pi\eta r v_1$

$$\therefore r = \sqrt{\frac{9\eta v_1}{2g(\rho_{oil} - \rho_{air})}}.$$

Fig. (c): **With an electric field,** the oil droplet rises with terminal speed v_2.
A uniform electric field is set up between plates P and Q by adjusting the potential difference V between the plates using a potentiometer circuit (not shown). The electric field strength E between the plates is $E = \frac{V}{d}$, where d is the plate separation.

The potential difference V causes the oil droplet to rise with a terminal speed v_2, such that

$$\frac{4}{3}\pi r^3 \rho_{oil} g + 6\pi\eta r v_2 = \frac{4}{3}\pi r^3 \rho_{air} g + qE$$

The electric charge q on the oil droplet can then be found by measuring v_2.

By repeating the procedure for different oil droplets carrying different charges, Robert Millikan was able to prove that

- the fundamental unit of electric charge e is 1.60×10^{-19} C;
- any charge q is always an integer multiple of e, i.e., electric charge is quantized;
- an electron is negatively charged.

Appendix B

Linear Accelerator (LINAC)

The most common method of accelerating a charged particle from rest is to use a uniform electric field.

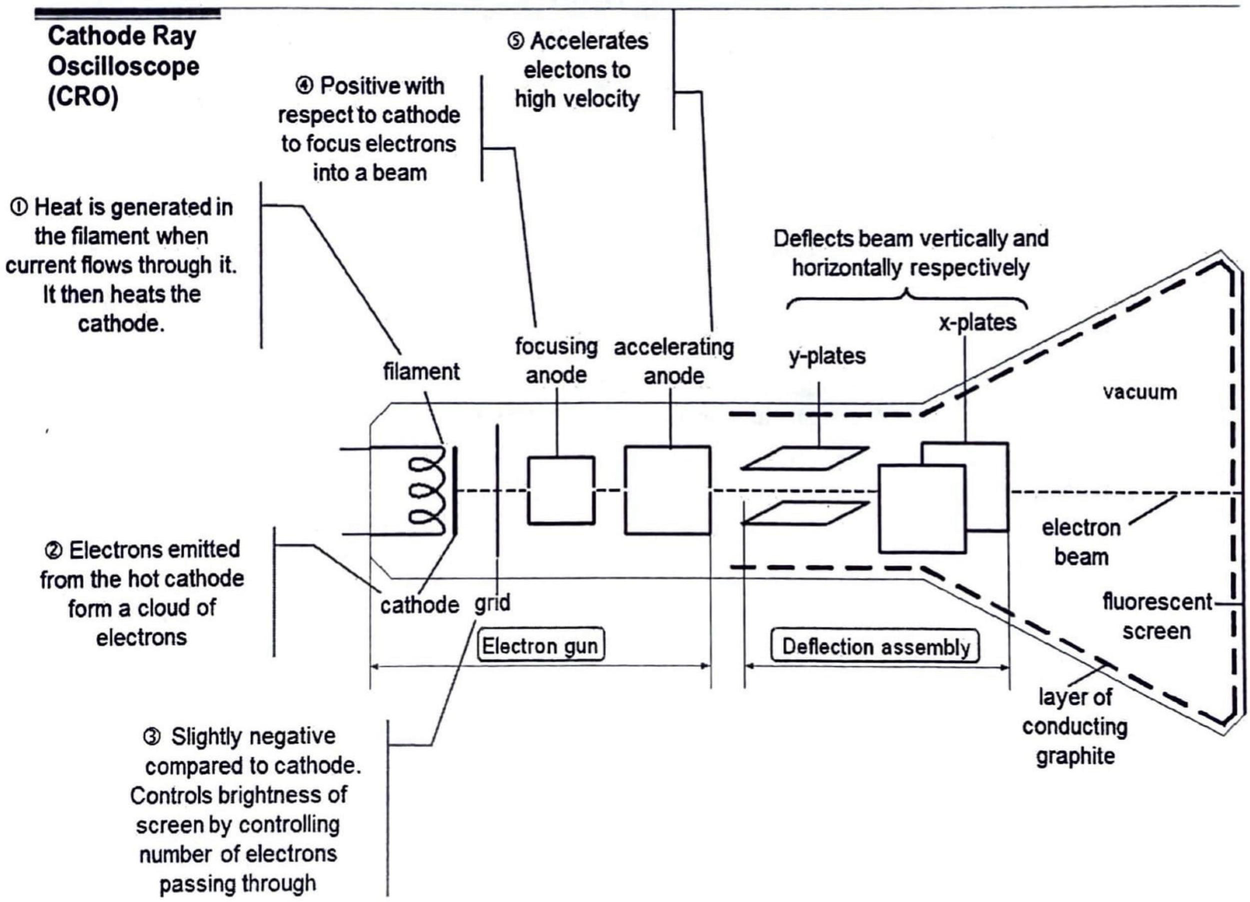
When a particle of charge Q and mass m is accelerated from rest through a potential difference V and distance d, its acceleration is $a = \frac{F_E}{m} = \frac{QE}{m} = \frac{QV}{md}$.

The kinetic energy it gained is $\Delta E_K = QV$,

and its final velocity is $\frac{1}{2}m\left(v_f^2 - v_i^2\right) = QV \Rightarrow v_f = \sqrt{\frac{2QV}{m}}$, if $v_i = 0$.

Appendix C

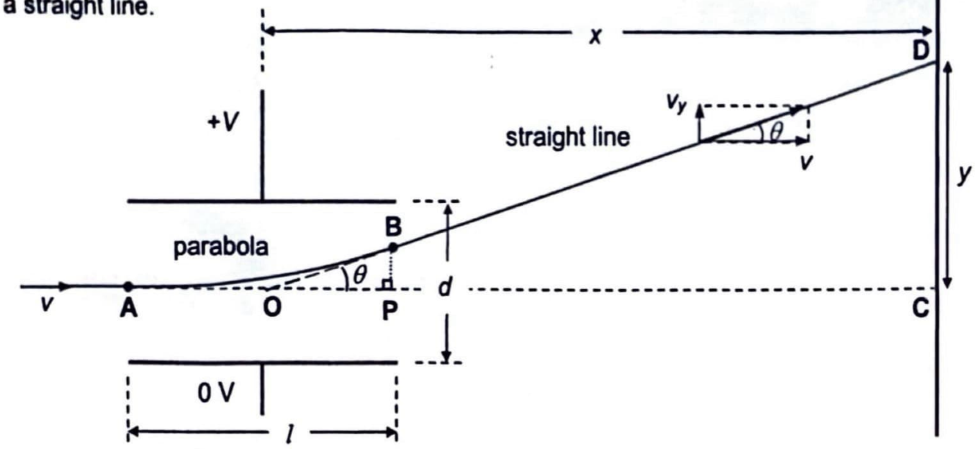
Cathode Ray Oscilloscope (CRO)



The main features of a cathode ray oscilloscope

Deflection by parallel plates

Consider a beam of electrons travelling between two horizontal parallel plates as shown below. The electrons experience a constant upward acceleration between the plates, causing them to move in a parabolic path with the component of the velocity perpendicular to the electric field being constant. On exiting the region between the plates, the electrons travel in a straight line.



