Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 Calculate $\sqrt{-\frac{35}{27}-\left(\frac{-11^2}{81}\right)}$.

Answer ..... [1]

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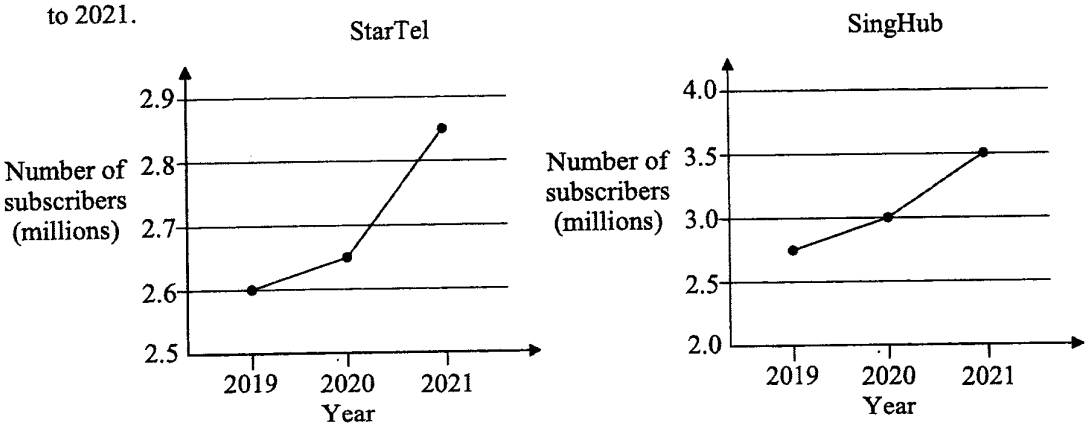
2 These charts show the number of subscribers for two telcommunication companies from 2019 to 2021.

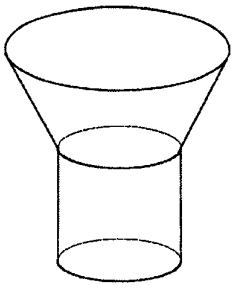


Explain why the charts give the impression that StarTel has a higher increase in the number of subscribers compared with SingHub from 2019 to 2021.

Answer .....

..... [1]

3



The diagram shows a container made from a cylinder and a frustum.
Water is poured into this container.

(a) Which of these diagrams represents the graph of d, the depth of water in centimetres, against t, the time in seconds?

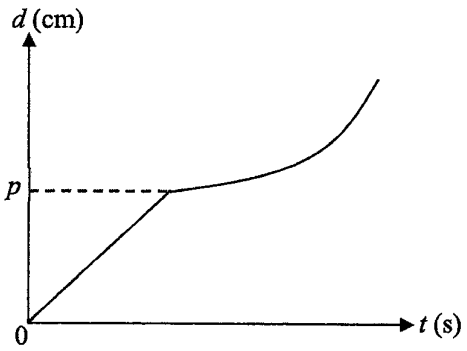


Diagram 1

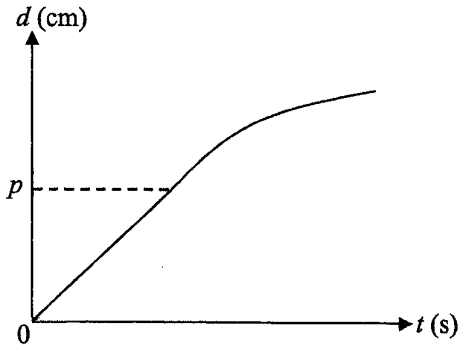


Diagram 2

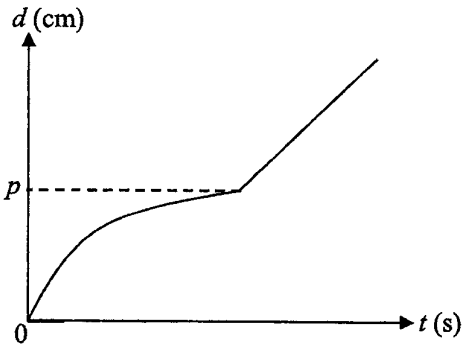


Diagram 3

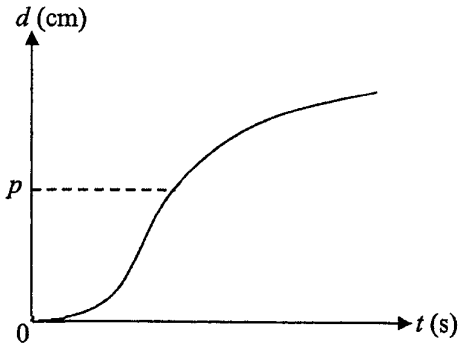


Diagram 4

Answer Diagram ..... [1]

(b) What does the value p in the graph represent?

