Q1

Method ①:

$2z+1=|w|$ (1)

$2w-z=4+24\text{i}$ (2)

From (2): $z=2w-4-24\text{i}$

Substitute into (1): $2(2w-4-24\text{i})+1=|w|$

$4w-7-48\text{i}=|w|$

Let $w=a+b\text{i}$

$4(a+b\text{i})-8-48\text{i}+1=\sqrt{a^2+b^2}$

$(4a-7)+(4b-48)\text{i}=\sqrt{a^2+b^2}$

Comparing Imaginary parts,

$4b-48=0$

$b=12$

Comparing Real parts,

$4a-7=\sqrt{a^2+b^2}$

$4a-7=\sqrt{a^2+12^2}$ **

$(4a-7)^2=a^2+144$

$15a^2-56a-95=0$

$\Rightarrow a=-\dfrac{19}{15}$ or $a=5$

From **, since $4a-7=$ a positive real number,

when $a=-\dfrac{19}{15}$, $4a-7=4\left(-\dfrac{19}{15}\right)-7<0$

$\Rightarrow$ reject $a=-\dfrac{19}{15}$

$\therefore$ $a=5$, $b=12$, $z=2(5+12\mathrm{i})-4-24\mathrm{i}=6$

$\Rightarrow w=5+12i$, $z=6$

Method ②:

$2z+1=|w|$ (1)

$2w-z=4+24\mathrm{i}$ (2)

$2z+1=$ a positive real number $\Rightarrow$ Let $z=x$ and $w=a+b\mathrm{i}$

From (2): $2(a+bi)-x=4+24\mathrm{i}$

Comparing Real and Imaginary parts,

$2a-x=4$

$2b=24\Rightarrow b=12$

From (1): $2x+1=\sqrt{a^2+b^2}$ (3)

Substitute $b=12$ and $x=2a-4$ into (3):

$2(2a-4)+1=\sqrt{a^2+12^2}$

$4a-7=\sqrt{a^2+12^2}$ **

$(4a-7)^2=a^2+144$

$16a^2-56a+49=a^2+144$

$15a^2-56a-95=0$

$\Rightarrow a=-\dfrac{19}{15}$ or $a=5$

$\Rightarrow x=-\dfrac{98}{15}$ or $x=6$

However $2z+1=$ a positive real number,

When $x=-\dfrac{98}{15}$, $2z+1=2\left(-\dfrac{98}{15}\right)+1<0$

$\Rightarrow$ reject $x=-\dfrac{98}{15}$ and $a=-\dfrac{19}{15}$

$\therefore\ x=6,\ \ a=5,\ \ b=12$

$\Rightarrow w=5+12i,\ \ z=6$

Q2	
(a)	When $x=\frac{1}{n}$, $y=\frac{\frac{1}{n}}{\sqrt{1+\left(\frac{1}{n}\right)^2}}$ When $x=\frac{2}{n}$, $y=\frac{\frac{2}{n}}{\sqrt{1+\left(\frac{2}{n}\right)^2}}$ $\dots\dots$ When $x=\frac{n-1}{n}$, $y=\frac{\frac{n-1}{n}}{\sqrt{1+\left(\frac{n-1}{n}\right)^2}}$ $A=\frac{1}{n}\left(\frac{\frac{1}{n}}{\sqrt{1+\left(\frac{1}{n}\right)^2}}\right)+\frac{1}{n}\left(\frac{\frac{2}{n}}{\sqrt{1+\left(\frac{2}{n}\right)^2}}\right)+\frac{1}{n}\left(\frac{\frac{3}{n}}{\sqrt{1+\left(\frac{3}{n}\right)^2}}\right)\dots+\frac{1}{n}\left(\frac{\frac{n-1}{n}}{\sqrt{1+\left(\frac{n-1}{n}\right)^2}}\right)$ $=\frac{1}{n^2}\left(\frac{1}{\sqrt{1+\left(\frac{1}{n}\right)^2}}+\frac{2}{\sqrt{1+\left(\frac{2}{n}\right)^2}}+\frac{3}{\sqrt{1+\left(\frac{3}{n}\right)^2}}+\dots+\frac{n-1}{\sqrt{1+\left(\frac{n-1}{n}\right)^2}}\right)$ $=\frac{1}{n^2}\left(\frac{n}{\sqrt{n^2+1^2}}+\frac{2n}{\sqrt{n^2+2^2}}+\frac{3n}{\sqrt{n^2+3^2}}+\dots+\frac{(n-1)n}{\sqrt{n^2+(n-1)^2}}\right)$ $=\frac{1}{n}\left[\frac{1}{\sqrt{n^2+1^2}}+\frac{2}{\sqrt{n^2+2^2}}+\frac{3}{\sqrt{n^2+3^2}}+\dots+\frac{(n-1)}{\sqrt{n^2+(n-1)^2}}\right]$ $=\frac{1}{n}\sum_{r=1}^{n-1}\frac{r}{\sqrt{n^2+r^2}}$ (shown)