Deflection of an electron beam through parallel plates

Within the parallel plates, an electron will experience an upward electrostatic force. This is the net force acting on the electron (as the weight of the electron is negligible compared to the electrostatic force acting on it).

$$F_E = eE = m_e a \quad \therefore a = \frac{eE}{m_e}$$

Horizontally, there is no acceleration, hence $u_x = v_x = v$. Using $s_x = u_x t + \frac{1}{2}a_x t^2$ the time taken for the electrons to traverse the length of the plates is

$$l = vt + 0 \quad \Rightarrow \quad t = \frac{l}{v}$$

Considering its vertical motion, its vertical velocity v_y after emerging from the electric field is

$$v_y = u_y + a_y t = 0 + \left(\frac{eE}{m_e}\right)\left(\frac{l}{v}\right) = \frac{eEl}{m_e v}$$

The electron traverses a parabolic path AB. When it emerges from the electric field, it no longer experiences an electrostatic force and hence travels in a straight line at an angle θ to the horizontal given by

$$\tan\theta = \frac{v_y}{v_x} = \frac{\left(eEl/m_e v\right)}{v} = \frac{eEl}{m_e v^2}$$

By using $s_y = u_y t + \frac{1}{2}a_y t^2$, the vertical displacement of the electron is

$$\text{BP} = 0 + \frac{1}{2}\left(\frac{eE}{m_e}\right)\left(\frac{l}{v}\right)^2 = \frac{eEl^2}{2m_e v^2}$$

Since $\tan\theta = \frac{\text{BP}}{\text{OP}} = \frac{eEl}{m_e v^2}$ and $\text{BP} = \frac{eEl^2}{2m_e v^2} \Rightarrow \text{OP} = \frac{eEl^2}{2m_e v^2} \div \frac{eEl}{m_e v^2} = \frac{l}{2}$

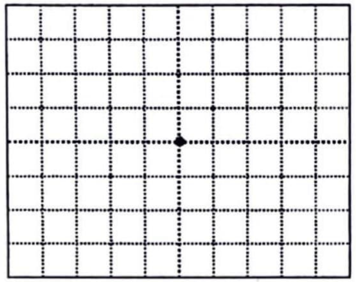
The deflected electron will eventually strike the screen at point D. ΔOBP and ΔODC are similar triangles, hence

$$\frac{y}{x} = \frac{\text{BP}}{\text{OP}} = \tan\theta \quad \Rightarrow \quad y = x\left(\frac{eEl}{m_e v^2}\right) = \frac{xe\left(V/d\right)l}{m_e v^2} = \left(\frac{xel}{m_e v^2 d}\right)V$$

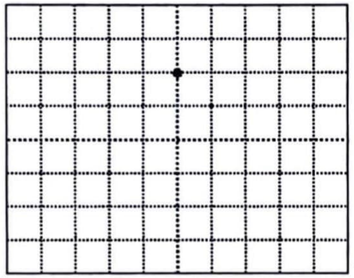
Hence, deflection y of the electrons is proportional to the p.d. V between the parallel plates.

How to read a CRO *(with the time-base off)*

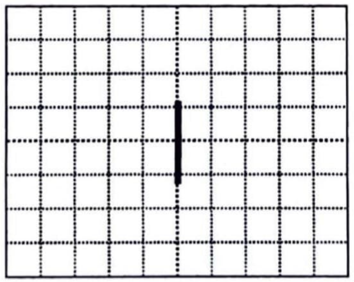
When the CRO is switched on, a spot of light appears at the centre of the screen if no voltage is connected to the *y*-input and the time base is switched off.



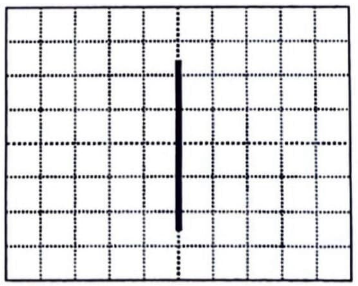
When a d.c. voltage is connected to the *y*-input, the spot of light is deflected as shown.
The CRO commonly found in the laboratory are already calibrated.
y-sensitivity = 2.0 V div^{-1}
$V_{d.c.}$ = 4.0 V



The voltage of an a.c. supply varies from $+V_0$ to $-V_0$ where V_0 is the amplitude.
When applied to the y-input, the spot oscillates with frequency equals to the frequency of a.c. supply. A vertical line is seen on the screen.
y-sensitivity = 2.0 V div^{-1}
V_0 = 2.5 V

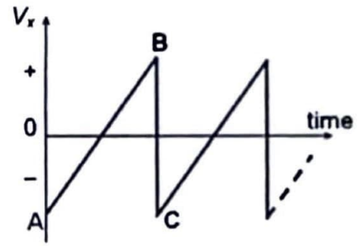


The screen on the right is obtained when the *y*-sensitivity is adjusted.
y-sensitivity = 1.0 V div^{-1}
V_0 = 2.5 V

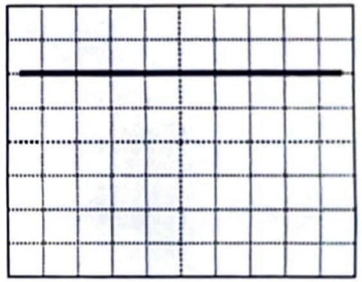


How to read a CRO *(with the time-base on)*

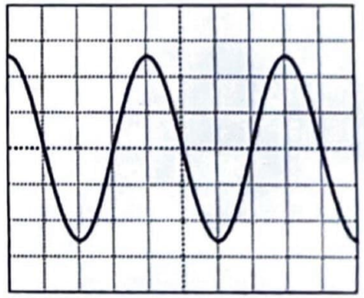
When the time-base is switched on, an internally generated saw-tooth voltage shown on the right is applied to the *x*-plates of the CRO.
As voltage V_x varies linearly with time from A to B, the spot moves at a constant speed from left to right of the screen. When the voltage changes suddenly from B to C, the spot returns to the left almost instantaneously.



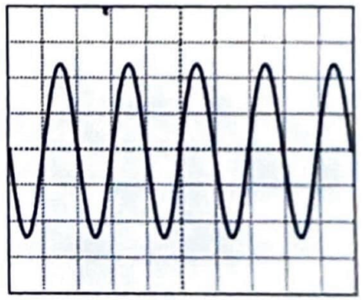
When a d.c. voltage is connected to the *y*-input and the time base is switched on, the spot of light will be deflected from left to right.



