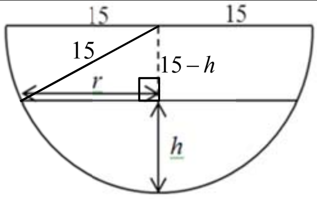
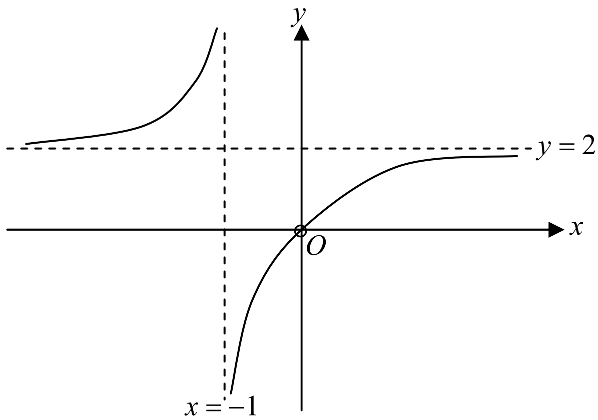
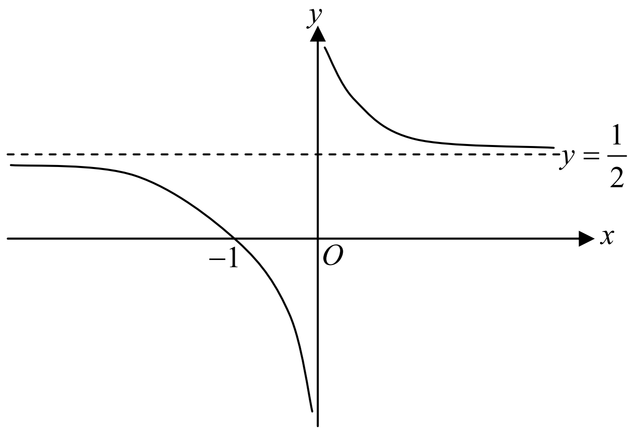
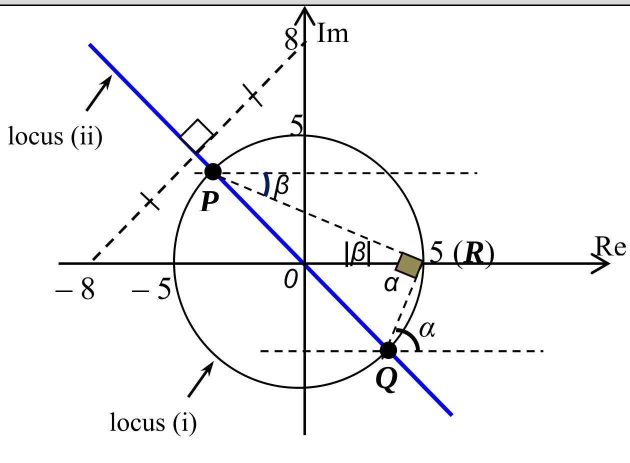
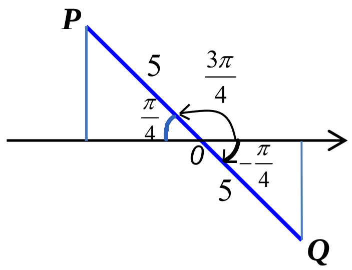
2012 Year 6 Preliminary Examination P2 Mark Scheme