(b)	$$\begin{aligned}\lim_{n\to\infty} A &= \int_0^1 \frac{x}{\sqrt{1+x^2}}\,\mathrm{d}x \\ &= \frac{1}{2}\int_0^1 2x\cdot\left(1+x^2\right)^{-\frac{1}{2}}\,\mathrm{d}x \\ &= \frac{1}{2}\left[\frac{\left(1+x^2\right)^{1/2}}{1/2}\right]_0^1 \\ &= \sqrt{2}-1\end{aligned}$$

Q3	
(a)	$\underset{\sim}{r} - \underset{\sim}{q}$ is parallel to $\underset{\sim}{p}$ $\Rightarrow \underset{\sim}{r} - \underset{\sim}{q} = \lambda \underset{\sim}{p}$ $\underset{\sim}{r} = \underset{\sim}{q} + \lambda \underset{\sim}{p}$ The point R lies on a line that passes through the point Q and is parallel to the vector $\underset{\sim}{p}$.
(b)	$\left(\underset{\sim}{r} - \underset{\sim}{q}\right) \cdot \underset{\sim}{p} = 0$ $\underset{\sim}{r} \cdot \underset{\sim}{p} - \underset{\sim}{q} \cdot \underset{\sim}{p} = 0$ $\underset{\sim}{r} \cdot \underset{\sim}{p} = \underset{\sim}{q} \cdot \underset{\sim}{p}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$ $2x - 5y + 3z = -7$ The point R lies on a plane that contains the point Q and is perpendicular to the vector $\underset{\sim}{p}$.

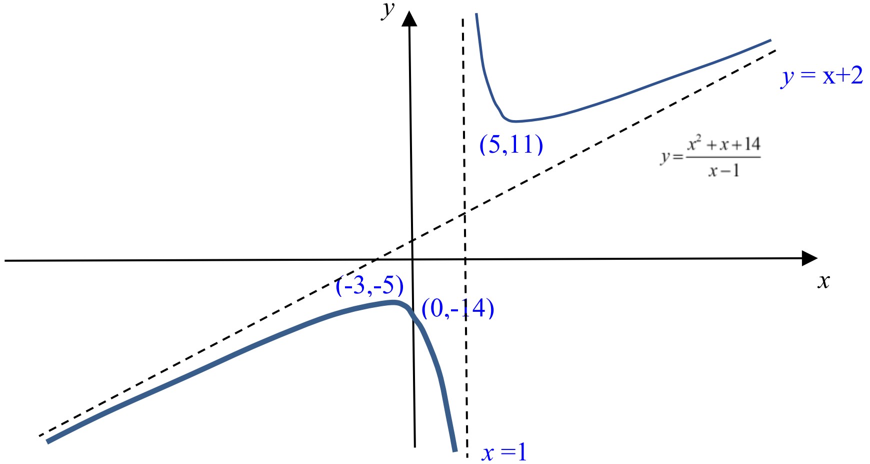
Q4	
(a)	There is only one(positive) real root in the equation $f(x)=0$. Since the equation has all real coefficients, then the two other roots must be a pair of complex conjugates.
(b)	Since $x=1-2\text{i}$ is a root of $2x^3-7x^2+16x+c=0$, $2(-11+2\text{i})-7(-3-4\text{i})+16(1-2\text{i})+c=0$ $15+c=0$ $\therefore c=-15$ (shown)
(c)	Since all the coefficients are real, $x=1+2\text{i}$ is another root of $2x^3-7x^2+16x+c=0$. $2x^3-7x^2+16x-15=0$ $\left[x-(1+2\text{i})\right]\left[x-(1-2\text{i})\right](2x-k)=0$ $\left[(x-1)+2\text{i}\right]\left[(x-1)-2\text{i}\right](2x-k)=0$ $\left[x^2-2x+5\right](2x-k)=0$ Comparing the coefficient of constant term (or by long division), $-5k=-15 \Rightarrow k=3$ Therefore, the last root is $x=\frac{3}{2}$ The roots are $x=1+2\text{i}$, $x=1-2\text{i}$, $x=\frac{3}{2}$
(d)	$2x^3-7x^2+16x+c=0$ Replace x with $\frac{1}{w}$ $2\left(\frac{1}{w}\right)^3-7\left(\frac{1}{w}\right)^2+16\left(\frac{1}{w}\right)-15=0$ $2-7w+16w^2-15w^3=0$