When an a.c. voltage is applied to the *y*-plates with the time-base switched on, the spot moves up and down as well as across the screen.
time-base = 1 ms div^{-1}
period = 4 ms
frequency = 250 Hz



The same a.c. voltage is applied to the *y*-input but the time base is adjusted.
time-base = 2 ms div^{-1}
period = 4 ms



Tutorial 13 ELECTRIC FIELDS

permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}
elementary charge	$e = 1.60 \times 10^{-19}$ C
rest mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
rest mass of proton	$m_p = 1.67 \times 10^{-27}$ kg
unified atomic mass constant	$u = 1.66 \times 10^{-27}$ kg

Self-Check Questions

S1 Define electric field strength and electric potential at a point.

S2 Sketch the electric field lines for
(a) two point charges of equal magnitude and same polarity.
(b) two point charges of equal magnitude and opposite polarity.

S3 Describe and explain the motion of a positively charged particle that enters a uniform electric field with a velocity that is perpendicular to the direction of the electric field.

S4 State the relationship between the electric field strength E at a point in an electric field and the potential V at that point.

S5 How is the magnitude of the electric field strength between charged parallel plates determined?

S6 What are the similarities and differences between electric force and gravitational force?

Self-Practice Questions

SP1 [J85/I/18]

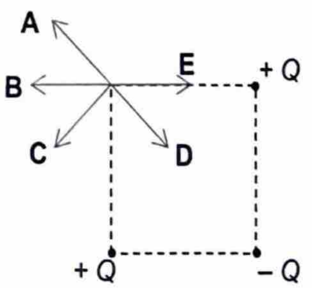
Two small conducting spheres A and B are hung by light, non-conducting threads from fixed points P_A and P_B, and are at the same level. A has mass M and carries charge q; B has mass $2M$ and carries charge $2q$. The repulsion between them causes the threads to make small angles θ_A and θ_B with the vertical.

What is the approximate value of the ratio $\theta_A : \theta_B$?

A 4.0 **B** 2.0 **C** 0.5 **D** 0.25

SP2 [N89/I/16]

Point charges, each of magnitude Q, are placed at three corners of a square as shown below. What is the direction of the resultant electric field at the fourth corner?



SP3 [N87/I/19]

In the direction indicated by an electric field line,

A the electric field strength must increase.

B the electric field strength must decrease.

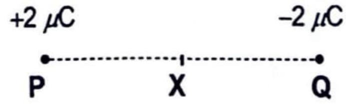
C the electric potential must remain constant.

D the electric potential must increase.

E the electric potential must decrease.

SP4 [N97/I/16]

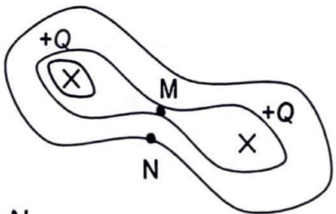
Two charges of +2 μC and –2 μC are situated at points P and Q respectively. X is midway between P and Q.



Which of the following correctly describes the electric field and the electric potential at point X?

	electric field	electric potential
A	towards **Q**	zero
B	towards **Q**	negative
C	towards **P**	zero
D	towards **P**	positive

SP5 The equipotential lines of a pair of point charges, each of charge +Q, are shown in the diagram.



If an electron moves from M to N,

A its electric potential energy falls.

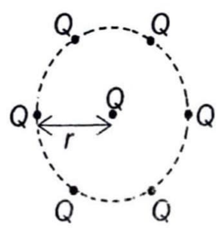
B it experiences a resultant force pulling it back to M.

C it experiences a resultant force pushing it away from M.

D it does not experience a resultant force.

SP6 [J89/I/16]

A point charge is surrounded symmetrically by six identical charges at distance *r* as shown in the diagram.



How much work is done by the forces of electrostatic repulsion when the point charge at the centre is removed to infinity?

A $\dfrac{6Q^2}{4\pi\varepsilon_0 r}$ **B** $\dfrac{6Q}{4\pi\varepsilon_0 r}$ **C** $\dfrac{6Q^2}{4\pi\varepsilon_0 r^2}$ **D** $\dfrac{6Q}{4\pi\varepsilon_0 r^2}$

SP7 [J82/II/21]

A metal sphere of radius 0.1 m was insulated from its surroundings and given a large positive charge.

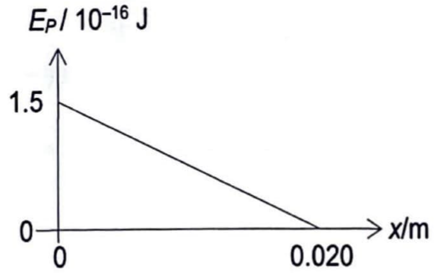
A small charge was brought from a distant point to a point 0.5 m from the sphere's centre. The work done against the electric field was *W* and the force on the small charge in its final position was *F*.

If the small charge is now placed at 1.0 m from the centre of the sphere, what are the values of work done and force?

	work done	force
A	$W/4$	$F/4$
B	$W/4$	$F/2$
C	$W/2$	$F/4$
D	$W/2$	$F/2$

SP8 [J00/I/6]

Two charged plates are 0.020 m apart, producing a uniform electric field. The potential energy E_P of an electron in the field varies with displacement x from one of the plates as shown.

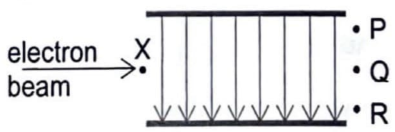


What is the magnitude of the force on the electron?

A 3.0×10^{-18} N **B** 7.5×10^{-17} N **C** 3.8×10^{-15} N **D** 7.5×10^{-15} N

SP9 [N88/I/27]

An electron enters a region in an evacuated tube in which there is a uniform electric field directed as shown in the diagram.

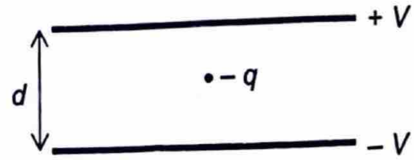


Which one of the following is a possible path for the beam?

A A curved line from X to P.
B A straight line from X to P.
C A curved line from X to R.
D A straight line from X to R.
E The straight line XQ.

SP10 [J94/I/15]

An oil droplet has a charge $-q$ and is situated between two parallel horizontal metal plates as shown in the diagram.



The separation of plates is d. The droplet is observed to be stationary when the upper plate is at potential $+V$ and the lower potential $-V$.

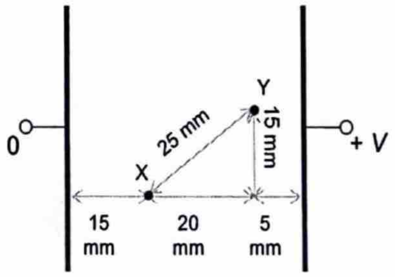
For this to occur, the weight of the droplet is equal in magnitude to

A $\frac{Vq}{d}$ **B** $\frac{2Vq}{d}$ **C** $\frac{Vd}{q}$ **D** $\frac{2Vd}{q}$

SP11 With reference to the diagram in question **SP10**, if the oil droplet acquires additional negative charge, which of the following changes should be made for the droplet to remain stationary?