Answer .....

..... [1]

4 **(a)** Express $4+7x-x^2$ in the form $-(x-h)^2+k$.

Answer ..... [2]

(b) Write down the maximum value of $4+7x-x^2$.

Answer ..... [1]

---

5 A box contains 6 red cubes, 10 blue cubes and n yellow cubes.

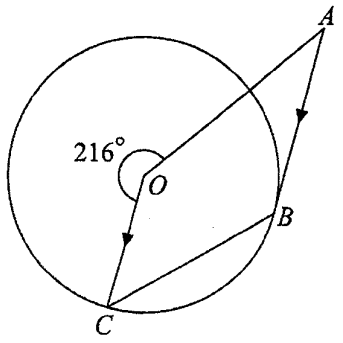
(a) A cube is chosen from the box at random and then replaced.
Write down, in terms of n, the probability that it is **not** a red cube.

Answer ..... [1]

(b) Given that another n red cubes are added into the box, the probability of not choosing a red cube is now $\frac{9}{16}$. Find the total number of cubes in the box.

Answer ..... cubes [3]

6



In the diagram, AB is a tangent to a circle, centre O.
C is a point on the circumference of the circle such that OC is parallel to AB.
Reflex angle $AOC = 216°$. Find angle AOB.

Answer Angle AOB = ... ° [3]

---

7 Solve the equation $\frac{3-2x}{4} = 6 - \frac{x+5}{7}$.

Answer x = ... [3]

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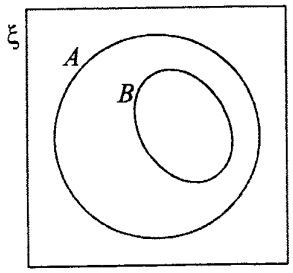
8 **(a)** On the Venn diagram, shade the region which represents $A \cap B'$.

Answer



[1]

(b) $\xi = \{\text{integers } x : 1 \leq x \leq 100\}$
$A = \{\text{perfect squares}\}$
$B = \{\text{odd numbers}\}$
$C = \{\text{integers ending with 3}\}$

(i) List the elements in $A \cap B$.

Answer ..... [1]

(ii) Find the number of elements in $B' \cap C$.

Answer ..... [1]

---

9 In a sequence, the same number is added each time to obtain the next term.
The first five terms of the sequence are

$$11, \ x, \ y, \ z, \ 27, \ \ldots$$

(a) Find the values of x, y, and z.

Answer $x =$ .....

$y =$ .....

$z =$ ..... [2]

(b) Write down an expression, in terms of n, for the nth term of this sequence.

Answer ..... [1]

---

10 **(a)** Given that $x^n = 10$, find the value of $2x^{-3n}$.

Answer ..... [1]

(b) Simplify $\left(\frac{8m^3}{n^{-6}}\right)^{\frac{1}{3}} \div \frac{m^{-4}}{n^3}$.

Answer ..... [2]

---

11 Given that $y = \sqrt{9-4x}$,

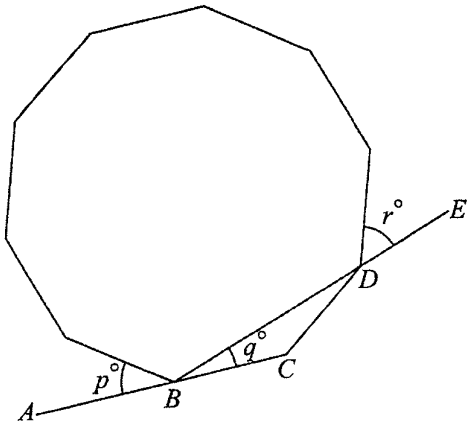
(a) find the value of y when $x = -10$,

Answer $y =$ ..... [1]

(b) rearrange the formula to make x the subject.