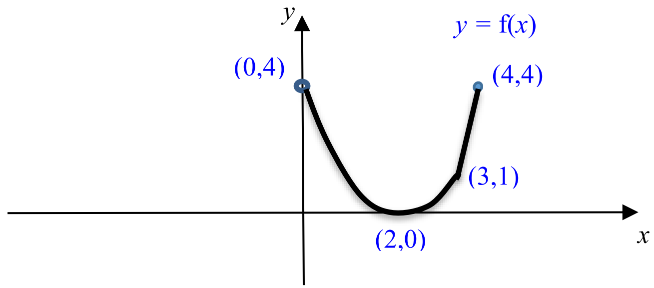
	Hence, the roots are $\frac{1}{w}=1+2\text{i}$; $\frac{1}{w}=1-2\text{i}$; $\frac{1}{w}=\frac{3}{2}$ $w=\frac{1}{1+2\text{i}}=\frac{1}{5}-\frac{2}{5}\text{i}$ $w=\frac{1}{1-2\text{i}}=\frac{1}{5}+\frac{2}{5}\text{i}$ $w=\frac{2}{3}$

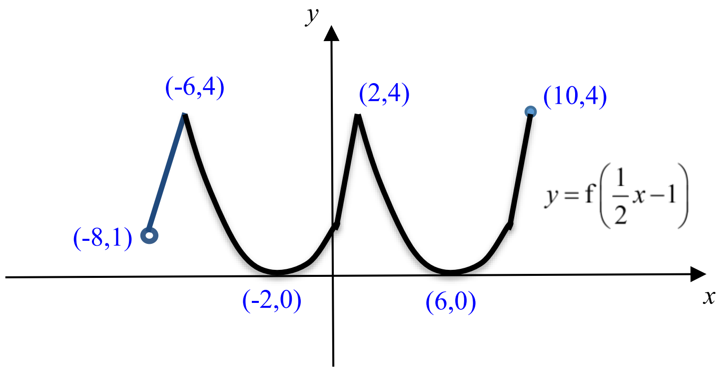
Q5	
(a)	**<u>Method ①:</u>** The *n*th term, u_n, is always one degree less than S_n since $u_n = S_n - S_{n-1}$. If S_n is quadratic, u_n would be linear but it is not since there is no common difference between consecutive terms. **<u>Method ②:</u> Proof by Contradiction** Suppose $S_n = an^2 + bn + c$ $S_n = an^2 + bn + c$ $a+b+c=-4$ $4a+2b+c=-4-2=-6$ $9a+3b+c=-6+12=6$ $16a+4b+c=6+38=44$ Using G.C., no solution found. Hence S_n cannot be a quadratic polynomial.
(b)	$S_n = an^3 + bn^2 + cn + d$ $a+b+c+d=-4$ $8a+4b+2c+d=-4-2=-6$ $27a+9b+3c+d=-6+12=6$ $64a+16b+4c+d=6+38=44$ Using G.C., $a=2,\ b=-5,\ c=-1,\ d=0$ $S_n = 2n^3 - 5n^2 - n$