A Move the plates closer together.
B Reverse the charges on the plates.
C Give the positive plate more positive charge.
D Move both plates the same distance upwards.
E Decrease the potential difference between the plates.

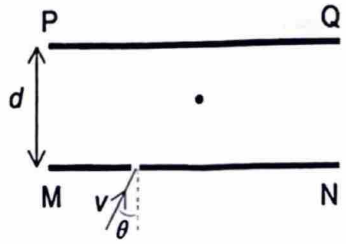
SP12 Two large plane parallel conducting plates are situated 40 mm apart as shown. The potential difference between the plates is V.



What is the potential difference between point X and point Y?

A $\frac{15}{40}V$ **B** $\frac{20}{40}V$ **C** $\frac{25}{40}V$ **D** $\frac{40}{40}V$

SP13 In an evacuated enclosure, a metal plate PQ is maintained at a negative potential V relative to a second plate MN. Electrons of velocity v enter the space between the plates as shown.



Given that the electron charge is e and that the electron mass is m_e, electrons just reach the plate PQ if

A $\frac{1}{2}m_e v^2 = \frac{eV}{d}$

B $\frac{1}{2}m_e (v\cos\theta)^2 = eV$

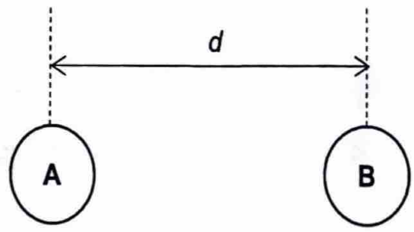
C $\frac{1}{2}m_e (v\cos\theta)^2 = \frac{eV}{d}$

D $\frac{1}{2}m_e (v\sin\theta)^2 = eV$

E $\frac{1}{2}m_e (v\sin\theta)^2 = \frac{eV}{d}$

SP14 A and B are two identical conducting spheres, each carrying a positive charge Q and are placed close to each other in a vacuum with their centres distance d apart as shown.

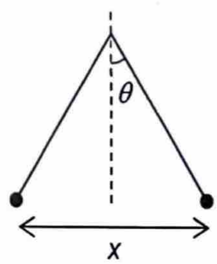
Explain why the electric force F between them is ***not*** given by the expression $F = \frac{Q^2}{4\pi\varepsilon_0 d^2}$.



[2]

SP15 Two similar conducting balls of mass m are hung from non-conducting threads of length l and carry similar charges q as shown. Using the small angle approximation $\tan\theta \approx \sin\theta \approx \theta$, show that the separation x, between the balls is given by

$$x = \left[\frac{q^2 l}{2\pi\varepsilon_0 mg}\right]^{\frac{1}{3}}.$$



[3]

SP16 Two electrons are fixed 2.0 cm apart. A third electron is shot from infinity and comes to rest midway between the two.

Determine the initial velocity of the third electron. [2]

Discussion Questions

D1 One particle has a mass of 3.00×10^{-3} kg and a charge of +8.00 μC. A second particle has a mass of 6.00×10^{-3} kg and the same charge. The two particles are initially held in place and then released. The particles fly apart, and when the separation between them is 0.100 m, the speed of the 3.00×10^{-3} kg particle is 125 m s^{-1}.

Determine the initial separation between the particles. [3]

D2 [N11/II/4]
A small metal sphere S is supported by an insulating string that is fixed at point P, as shown in Fig. 2.1.

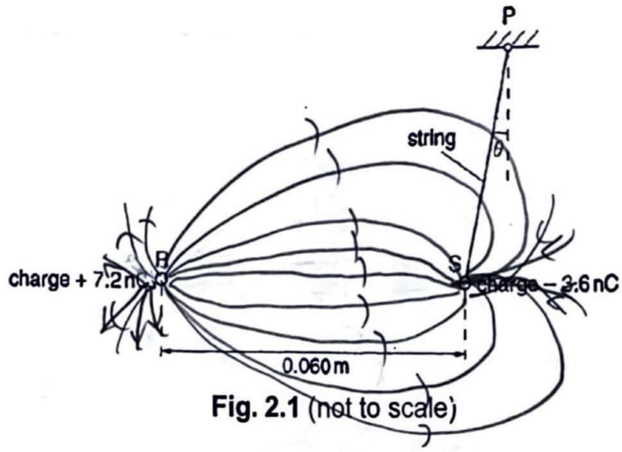


Fig. 2.1 (not to scale)

Sphere S has a weight of 0.49×10^{-3} N and charge −3.6 nC. Another metal sphere R with a charge of +7.2 nC is placed 0.060 m from S. The diameter of the spheres is negligible compared to the distance between them.

A line joining R and S is horizontal. The string makes an angle θ with the vertical.

(a) **(i)** On Fig. 2.1, draw electric field lines to represent the electric field in the region between R and S. [3]

(ii) Calculate the magnitude of the electric field strength at the mid-point between R and S. [2]

(iii) Show that the electric force acting on S is 6.5×10^{-5} N. [2]

(b) Determine the angle θ and the tension in the string. [4]

(c) The horizontal distance from R towards S is x. On Fig. 2.2, sketch the variation with x of the potential V. [2]

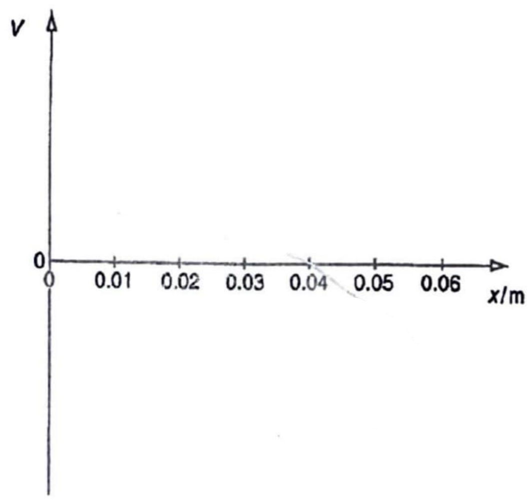


Fig. 2.2

D3 [N14/III/6(part)]

(b) Two charged solid metal spheres A and B are situated in a vacuum. Their centres are separated by a distance of 30.0 cm, as illustrated in Fig. 3.1. The diagram is not to scale.

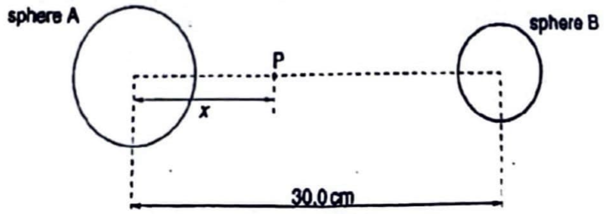


Fig. 3.1

Point P is a point on the line joining the centres of the two spheres. Point P is a distance x from the centre of sphere A.