Section A

Qn	Suggested Solution	Marking Scheme
1	By Pythagoras' Theorem, $r^2 = 15^2 - (15-h)^2$ $r = \sqrt{30h - h^2}$ (shown)  	**AG1 –** Correct formulation leading to AG
1(i)	$V = \frac{\pi}{3}\left(45h^2 - h^3\right)$ $\frac{\mathrm{d}V}{\mathrm{d}t} = \left(30\pi h - \pi h^2\right)\frac{\mathrm{d}h}{\mathrm{d}t}$ $-20 = \left(30\pi(5) - \pi(5)^2\right)\frac{\mathrm{d}h}{\mathrm{d}t}$ $\frac{\mathrm{d}h}{\mathrm{d}t} = -0.050929 = -0.0509$ cm/min (3.s.f) **Alternative** $\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{\pi}{3}\left(90h - 3h^2\right) = 30\pi h - \pi h^2$ $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$ $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{\left(30\pi(5) - \pi(5)^2\right)}(-20)$ $\frac{\mathrm{d}h}{\mathrm{d}t} = -0.050929 = -0.0509$ cm/min (3.s.f)	**M1** – Correct differentiation **A1** **Alternative** **M1** – Differentiation & chain rule **A1**
1(ii)	$r = \sqrt{30h - h^2}$ $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{2}\left(30h - h^2\right)^{-\frac{1}{2}}(30 - 2h)$ $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t}$ When $h = 5$, $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{15-(5)}{\sqrt{30(5)-(5)^2}}(-0.050929)$ $\frac{\mathrm{d}r}{\mathrm{d}t} = -0.045552 = -0.0456$ cm/min (3.s.f)	**M1** – Evaluate r **M1**– Correct differentiation **A1** – Rate of <u>decrease</u> = 0.0456 cm/min ***or*** $\frac{\mathrm{d}r}{\mathrm{d}t} = -0.0456$ cm/min **Total: 6 marks**

Qn	Suggested Solution	Marking Scheme
2		**G1** – Asymptotes **G1** – Shape + exclude origin
	C_1 is transformed to C_2 by i) a reflection in the y-axis and ii) scaling by a factor of $\frac{1}{2}$ along the x-axis. Note: Order of (i) and (ii) can be interchanged.	**B1** – (i) **B1** – (ii) Note: Order of (i) and (ii) can be interchanged.
		**G1** – Horizontal asymptote **G1**– Shape and x-intercept **Total: 6 marks**

Qn	Suggested Solution	Mark Scheme
3 **(i)** **(ii)**		**G1** – Circle with centre at origin, radius 5 **G1 –** Perpendicular bisector of the line segment joining points representing (0,8) and (-8,0); passes through origin

	 From diagram, $p=5e^{i\frac{3\pi}{4}}=5\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)=\frac{5}{\sqrt{2}}(-1+i)$ $q=5e^{-i\frac{\pi}{4}}=5\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)=\frac{5}{\sqrt{2}}(1-i)$ **<u>Alternative</u>** $\arg\left(\frac{p}{q}\right)>0\Rightarrow\arg(p)-\arg(q)>0\Rightarrow\arg(p)>\arg(q)$ Equation of circle: $x^2+y^2=5^2$ -------- (i) Equation of perpendicular bisector: $y=-x$ -------- (ii) Solving (i) and (ii) : $p=\frac{5}{\sqrt{2}}(-1+i)$ $q=\frac{5}{\sqrt{2}}(1-i)$ From diagram $\arg(5-q)=\alpha>0$ $\arg(5-p)=-\lvert\beta\rvert$ $\therefore\arg\frac{(5-p)}{(5-q)}$ $=\arg(5-p)-\arg(5-q)$ $=-(\lvert\beta\rvert+\alpha)$ (= $\angle PRQ$; using corresponding angles) $=-\frac{\pi}{2}$ ($\angle$ at circumference, semi-circle)	**M1** – Use polar forms with correct radius and angle for p and q **A1** **A1** **M1** – Solve simultaneous eqns (must see correct equations for both) **A1** **A1** **B1 :** $-\frac{\pi}{2}$ **Total: 6 marks**