(c)	$u_n = S_n - S_{n-1}$ $= 2n^3 - 5n^2 - n - \left[2(n-1)^3 - 5(n-1)^2 - (n-1)\right]$ $= 2n^3 - 5n^2 - n - \left[2\left(n^3 - 3n^2 + 3n - 1\right) - 5\left(n^2 - 2n + 1\right) - n + 1\right]$ $= 2n^3 - 5n^2 - n - \left(2n^3 - 11n^2 + 15n - 6\right)$ $= 6n^2 - 16n + 6$
(d)	$\sum_{n=10}^{2m} \left(u_n - u_{n-1}\right)$ $= \cancel{u_{10}} - u_9$ $+ \cancel{u_{11}} - \cancel{u_{10}}$ $+ \cancel{u_{12}} - \cancel{u_{11}}$ $+ \ldots$ $+ \cancel{u_{2m-1}} - \cancel{u_{2m-2}}$ $+ u_{2m} - \cancel{u_{2m-1}}$ $= u_{2m} - u_9$ $= 6(2m)^2 - 16(2m) + 6 - \left[6(9)^2 - 16(9) + 6\right]$ $= 24m^2 - 32m - 342$

Q6	
(a)	By observation, $d=1$ Hence, $y=x+2+\frac{n}{x-1}=\frac{(x+2)(x-1)+n}{x-1}=\frac{x^2+x-2+n}{x-1}$ By comparison, $a=b=1$
(b)	$y=\frac{x^2+x+c}{x-1}$ $\frac{dy}{dx}=\frac{(2x+1)(x-1)-(x^2+x+c)}{(x-1)^2}=\frac{x^2-2x-1-c}{(x-1)^2}$ For stationary points to occur, $\frac{dy}{dx}=0$ $x^2-2x-1-c=0$ Hence, equation must yield 2 real roots, i.e $D>0$ $(-2)^2-4(1)(-c-1)>0$ $4+4c+4>0$ $c>-2$

(c)	
(d)	Maximum number of solutions = 2 For 2 solutions to occur, $k > 8$

Q7	
(a)(i)	 The line $y = 2$ cuts the graph of $y = \mathrm{f}(x)$ twice. Hence f is not a one-one function and so f^{-1} does not exist.
(a)(ii)	$k = 2$ $y = (x-2)^2$ $x = 2 - \sqrt{y}$ or $2 + \sqrt{y}$ (reject as $0 < x \le 2$) $\therefore\ \mathrm{f}^{-1}(x) = 2 - \sqrt{x}\ ,\ 0 \le x < 4$
(a)(iii)	For composite fg to exist, $\mathrm{R_g} \subseteq \mathrm{D_f}$ From graph/observation, $\mathrm{R_g} = (3, 4]$. Hence, only second 'piece' of f is relevant: $3x - 8,\ 3 < x \le 4$ Therefore, $\mathrm{fg} : x \mapsto 3(\mathrm{e}^x + 3) - 8,\ \ x \in \mathbb{R}\ ,\ \ x \le 0$
(b)(i)	$\mathrm{f}(25) = \mathrm{f}(1 + 6 \times 4) = \mathrm{f}(1) = 1$ $\mathrm{f}(-8) = \mathrm{f}(4 - 3 \times 4) = \mathrm{f}(4) = 4$

(b)(ii)	y (-6,4) (2,4) (10,4) $y=\mathrm{f}\left(\frac{1}{2}x-1\right)$ (-8,1) (-2,0) (6,0) x

Q8

(a)

$$\int \sqrt{15-x^2}\,\mathrm{d}x$$

$$= \int \sqrt{15-x^2}\,\frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta$$

$$= \int \sqrt{15-15\sin^2\theta}\,\sqrt{15}\cos\theta\,\mathrm{d}\theta$$

$$= \int \sqrt{15\left(1-\sin^2\theta\right)}\,\sqrt{15}\cos\theta\,\mathrm{d}\theta$$

$$= 15\int \cos^2\theta\,\mathrm{d}\theta$$

$$= 15\int \frac{\cos 2\theta+1}{2}\,\mathrm{d}\theta$$

$$= \frac{15}{2}\left(\frac{\sin 2\theta}{2}+\theta\right)+C$$

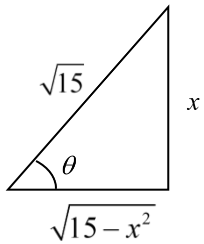
$$= \frac{15}{2}\left(\sin\theta\cos\theta+\theta\right)+C$$

$$= \frac{15}{2}\left[\frac{x}{\sqrt{15}}\cdot\frac{\sqrt{15-x^2}}{\sqrt{15}}+\sin^{-1}\left(\frac{x}{\sqrt{15}}\right)\right]+C$$

$$= \frac{1}{2}x\sqrt{15-x^2}+\frac{15}{2}\sin^{-1}\left(\frac{x}{\sqrt{15}}\right)+C$$

$$x = \sqrt{15}\sin\theta$$

$$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \sqrt{15}\cos\theta$$



(b)