The variation with distance x of the electric field strength E at point P is shown in Fig. 3.2.

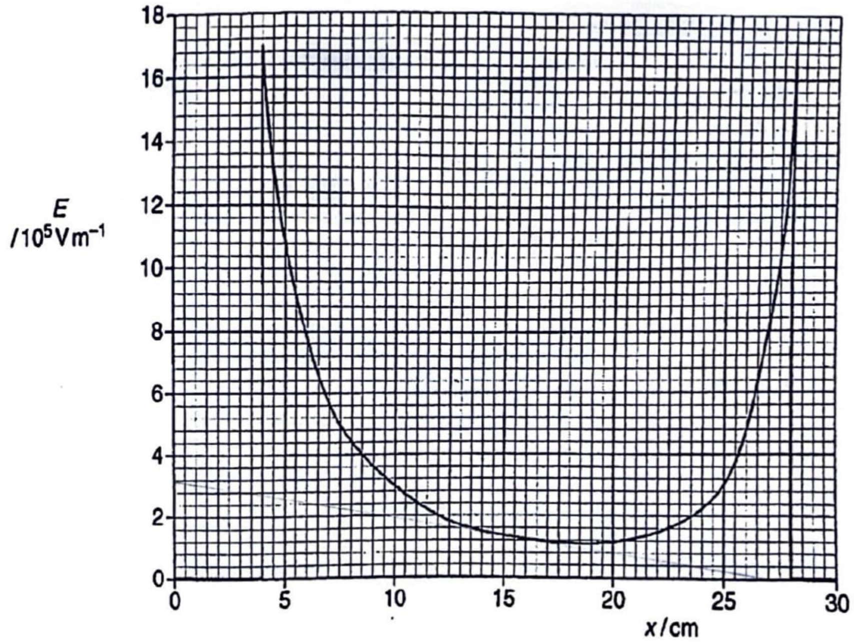


Fig. 3.2

(i) Suggest why the electric field strength is zero for two regions of x. [1]

(ii) Use Fig. 3.2 to

1. determine the radius of each sphere, [1]
2. state and explain whether the spheres have charges of the same, or opposite, sign. [2]

(iii) A lithium-7 $\left(^{7}_{3}\text{Li}\right)$ nucleus moves along the line joining the centres of the two spheres.

1. Estimate the energy gained by this nucleus as it moves from point P where $x = 16.0$ cm to the point P where $x = 21.0$ cm. [5]
2. Calculate the acceleration of the nucleus at point P where $x = 25.0$ cm. [2]
3. The nucleus is at rest at point P where $x = 4.0$ cm.
   Describe qualitatively the variation with x of the acceleration of the nucleus for $x = 4.0$ cm to $x = 28.0$ cm. [3]

D4 [N12/III/7(part)]

(b) (i) State the relation between electric field strength E and potential V. [1]

(ii) Two charged metal spheres A and B, of diameters 18 cm and 12 cm respectively, are isolated in space, as shown in Fig. 4.1.

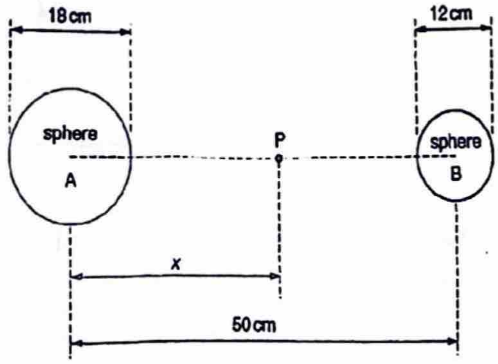


Fig. 4.1

The centres of the spheres are separated by a distance of 50 cm. Point P is at a distance x from the centre of sphere A along the line joining the centres of the two spheres.

The variation with x of the electric potential V at P is shown in Fig. 4.2.

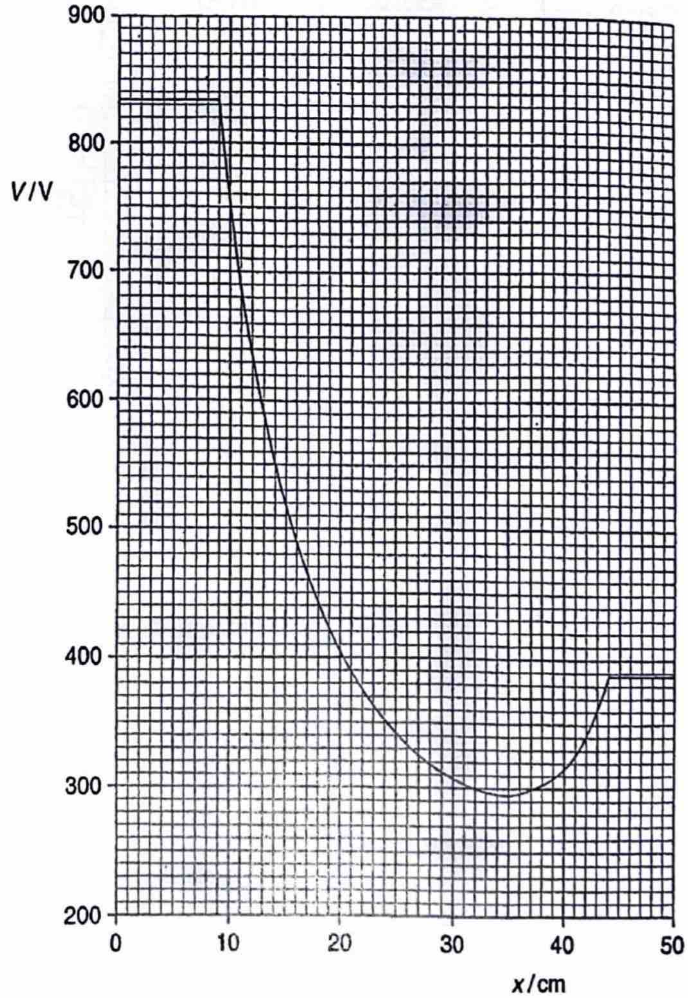


Fig. 4.2

1. State and explain the direction of the electric field at the point P, where $x = 25.0$ cm. [2]
2. Use Fig. 4.2 to determine the force on an electron placed at point P, where $x = 35.0$ cm. [3]
3. By making reference to electric fields, explain why the potential is constant for distances between $x = 0$ and $x = 9.0$ cm. [2]

(c) A student states that the potential V decreases with distance x for distances between $x = 10$ cm and $x = 25$ cm according to the expression

$$Vx = \text{constant}.$$

(i) Without drawing a graph, use data from Fig. 4.2 to show whether the student is correct. [3]

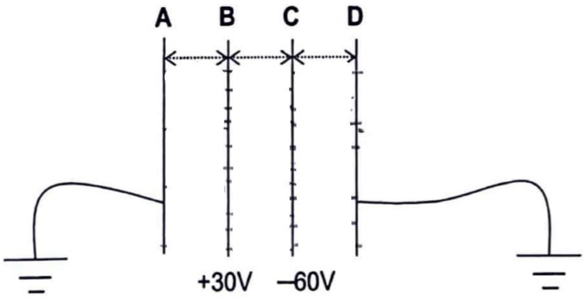
(ii) Suggest an explanation for your conclusion in **(i)**. [1]

(d) An electron, initially at rest a long distance from the spheres in **(b)**, approaches the spheres and passes between the two spheres.