Qn	Suggested Solution	Mark Scheme
4 (i)	$a_{50} = 99a_1$ $a_1 + 49d = 99a_1$ $a_1 = \frac{1}{2}(0.15) = 0.075$ (Shown)	**M1** – Correct expression for a_{50} in terms of a_1 **AG1**
4 (ii)	$\sum_{n=1}^{50} a_n = \frac{50}{2}(0.075 + 99 \times 0.075) = 187.5$	**M1** – Correct formula **A1**
4 (iii)	$b_k < a_{25}$ $(99 \times 0.075)(0.98)^{k-1} < (0.075) + 24(0.15)$ $k > 35.8$ least $k = 36$	**M1** – For b_k **A1**
4 (iv)	Consider $\frac{b_1(1-0.98^h)}{1-0.98} > 0.99\frac{b_1}{1-0.98}$ $0.98^h < 0.01$ $h > 227.9$ least $h = 228$	**M1** – Correct inequality (must see inequality) **A1**
4 (v)	$\sum_{m=0}^{\infty} b_{1+3m}$ $= b_1 + b_4 + b_7 + \ldots$ $= \frac{99 \times 0.075}{1-(-0.98)^3}$ $= 3.82$ (3s.f.)	**M1** – Correct formula **A1** **Total: 10 marks**

Qn	Suggested Solution	Marking Scheme
5 (i)	Subst. $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ into LHS of equation of p_1, we have LHS = $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ 7 \\ 2 \end{pmatrix} = \alpha + 20 =$ RHS. $\therefore$ A lies in p_1 (Shown) Subst. $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ into LHS of equation of p_2, we have	 **B1**

	LHS = $\begin{pmatrix}1\\2\\3\end{pmatrix}\cdot\begin{pmatrix}3\\5\\\beta\end{pmatrix}=7 \Rightarrow \beta=-2$	**B1**
5 (ii)	Consider $\begin{pmatrix}\alpha\\7\\2\end{pmatrix}\times\begin{pmatrix}3\\5\\-2\end{pmatrix}=\begin{pmatrix}-24\\2\alpha+6\\5\alpha-21\end{pmatrix}$ $\therefore\ l:\mathbf{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}+m\begin{pmatrix}-24\\2\alpha+6\\5\alpha-21\end{pmatrix},\ m\in\mathbb{R}$ Here, $\begin{pmatrix}-24\\2\alpha+6\\5\alpha-21\end{pmatrix}=k\begin{pmatrix}4\\-2\\1\end{pmatrix}$ $\Rightarrow k=-6,\ \alpha=3$	**M1** – Consider cross product of the normals **A1** – Correct equation formed **AG1** – Correct method leading to AG
5 (iii)	Acute angle $=\cos^{-1}\dfrac{\left\lvert\begin{pmatrix}3\\7\\2\end{pmatrix}\cdot\begin{pmatrix}3\\5\\-2\end{pmatrix}\right\rvert}{\sqrt{62}\sqrt{38}}=34.5^\circ$	**M1** – Correct formula used (condone w/o modulus) **A1**
5 (iv)	Let foot of perpendicular be F. $\overrightarrow{OF}=\begin{pmatrix}1+4k\\2-2k\\3+k\end{pmatrix}\Rightarrow\overrightarrow{BF}=\begin{pmatrix}-1+4k\\6-2k\\-5+k\end{pmatrix}$ $\begin{pmatrix}-1+4k\\6-2k\\-5+k\end{pmatrix}\cdot\begin{pmatrix}4\\-2\\1\end{pmatrix}=0$ $\Rightarrow k(16+4+1)=4+12+5$ $\therefore k=1$ Hence $\overrightarrow{OF}=\begin{pmatrix}5\\0\\4\end{pmatrix}$. **<u>Alternatively</u>**, $\overrightarrow{AF}=\overrightarrow{AB}\cdot\dfrac{\begin{pmatrix}4\\-2\\1\end{pmatrix}}{\sqrt{21}}\dfrac{\begin{pmatrix}4\\-2\\1\end{pmatrix}}{\sqrt{21}}=\dfrac{\left[\begin{pmatrix}2\\-4\\8\end{pmatrix}-\begin{pmatrix}1\\2\\3\end{pmatrix}\right]\cdot\begin{pmatrix}4\\-2\\1\end{pmatrix}}{21}\begin{pmatrix}4\\-2\\1\end{pmatrix}$ $=\begin{pmatrix}4\\-2\\1\end{pmatrix}$	**M1** – Find either $\overrightarrow{OF}$ or $\overrightarrow{BF}$ **M1** – Use dot product for appropriate pair of vectors and set to zero **A1** **M1** – Correct projection used