Method ①:

$x = 6\cos\theta,\quad y = 2\sqrt{2}\sin\theta$ — ①

$x^2+y^2 = 15$ — ②

$$(6\cos\theta)^2+\left(2\sqrt{2}\sin\theta\right)^2=15$$

$$36\cos^2\theta+8\sin^2\theta=15$$

$$36\left(1-\sin^2\theta\right)+8\sin^2\theta=15$$

$$36-36\sin^2\theta+8\sin^2\theta=15$$

$$28\sin^2\theta=21$$

$$\sin^2\theta=\frac{3}{4}$$

$$\sin\theta=\frac{\sqrt{3}}{2}\ \text{ or }-\frac{\sqrt{3}}{2}\ \left(\text{rej.}\because\sin\theta>0\text{ for }P\right)$$

$$\theta=\frac{\pi}{3}$$

When $\theta=\frac{\pi}{3}$, $\quad x=6\cos\frac{\pi}{3}=6\left(\frac{1}{2}\right)=3$

$$y=2\sqrt{2}\sin\frac{\pi}{3}=2\sqrt{2}\left(\frac{\sqrt{3}}{2}\right)=\sqrt{6}$$

$\therefore P\left(3,\ \sqrt{6}\right)$

<u>Method ②:</u>

$x=6\cos\theta,\quad y=2\sqrt{2}\sin\theta$

Using $\cos^2\theta+\sin^2\theta=1$,

$$\left(\frac{x}{6}\right)^2+\left(\frac{y}{2\sqrt{2}}\right)^2=1$$

$$\frac{x^2}{36}+\frac{y^2}{8}=1 \qquad \text{— ①}$$

$x^2+y^2=15$

	$y^2 = 15 - x^2$ — ② Substitute ② into ①: $\frac{x^2}{36} + \frac{15 - x^2}{8} = 1$ $8x^2 + 36(15 - x^2) = 288$ $28x^2 = 252$ $x^2 = 9$ $x = 3$ or -3 $(\text{rej.} \because x > 0 \text{ for } P)$ When $x = 3$, $y^2 = 15 - 9 = 6$ $y = \sqrt{6}$ or $-\sqrt{6}$ $(\text{rej.} \because y > 0 \text{ for } P)$ $\therefore P(3, \sqrt{6})$
(c)	**<u>Method ①:</u>** Area of region $R = \underbrace{\int_0^3 y_2 \, \mathrm{d}x}_{\text{curve } C_2} - \underbrace{\int_0^3 y_1 \, \mathrm{d}x}_{\text{curve } C_1}$ **For C₂:** $\int_0^3 y_2 \, \mathrm{d}x = \int_0^3 \sqrt{15 - x^2} \, \mathrm{d}x$ $= \left[\frac{1}{2} x \sqrt{15 - x^2} + \frac{15}{2} \sin^{-1}\left(\frac{x}{\sqrt{15}} \right) \right]_0^3$ $= \frac{3}{2}\sqrt{6} + \frac{15}{2} \sin^{-1}\left(\frac{3}{\sqrt{15}} \right)$ $= \frac{3}{2}\sqrt{6} + \frac{15}{2} \sin^{-1}\left(\frac{\sqrt{15}}{5} \right)$