(i) Calculate the minimum speed of the electron as it crosses the line joining the centres of the two spheres. [2]

(ii) Describe the path of the electron for the minimum speed in **(i)**. [2]

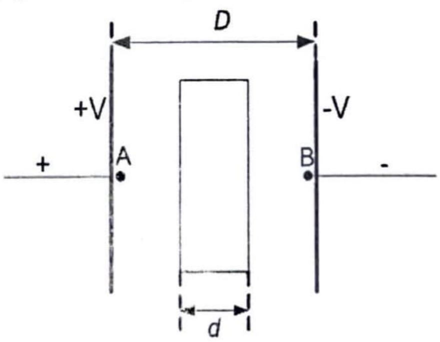
D5 Four large, square conducting plates **A**, **B**, **C** and **D**, of negligible thickness, are arranged as shown in the figure. The distance between adjacent plates is 15 mm. The outer plates **A** and **D** are earthed, **B** is maintained at a potential of +30 V, and C at –60 V.



Sketch labelled graphs to show how the electric potential and electric field strength varies along a line through the centre of the plates perpendicular to their planes.

[3]

D6 **[J87/III/6]**
Two large metal plates are oppositely charged and placed a distance D apart. A conductor of thickness d is situated centrally between the plates.



Sketch graphs, one in each case, to show the variation from A to B of

(a) the electric potential V, [2]

(b) the electric field strength E. [1]

D7 [N07/II/2]

Electrons are emitted from cathode C and are accelerated towards anode A, as shown in Fig. 7.1.

Fig. 7.1

The anode is earthed. CX is a line drawn from C normal to the anode A. The distance CX is 4.0 cm.

The variation with distance *d* from C along CX of the magnitude of the electric field strength *E* is shown in Fig 7.2.

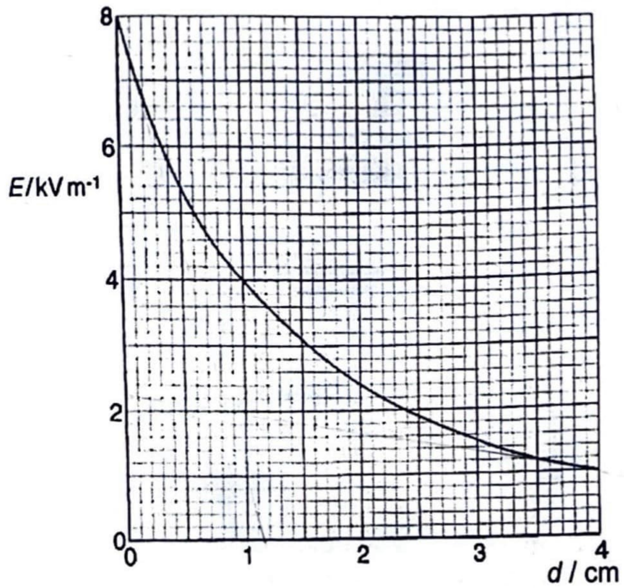


Fig. 7.2

(a) Define *electric field strength* at a point. [1]

(b) **(i)** On Fig. 7.1, mark with an arrow the direction of the electric field along CX. [1]

(ii) Use Fig. 7.2 to determine the force *F* on an electron at a point mid-way between C and X. [2]

(c) **(i)** A student assumes that the force *F* on the electron remains constant as the electron moves from C to X. Use the value of *F* calculated in **(b)(ii)** to estimate, on the basis of this assumption, the potential difference between C and X. [2]

(ii) Suggest, with a reason, whether the magnitude of the potential difference calculated in **(c)(i)** will be an over-estimate or an under-estimate of the actual potential difference. [1]

D8 [N07/III/2]

Fig. 8.1 is drawn to full scale and shows the pattern of the electric field (solid lines) in and around a pair of parallel, charged metal plates. It also shows lines joining points at the same potential (dotted lines).