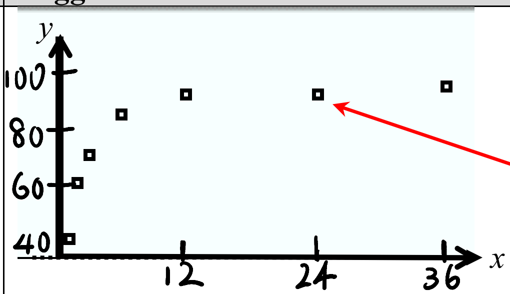
	$\therefore \overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{AF} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix}$ Consider $\overrightarrow{AB'} = \overrightarrow{AB} + 2\overrightarrow{BF}$ where *B'* is reflection of *B* in *l*. $\overrightarrow{AB'} = \begin{pmatrix} 1 \\ -6 \\ 5 \end{pmatrix} + 2\begin{pmatrix} 3 \\ 4 \\ -4 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix}$ Hence required line is $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + m\begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix}$, $m \in \mathbb{R}$.	**M1** – Correct method to find $\overrightarrow{OF}$ from $\overrightarrow{AF}$ **A1** **M1** – Correct vector equation formed using previous part **A1** **Total: 12 marks**

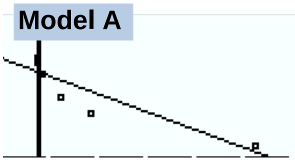
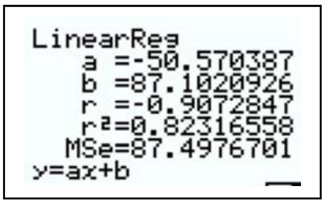
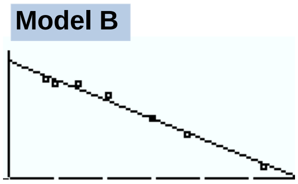
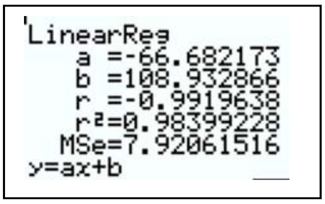
Section B

Qn	Solution	Mark Scheme
6 (i)	$k = \frac{810}{30} = 27$ List the applications in some order e.g. by applicants' names or loan amount, and randomly select the first application to process. Subsequently, select every 27th application until a sample of 30 applications is selected, going back to the front of the list if necessary. Alternatively (only applicable if *k* is an integer), List the applications in some order and divide the applications into 30 groups of 27 each. Randomly select a number from 1 to 27, e.g. 5, and pick the 5th application in each of the 30 groups.	**B1** for every 2 points: • List applications in a certain order • Randomly select first application • Select every 27th application thereafter • Go back to the front of the list if necessary
6 (ii)	A systematic random sample may not ensure that all categories of loan applications were processed within that day if the sampling interval coincides with a cyclic pattern in the list of applications.	**B1** – Disadvantage of a systematic random sample
6 (iii)	Stratified random sampling ensures that each category of loan is proportionately represented in the sample.	**B1** – Advantage of stratified random sample (answer in context) **Total : 4 marks**