	For C_1: $\int_0^3 y_1\,\mathrm{d}x = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 2\sqrt{2}\sin\theta \frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta$ $= \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 2\sqrt{2}\sin\theta(-6\sin\theta)\mathrm{d}\theta$ $= -6\sqrt{2}\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 2\sin^2\theta\,\mathrm{d}\theta$ $= -6\sqrt{2}\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} (1-\cos 2\theta)\mathrm{d}\theta$ $= -6\sqrt{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_{\frac{\pi}{2}}^{\frac{\pi}{3}}$ $= -6\sqrt{2}\left[\left(\frac{\pi}{3} - \frac{\sin\left(\frac{2\pi}{3}\right)}{2}\right) - \left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right)\right]$ $= -6\sqrt{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} - \frac{\pi}{2}\right)$ $= -6\sqrt{2}\left(-\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$ $= \sqrt{2}\pi + \frac{3\sqrt{6}}{2}$ Area of region $R = \frac{3}{2}\sqrt{6} + \frac{15}{2}\sin^{-1}\left(\frac{\sqrt{15}}{5}\right) - \sqrt{2}\pi - \frac{3}{2}\sqrt{6}$ $= \frac{15}{2}\sin^{-1}\left(\frac{\sqrt{15}}{5}\right) - \sqrt{2}\pi$

where $m=\dfrac{15}{2}$ and $n=\dfrac{1}{5}$

<u>**Method ②:**</u> **[Not possible]**

Area of region $R=\underbrace{\int_0^3 y_2\,\mathrm{d}x}_{\text{curve } C_2}-\underbrace{\int_0^3 y_1\,\mathrm{d}x}_{\text{curve } C_1}$

For C_2:

$$\int_0^3 y_2\,\mathrm{d}x=\int_0^3\sqrt{15-x^2}\,\mathrm{d}x$$

$$=\left[\frac{1}{2}x\sqrt{15-x^2}+\frac{15}{2}\sin^{-1}\left(\frac{x}{\sqrt{15}}\right)\right]_0^3$$

$$=\frac{3}{2}\sqrt{6}+\frac{15}{2}\sin^{-1}\left(\frac{3}{\sqrt{15}}\right)$$

$$=\frac{3}{2}\sqrt{6}+\frac{15}{2}\sin^{-1}\left(\frac{\sqrt{15}}{5}\right)$$

For C_1:

$$\frac{y^2}{8}=1-\frac{x^2}{36}$$

$$y^2=8-\frac{2}{9}x^2$$

$$y=\sqrt{8-\frac{2}{9}x^2}\ \left(\text{rej}-\text{ve}\because y>0\right)$$

$$\int_0^3 y_1\,\mathrm{d}x=\int_0^3\sqrt{8-\frac{2}{9}x^2}\,\mathrm{d}x$$

$$=\frac{\sqrt{2}}{3}\int_0^3\sqrt{36-x^2}\,\mathrm{d}x$$

$$=\ldots$$

Q9

(a)

Length of $BC = 2r\cos\theta$

Length of $CD = 2r\sin\theta$

$$
\begin{aligned}
P &= 2[AB + BC + CD] \\
&= 2[4r + 2r\cos\theta + 2r\sin\theta] \\
&= 4r(2 + \cos\theta + \sin\theta) \quad (\text{shown})
\end{aligned}
$$

(b)

$$\frac{dP}{d\theta} = 4r(-\sin\theta + \cos\theta)$$

At maximum P, $4r(-\sin\theta + \cos\theta) = 0$

$$
\begin{aligned}
\sin\theta &= \cos\theta \\
\tan\theta &= 1 \\
\theta &= \frac{\pi}{4}
\end{aligned}
$$

<u>**Method ①:**</u>

θ	$\left(\frac{\pi}{4}\right)^-$	$\frac{\pi}{4}$	$\left(\frac{\pi}{4}\right)^+$
$\frac{dP}{d\theta}$	>0	0	<0
	/	—	\

Hence, P is maximum when $\theta = \frac{\pi}{4}$.

	Method ②: $\frac{\mathrm{d}^2P}{\mathrm{d}\theta^2}=4r\left(-\cos\theta-\sin\theta\right)$ When $\theta=\frac{\pi}{4}$, $\frac{\mathrm{d}^2P}{\mathrm{d}\theta^2}=4r\left(-\cos\frac{\pi}{4}-\sin\frac{\pi}{4}\right)<0$ Hence, P is maximum when $\theta=\frac{\pi}{4}$. $\text{Maximum Distance}=4r\left(2+\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\right)$ $=4r\left(2+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)$ $=4r\left(2+\sqrt{2}\right)$ metres