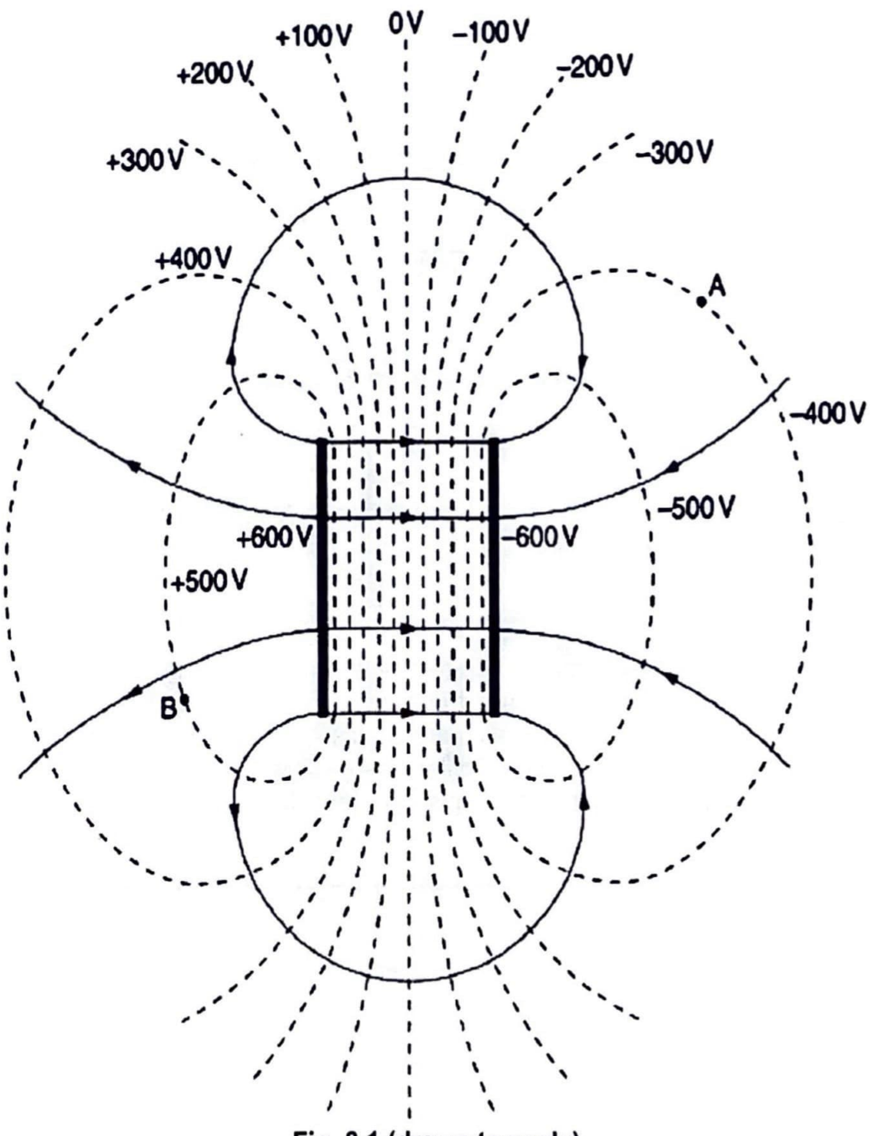


Fig. 8.1 (drawn to scale)

(a) From the diagram, deduce

(i) the value of the electric field strength between the plates, [3]

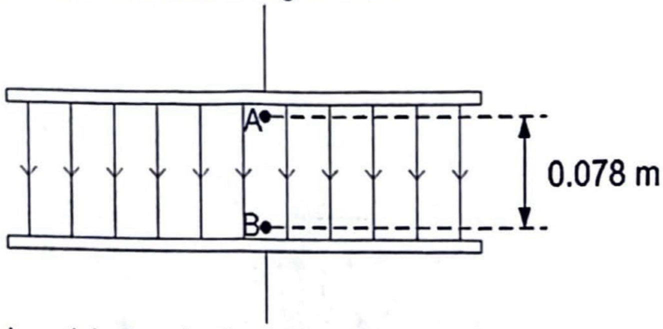
(ii) the work that needs to be done to move a charge of 8.0×10^{-19} C from point A to point B. [2]

(b) Explain why, in the absence of any other charged bodies, the potential will be zero along the centre line between the plates. [2]

D9 [J93/III/14]

(a) Calculate the force exerted on a proton, which has a charge of $+1.6\times10^{-19}$ C, when it is in a uniform field strength of 2.7×10^{5} N C^{-1}. [2]

(b) A proton is moved in a vacuum by a uniform electric field of 2.7×10^{5} N C^{-1} from A to B, a distance of 0.078 m as shown in the figure below.

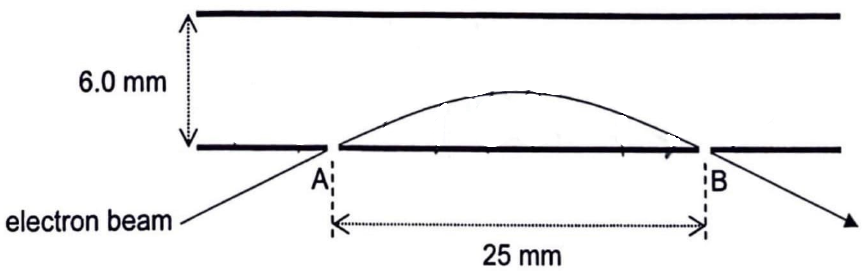


(i) How much work is done by the field on the proton? [2]

(ii) What is the gain in the kinetic energy of the proton? [1]

(iii) Calculate the difference in potential between A and B. State whether A or B is at the higher potential. [3]

D10 The figure below shows the principle of a type of velocity selector for electrons. Two parallel metal plates are arranged 6.0 mm apart in a vacuum. The lower plate is at a potential of +45 V relative to the upper, and has two parallel slits A and B in it which are 25 mm apart. A collimated beam containing electrons of different speeds is directed towards slit A at an angle of 15° with the plate. Electrons emerging from B have the same velocity. Neglect the effect of gravity.



(a) (i) What is the magnitude and direction of the acceleration of an electron while it is between the plates? [2]

(ii) Draw the electric field lines between the plates. [1]

(iii) Describe the path of the beam in between the plates. [2]

(b) Determine the speed of the electrons which come out of slit B. [3]

Challenging Questions

C1 A certain molecule consists of two singly-charged ions, A^+ and B^-. An equation for the potential energy U_P of the molecule is

$$U_P = -\frac{e^2}{4\pi\varepsilon_0 r} + \frac{C}{r^9}$$

where r is the separation of the ions and C is a positive constant.

(a) Give a physical interpretation of each of the two terms on the right-hand side of this equation. Explain why the exponent of r in the first term is –1.

(b) Given that force F and potential energy U_P are related by the expression $F = -\frac{dU_P}{dr}$, show that $C = \frac{e^2 r_0^{\ 8}}{36\pi\varepsilon_0}$, where r_0 is the equilibrium separation of the ions.

(c) Determine U_{min}, the minimum potential energy of the molecule, in terms of r_0, e and ε_0.

(d) It is known that, in this molecule, r_0 is about 0.3 nm. Deduce the energy required to break the bond between the ions.

Answers

D1 0.0141 m

D2 **(a)(ii)** 1.08×10^5 N C^{-1}

(b) 7.53° , 4.94×10^{-4} N

D3 **(b)(ii)1.** radius of A = 4.00 cm , radius of B = 2.00 cm

(b)(iii)1. 2.88×10^{-15} J , **2.** 1.24×10^{13} m s^{-2}

D4 **(b)(ii)2.** 0 N

(d)(i) 1.02×10^7 m s^{-1}

D7 **(b)(ii)** 3.84×10^{-16} N

(c)(i) 96 V

D8 **(a)(i)** 48000 V m^{-1}

(a)(ii) 7.2×10^{-16} J

D9 **(a)** 4.32×10^{-14} N

(b)(i) 3.37×10^{-15} J

(b)(ii) 3.37×10^{-15} J

(b)(iii) 21.1 kV

D10 **(a)(i)** 1.32×10^{15} m s^{-2}, downwards

(b) 8.12×10^6 m s^{-1}

C1 **(c)** $-\frac{2}{9}\frac{e^2}{\pi\varepsilon_0 r_0}$

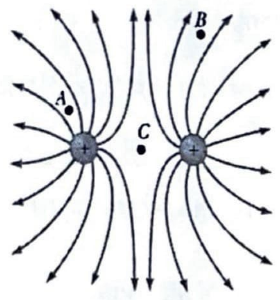
(d) 6.8×10^{-19} J

Tutorial 13 Electric Fields Suggested Solutions

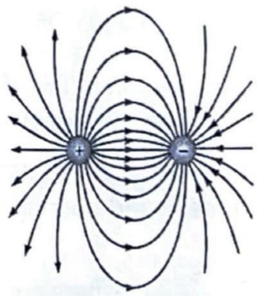
S1 The **electric field strength** at a point is defined as the electric force exerted per unit positive charge placed at that point.