Qn	Suggested Solution	Mark Scheme
7(i)	Let G and E be the number of Green Top taxis and EZCab taxis arriving in a randomly chosen 10-minute period respectively. $G \sim \mathrm{P_o}(3)$, $E \sim \mathrm{P_o}(5)$ $G + E \sim \mathrm{P_o}(3+5)$ $\mathrm{P}(G+E \geq 7) = 1 - \mathrm{P}(G+E \leq 6)$ $= 1 - 0.31337 = 0.68663 = 0.687$ (shown) (3 s.f.)	**B1 -** $G + E \sim \mathrm{P_o}(3+5)$ **AG1**
7(ii)	Required probability $=\mathrm{P}(\text{all taxis arrived were EZCab} \mid \text{at least 7 taxis arrived})$ $= \dfrac{\mathrm{P}(\text{all taxis arrived were EZCab and at least 7 taxis arrived})}{\mathrm{P}(\text{at least 7 taxis arrived})}$ $= \dfrac{\mathrm{P}(E \geq 7) \cdot \mathrm{P}(G = 0)}{\mathrm{P}(G + E \geq 7)}$ $= \dfrac{[1 - \mathrm{P}(E \leq 6)] \cdot \mathrm{P}(G = 0)}{\mathrm{P}(G + E \geq 7)}$ $= \dfrac{(1 - 0.76218) \cdot (0.049787)}{0.68663}$ $= 0.0172$ (3 s.f.)	**M1 –** $\dfrac{\mathrm{P}(E \geq 7) \cdot \mathrm{P}(G = 0)}{\mathrm{P}(G + E \geq 7)}$ *Note*: Accept 0.687 for denominator (given) **A1** **Total : 4 marks**

Qn	Suggested Solution	Marking Scheme
8 (i)	Number of six-figure number that can be formed $= {}^7P_6 = 5040$	**B1** – accept w/o working
8 (ii)	Number of six-figure number that can be formed $= 7^6 = 117649$	**B1** – accept w/o working
8 (iii)	Case 1: xx yy zz Choose 3 digits out of 7 digits = ${}^7C_3$ Arrange 6 digits with 3 pairs of identical digits = $\frac{6!}{2!2!2!}$ Number of six-figure number = ${}^7C_3 \times \frac{6!}{2!2!2!} = 3150$ Case 2: xxx yyy Number of six-figure number = ${}^7C_2 \times \frac{6!}{3!3!} = 420$ Case 3: xxxx yy Choose 3 digits out of 7 digits = ${}^7C_3$ There are 2 possible cases: xxxx yy or yyyy xx Number of six-figure number = ${}^7C_2 \times 2 \times \frac{6!}{4!2!} = 630$ Case 4: xxxxxx Number of six-figure number = 7 Number of six-figure number such that every digit in the number appears at least twice = 3150 + 420 + 630+7 = 4207	For either case 1, 2 or 3 **M1** – Choose required number of digits **M1** – Arrange digits taking into consideration identical digits **M1** – 3 out of 4 cases correctly worked out (but condone wrong answer computed) **A1** **Total: 6 marks**

Qn	Suggested Solution	Mark Scheme
9 (i)		**G1** – Correct horz/vert axes & labelled; 7 points with correct relative positions; create equally-spaced intervals for both axes (Note: Check 2nd last point must not be lower than the previous)
	r =0.729 (3sf) The diagram shows a non-linear relationship between x and y. Also, the value of $\|r\|$ is not very close to 1. Hence a linear model is not appropriate.	**B1** – r value **B1** – Comment based on scatter plot (comment based on r is optional) (Note: Do not reward for commenting on r value alone.)

9 (ii)	    Since $\|r_B\| > \|r_A\|$ and is <u>closer to 1</u> (or the scatter plot shows a more linear relationship between the variables), Model B is better.	**M1** – Either one of $\|r_B\|$ or $\|r_A\|$ correct or draw either line of best fit correctly **A1** – Choose model B with correct explanation using r or scatter diagram
9 (iii)	Regression line for Model B: $y = -\dfrac{66.682}{\sqrt{x}} + 108.93$ When $x = 65$ $y = -\dfrac{66.682}{\sqrt{65}} + 108.93 = 100.66$ The estimated exam score is not valid as the value of $x = 65$ is beyond the data range of 36 hours	√**M1** – (ecf) Use equation for substitution **A1** – Correct y value **B1** – x beyond data range [**Note: Do not award if comment given as "score cannot be more than 100"**] **Total: 8 marks**