(c)	3 minutes $=180$ seconds Time to complete one loop $=\dfrac{4r\left(2+\sqrt{2}\right)}{6}$ To find maximum value of r, $\dfrac{4r\left(2+\sqrt{2}\right)}{6}=180$ $r=\dfrac{180\times 6}{4\left(2+\sqrt{2}\right)}$ $=\dfrac{270}{\left(2+\sqrt{2}\right)}\times\dfrac{2-\sqrt{2}}{2-\sqrt{2}}$ $=135\left(2-\sqrt{2}\right)$ $=270-135\sqrt{2}$
(d)	Area of $\Delta BCD=\dfrac{1}{2}(2r)\left(2r\cos\theta\sin\theta\right)$ $=2r^2\cos\theta\sin\theta$ Area of $ABCDEF=(4r)(2r)+2\left(2r^2\cos\theta\sin\theta\right)$ $=8r^2+4r^2\cos\theta\sin\theta$ When $\theta=\dfrac{\pi}{4}$ and $r=270-135\sqrt{2}$, Cost of planting grass for $ABCDEF$ $=0.15\times\left[8\left(270-135\sqrt{2}\right)^2+4\left(270-135\sqrt{2}\right)^2\cos\dfrac{\pi}{4}\sin\dfrac{\pi}{4}\right]$ $=\$9380.75<\$10000$ Hence, management can afford to cover the entire shape $ABCDEF$ with grass.

Q10	
(a)	<table><tr><td>nth day</td><td>Number of daily views at the end of nth day</td></tr><tr><td>1</td><td>1196</td></tr><tr><td>2</td><td>$3(1196)$</td></tr><tr><td>3</td><td>$3^2(1196)$</td></tr></table> G.P. with first term = 1196 and common ratio = 3 Number of daily views at the end of the third day $=3^2(1196)$ $=10764$
(b)	Total number of views at the end of the 7th day $=\dfrac{1196(3^7-1)}{3-1}$ $=1307228$ $<5000\ 000$ The video will not go viral.
(c)	$\dfrac{n}{2}\left[2(576)+(n-1)(780)\right]>100\ 000$ $576n+390n^2-390n>100\ 000$ $390n^2+186n-100\ 000>0$ Using G.C., $n<-16.253$ or $n>15.776$ OR

	<table><tr><td>n</td><td>$390n^2+186n-100\ 000$</td><td></td></tr><tr><td>15</td><td>–9460</td><td><0</td></tr><tr><td>16</td><td>2816</td><td>>0</td></tr><tr><td>17</td><td>15872</td><td>>0</td></tr></table> Least $n=16$
(d)	Total number of comments at the end of Day 16 $=\frac{16}{2}\left[2(576)+(16-1)(780)\right]$ OR $=100\ 000+2816$ (from G.C. table) $=102816$ $\qquad$ $=102816$ <table><tr><td>n</td><td>Start of Day</td><td>End of Day</td></tr><tr><td>1</td><td>$102\ 816-w$</td><td>$1.03(102\ 816-w)$ $=1.03(102\ 816)-1.03w$</td></tr><tr><td>2</td><td>$1.03(102\ 816)-1.03w-w$</td><td>$1.03\left[1.03(102\ 816)-1.03w-w\right]$ $=(1.03)^2(102\ 816)-(1.03)^2w-1.03w$</td></tr><tr><td>3</td><td>$(1.03)^2(102\ 816)-(1.03)^2w-1.03w-w$</td><td>$(1.03)^3(102\ 816)-(1.03)^3w-(1.03)^2w-1.03w$</td></tr></table> Number of comments by the end of Day n $=(1.03)^n(102\ 816)-(1.03)^n w-(1.03)^{n-1}w-\ldots-1.03w$ $=(1.03)^n(102\ 816)-\left[(1.03)^n w+(1.03)^{n-1}w+\ldots+1.03w\right]$ $=(1.03)^n(102\ 816)-\frac{1.03w\left[(1.03)^n-1\right]}{1.03-1}$ $=(1.03)^n(102\ 816)-\frac{103w}{3}\left[(1.03)^n-1\right]$ (shown) where $M=102816$

(e)	$(1.03)^{31}(102\ 816)-\frac{103w}{3}\left[(1.03)^{31}-1\right]\leq 0$ $\frac{103w}{3}\left[(1.03)^{31}-1\right]\geq (1.03)^{31}(102\ 816)$ $w\geq 4990.961031$ $w\geq 4991$