The **electric potential** at a point is defined as the work done per unit positive charge by an external force in bringing a small test charge from infinity to that point.

S2 (a)



(b)



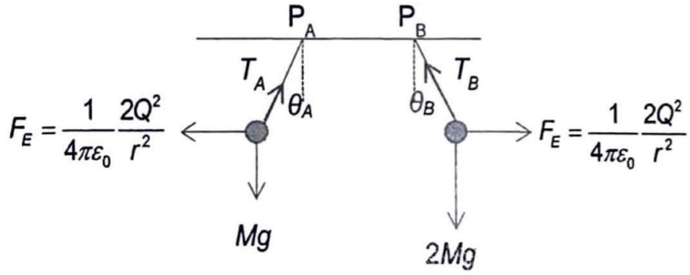
S3 The positively charged particle will experience a constant acceleration towards the plate of lower potential, while the component of the velocity perpendicular to the electric field remains constant. Thus, the path is parabolic between the plates. On exiting the region between the plates, the particle moves in a straight line.

S4 The electric field strength at a point is numerically equal to the potential gradient at a point; or $E = -\frac{dV}{dx}$.

S5 The magnitude of the uniform electric field strength between charged parallel plates is determined by the potential difference between the two plates divided by the separation between them. $|E| = \left|\frac{\Delta V}{d}\right|$

S6 **Similarity**: Both forces are inverse-square laws.
Differences: Electric force can be attractive and repulsive in nature, while gravitational force is only attractive. Electric force exists between charges, while gravitational force exists between masses.

SP1



From the triangle of forces, $\tan\theta_A = \frac{\frac{1}{4\pi\varepsilon_0}\frac{2Q^2}{r^2}}{Mg}$ & $\tan\theta_B = \frac{\frac{1}{4\pi\varepsilon_0}\frac{2Q^2}{r^2}}{2Mg}$.

Since the angles are very small, $\tan\theta \approx \theta$, $\theta_A : \theta_B \approx 2.0$.

Answer: B

SP2 If a positive test charge is placed at the fourth corner, it will be repelled by +Q and attracted by –Q. The resultant force exerted by the two +Q (direction of A) will be larger than the force exerted by –Q (direction of D). Hence, resultant force due to the three charges is in the direction of A.
Answer: A

SP3 By definition, the direction of the electric field is the direction that a positive charge will move i.e. towards lower potential, when placed at that point in the field. A positive charge will accelerate in the direction of the field and electric potential will decrease.
Answer: E

SP4 Electric Field points from high to low potential, towards Q.

$$V_X = V_P + V_Q$$
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{+2}{r} + \frac{-2}{r}\right)\times 10^{-6} = 0$$

Answer: A

SP5 $\Delta V = V_N - V_M = -\text{ve} \;\Rightarrow\; \Delta U = q\Delta V = +\text{ve}$ (option A is out)
Since E points from M to N, a –ve charge will experience a resultant force from N to M.
Answer: B

SP6
$$W_{\text{by electric force}} = -U = -q\Delta V = Q(V_i - V_f)$$
$$= Q\left(6\times\frac{Q}{4\pi\varepsilon_0 r} - 0\right) = \frac{6Q^2}{4\pi\varepsilon_0 r}$$

Answer: A

SP7
$$W \propto \frac{1}{r} \;\Rightarrow \frac{W_{1.0}}{W_{0.5}} = \frac{0.5}{1.0} = \frac{1}{2}$$
$$F_E \propto \frac{1}{r^2} \;\Rightarrow \frac{F_{1.0}}{F_{0.5}} = \left(\frac{0.5}{1.0}\right)^2 = \frac{1}{4}$$

Answer: C

SP8
$$|F| = \left|-\frac{dE_P}{dx}\right| = \frac{1.5\times 10^{-16}}{0.020} = 7.5\times 10^{-15}\text{ N}$$

Answer: D

SP9 Electrons experience a constant vertical, upward force, resulting in a parabolic path upwards.
Answer: A

SP10
$$mg = qE = q\frac{\Delta V}{d} = q\frac{(V-(-V))}{d} = \frac{2Vq}{d}$$

Answer: B

SP11 Based on above equation, the electric force on the larger charge can be reduced (to achieve equilibrium) by decreasing V or increasing d.
Answer: E

SP12 Since the field is uniform, equipotential lines are parallel to the plates and equally spaced. Potential gradient is constant.
Since 40 mm ≡ V, 20 mm ≡ 20/40 V (the slanted 25mm serves no purpose).
Answer: B

SP13 For the electron to just reach PQ, it's velocity should be solely horizontal by then (i.e. vertically, velocity is zero).
By the principle of conservation of energy
decrease in KE = increase in EPE

$$\frac{1}{2}m_e v^2 - \frac{1}{2}m_e (v\sin\theta)^2 = q\Delta V$$

$$\frac{1}{2}m_e v(1-\sin^2\theta) = q\Delta V$$

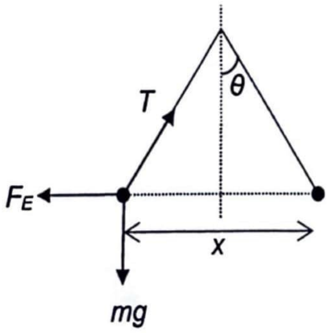
$$\frac{1}{2}m_e (v\cos\theta)^2 = eV$$

Answer: B

SP14
- The charges on A will repel those on B (and vice versa) such that the charges are concentrated on the outer surfaces of both spheres.
- Since the charges are no longer uniformly distributed on the surface of the spheres, there is no longer spherical symmetry.
- $F = \frac{Q^2}{4\pi\varepsilon_0 d^2}$ only applies if the charges are uniformly distributed on the surfaces of the spheres as d is the distance between the centres of the spheres.

Hence, the force between them cannot be represented by $F = \frac{Q^2}{4\pi\varepsilon_0 d^2}$.

SP15



$$T\cos\theta = mg$$

$$T\sin\theta = F_E$$

$$\tan\theta = \frac{F_E}{mg} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2}}{mg} = \frac{q^2}{mg4\pi\varepsilon_0 x^2}$$

By small angle approximation,

$$\sin\theta = \frac{\frac{1}{2}x}{l} \approx \tan\theta \Rightarrow \frac{x}{2l} = \frac{q^2}{mg4\pi\varepsilon_0 x^2}$$

$$x^3 = \frac{2lq^2}{mg4\pi\varepsilon_0} \Rightarrow x = \left[\frac{q^2 l}{2\pi\varepsilon_0 mg}\right]^{\frac{1}{3}}$$

SP16 By the principle of conservation of energy,

decrease in K.E. = increase in E.P.E.

$$\frac{1}{2}m_e u^2 - 0 = 2\left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{1.0\times10^{-2}}\right) - 0$$

$$u = 318 \text{ m s}^{-1}$$