Qn	Solution	Mark Scheme
10 (i)	Unbiased estimate of population mean, $\bar{t} = \dfrac{\sum(t-15)}{120} + 15$ $= \dfrac{123}{120} + 15$ $= 16.025 \text{ (exact)} = 16.0 \ \ (3 \text{ s.f.})$ Unbiased estimate of population variance	**B1** – Accept 3 s.f.

	$s^2=\frac{1}{n-1}\left(\sum(t-15)^2-\frac{\left(\sum(t-15)\right)^2}{n}\right)$ $=\frac{1}{119}\left(2504-\frac{123^2}{120}\right)=19.98256=19.983$ (shown)	**AG1 –** Must see values substituted
10 (ii)	Test $H_0:\mu=15$ Against $H_1:\mu\neq 15$ Conduct a 2-tailed test at 5% significance level. Since $n=120$ is large, by Central Limit Theorem and under H_0, $\bar{T}\sim \mathrm{N}\left(15,\frac{19.983}{120}\right)$ approximately Using z-test, since $\bar{t}=16.025$, p-value = 0.0120 (3 s.f) Since p-value = 0.0120 < 0.05, we reject H_0 and conclude that there is sufficient evidence at 5% significance level to show that the mean waiting time differed from 15 minutes.	**B1** – H_0 and H_1 correctly stated **B1** – Correct distribution for $\bar{T}$ with statement of CLT **B1** – Correct p value, comparison and conclusion
10 (iii)	Test $H_0:\mu=15$ Against $H_1:\mu>15$ Since n is large, by Central Limit Theorem, $\bar{T}\sim \mathrm{N}\left(15,\frac{19.983}{n}\right)$ approximately Since H_0 is not rejected, $\mathrm{P}(\bar{T}>15.5)>0.06$ [or : p value > 0.06] $\mathrm{P}\left(Z>\frac{15.5-15}{\sqrt{\frac{19.983}{n}}}\right)>0.06$ $0.5\sqrt{\frac{n}{19.983}}<1.5547$ $n<193.20$ $\therefore 50\le n\le 193$, where $n\in\mathbb{Z}^+$.	**B1** – Correct statement or equivalent **M1** –Inequality with z_c **A1** – Correct answer in terms of integer (condone without lower limit "$50\le n$" or "$n\in\mathbb{Z}^+$") **Total: 8 marks**

Qn	Suggested Solution	Mark Scheme
11 (a) (i)	$\text{P}\left(\text{Abbey is first and Betty is sixth in the queue}\right)=\frac{1}{8}\left(\frac{1}{7}\right)=\frac{1}{56}$ Or $\frac{6!}{8!}=\frac{1}{56}$ Or $\left(\frac{1}{8}\right)\left(\frac{6}{7}\right)\left(\frac{5}{6}\right)\left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right)=\frac{1}{56}$	**B1** – Accept w/o working
11 (a) (ii)	Required probability =P(Abbey is first) + P(Betty is second) – P(Abbey is first and Betty second) $=\frac{1}{8}+\frac{7}{8}\left(\frac{1}{7}\right)-\frac{1}{8}\left(\frac{1}{7}\right)$ or $\frac{7!}{8!}+\frac{7!}{8!}-\frac{6!}{8!}$ $=\frac{13}{56}$ <u>Alternatively</u>, Required probability = P(Abbey is first but Betty is not second) + P(Abbey is not first but Betty is second) + P(Abbey is first and Betty second) $=\frac{6\times 6!}{8!}+\frac{6\times 6!}{8!}+\frac{6!}{8!}=\frac{13}{56}$	**M1** – Make use of formula & obtain 2 out of 3 terms correct **A1**
11 (b) **(i)**	Let A, B & C be the events a drawer containing: 2 gold coins; 1 gold coin and 1 silver coin; and 2 silver coins is selected, respectively. Let G & S be the events: a gold coin is selected; and a silver coin is selected, respectively. $\text{P}(A)=\text{P}(B)=\text{P}(C)=\frac{1}{3}$ $\text{P}(G)=\frac{1}{3}+\frac{1}{3}\cdot\frac{1}{2}=\frac{1}{2}$ and $\text{P}(S)=1-\text{P}(G)=1-\frac{1}{2}=\frac{1}{2}$ P(Drawer containing 2 gold coins is selected given that the coin selected is gold) $=\text{P}(A\mid G)$ $=\frac{\text{P}(A\cap G)}{\text{P}(G)}$ $=\frac{1/3}{1/2}=\frac{2}{3}$	**B1** – Awarded to value of $\text{P}(G)$ presented in any part of the working **M1** – Recognise & write down conditional probability $\frac{\text{P}(A\cap G)}{\text{P}(G)}$ with values substituted **A1**

11 (b) (ii)	P(a gold coin is selected, followed by a silver coin) $=\mathrm{P}(A\cap G).\mathrm{P}(S\mid\{A\cap G\})+\mathrm{P}(B\cap G).\mathrm{P}(S\mid\{B\cap G\})$ $=\left(\frac{1}{3}\cdot\frac{2}{2}\right)\left(\frac{1}{3}\cdot\frac{1}{2}+\frac{1}{3}\cdot\frac{2}{2}\right)+\left(\frac{1}{3}\cdot\frac{1}{2}\right)\left(\frac{1}{3}\cdot\frac{1}{1}+\frac{1}{3}\cdot\frac{2}{2}\right)$ $=\frac{1}{3}\left(\frac{1}{2}\right)+\frac{1}{6}\left(\frac{2}{3}\right)$ $=\frac{5}{18}$	**M1 –** Consider 2 <u>main</u> cases correctly **M1 –** Probability of any 2 <u>sub-cases</u> correct **M1 –** Probability of other 2 <u>sub-cases</u> correct **A1** **Total: 10 marks**

<table>
<tr><th>Qn</th><th>Suggested Solution</th><th>Mark Scheme</th></tr>
<tr><td>12(i)</td><td>$\mathrm{P}(X=0)+\mathrm{P}(X=19)=0.1$
$(1-p)^{20}+\binom{20}{19}p^{19}(1-p)=0.1$

By GC, $p = 0.10874 = 0.109$ (3 s.f.)</td><td>M1 – Condone error in coefficient

A1</td></tr>
<tr><td>12(ii)</td><td>Reasons should address either one of these behaviour:
1: The probability of "success" p is the same for each outcome.
2: The responses are independent of each other.

<table>
<tr><th></th><th>Behaviour of X</th><th>Behaviour of Y</th></tr>
<tr><td>1.</td><td>Probability of responding positively to the email (“success”) is unlikely to be the same for each student since
- not everyone is a fan of action thrillers, or
- not everyone reads email.</td><td>Probability of picking a girl (“success”) is the same for each of the 5 weekdays since all names are in the bag.</td></tr>
<tr><td>OR
2.</td><td>Responses may not be independent since decision of a student is likely to influence or be influenced by other students in his/her clique.</td><td>The trials of picking a student on each of the five day are independent since the names are drawn from a bag at random.</td></tr>
</table></td><td>B1 – Describe one characteristic of Y in context

B1 – Compare to characteristic of X in context</td></tr>
<tr><td>12
(iii)</td><td>$Y \sim \mathrm{B}(5, \frac{12}{21})$ i.e. $Y \sim \mathrm{B}(5, \frac{4}{7})$
$\mathrm{P}(Y \geq 3) = 1 - \mathrm{P}(Y \leq 2)$
$= 1 - 0.36788 = 0.63212 = 0.632$ (3 s.f.)</td><td>B1 – Correct distribution

M1 – Use complement
A1</td></tr>
<tr><td>12
(iv)</td><td>Let W be the number of weeks in which a girl is on duty more often than a boy, out of 20.

$W \sim \mathrm{B}(20, 0.63212)$
Since n=20 is sufficiently large such that $np = 12.642 > 5$ and $nq = 7.3576 > 5$,

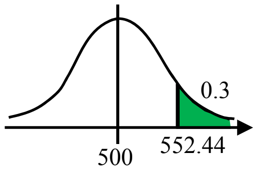
$W \sim \mathrm{N}(12.642, 4.6508)$ approximately.

$\mathrm{P}(Y < 11) = \mathrm{P}(Y < 10.5)$ after continuity correction
$= 0.16029 = 0.160$ (3 s.f.)</td><td>M1 – Check conditions based on original binomial distribution

B1 - Define approx. distribution

A1

Total: 10 marks</td></tr>
</table>

Qn	Suggested Solution	Mark Scheme
13(i)	Let X & Y be random variables that denote the score of a prospective applicant of Kingdom University and a prospective applicant of Island University respectively. Given $X \sim \mathrm{N}\left(500, 100^2\right)$ and $Y \sim \mathrm{N}\left(480, 60^2\right)$. $\mathrm{P}(X > a) \le 0.3$  Alternative $\mathbf{P}(X < a) \ge 0.7$ $a \ge 552.44...$ The minimum score a randomly chosen student must achieve is 553 (3 sf).	**M1** – Either $\mathrm{P}(X > a) \le 0.3$ or $\mathrm{P}(X < a) \ge 0.7$ **A1** – 553
13(ii)	$\overline{X} \sim \mathrm{N}\left(500, \frac{100^2}{5}\right)$ and $\overline{Y} \sim \mathrm{N}\left(480, \frac{60^2}{5}\right)$ $\overline{X} - \overline{Y} \sim \mathrm{N}\left(500 - 480, \frac{100^2}{5} + \frac{60^2}{5}\right)$ $\overline{X} - \overline{Y} \sim \mathrm{N}(20, 2720)$ Required Probability $= \mathrm{P}\left(\left\lvert\overline{X} - \overline{Y}\right\rvert > 50\right)$ $= \mathrm{P}\left(\overline{X} - \overline{Y} < -50\right) + \mathrm{P}\left(\overline{X} - \overline{Y} > 50\right)$ $= 0.089767 + 0.28256$ $= 0.37232$ $= 0.372 \text{ (3sf)}$	**B1** – Distribution of $\overline{X} - \overline{Y}$ **M1** – $\mathrm{P}\left(\overline{X} - \overline{Y} < -50\right) + \mathrm{P}\left(\overline{X} - \overline{Y} > 50\right)$ or $1 - \mathrm{P}\left(-50 < \overline{X} - \overline{Y} < 50\right)$ **A1** – 0.372
13(iii)	Required Probability $= \left[\mathrm{P}(X > 500)\right]^5 \left[\mathrm{P}(Y > 500)\right]^5$ $= (0.5)^5 (0.36944)^5$ $= 2.15 \times 10^{-4}$ or 0.000215 (3 s.f.)	**M1** – Either 0.5^5 or 0.36944^5 correct **A1**
13(iv)	Let $T = Y_1 + Y_2 + ... + Y_5 \sim \mathrm{N}\left(5 \times 480, 5 \times 60^2\right)$ $5X \sim \mathrm{N}\left(5 \times 500, 5^2 \times 100^2\right)$ $T - 5X \sim \mathrm{N}(-100, 268000)$ $\mathrm{P}(T - 5X > 0) = 0.42341 = 0.423$ (3 s.f.)	**M1** – Either distribution of T **or** $5X$ correct **B1** – Distribution of $T - 5X$ correct **A1** – 0.423 **Total: 10 marks**