Answer $x =$ ..... [2]

12



The diagram shows a regular decagon. The side BC is produced to A and BDE is a straight line. Find the values of p, q and r.

Answer p = ...

q = ...

r = ... [3]

---

13 The table shows the number of hours spent on social media by 100 people who responded to a survey.

Number of hours	1	2	3	4	5
Number of people	19	x	26	13	y

(a) If the mode is 2 hours, write down the smallest value of x and the corresponding value of y.

Answer x = ...

y = ... [2]

(b) If the median is 2.5 hours, find the value of y.

Answer y = ... [1]

---

14 In January 2020, the exchange rate between US dollars and Singapore dollars is US$1 = S$1.3453.

(a) Mr Lim invested US$5000 in an account paying compound interest at 2.5% per year. Calculate the amount of US dollars in the account after two years.

Answer US$ ..... [2]

In January 2022, the exchange rate between Singapore dollars and US dollars is S$1 = US$0.7415.

(b) Mr Lim exchanged all the US dollars in the account to Singapore dollars. Did he make a gain or loss? Show your working clearly.

Answer ..... [3]

15 An area of 36 cm^2 on the map represents an actual area of 9 km^2.

(a) The scale of the map is in the form 1 : n. Find n.

Answer $n =$ .....[2]

(b) Calculate the length of a road on the map, in centimetres, which has an actual length of 1.64 km.

Answer ..... cm [2]

---

16 **(a)** Simplify $(2a-3)^2-4a(a-4)$.

Answer ..... [2]

(b) Factorise completely $14x^2-7xy+3ay-6ax$.

Answer ..... [2]

---

17 **(a)** Express 1188 as the product of its prime factors.

Answer ..... [1]

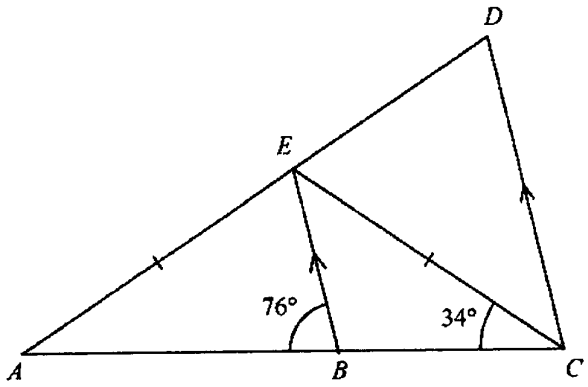
(b) The number $1188m$ is a perfect cube.
Find the smallest positive integer value of m.

Answer ..... [1]

(c) Find the greatest integer that will divide both 1188 and 360 exactly.

Answer ..... [1]

18 In the diagram, AED and ABC are straight lines.
$AE = EC$ and BE is parallel to CD.



(a) Complete these statements.

(i) Angle ECD = ... ° because .....

..... [2]

(ii) Angle EDC = ... ° because .....

..... [2]

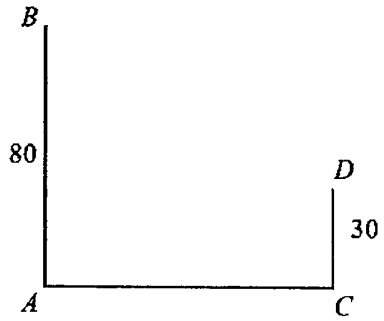
(b) Complete the statement.

..... is the longest side of the triangle EDC because

.....

..... [1]

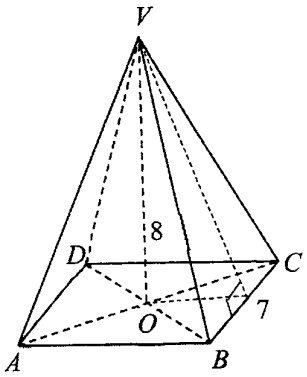
19



In the diagram, CD is a building directly opposite a tower AB, both which are built on horizontal ground, AC.
AB is 80 m high and CD is 30 m high.
The angle of elevation of B from C is $50.6°$.
Calculate the angle of elevation of the top of the tower from the point D.

Answer .....° [4]

20 The figure below shows a square pyramid. VO is vertical to the base $ABCD$, $VO = 8$ cm and $BC = 7$ cm.



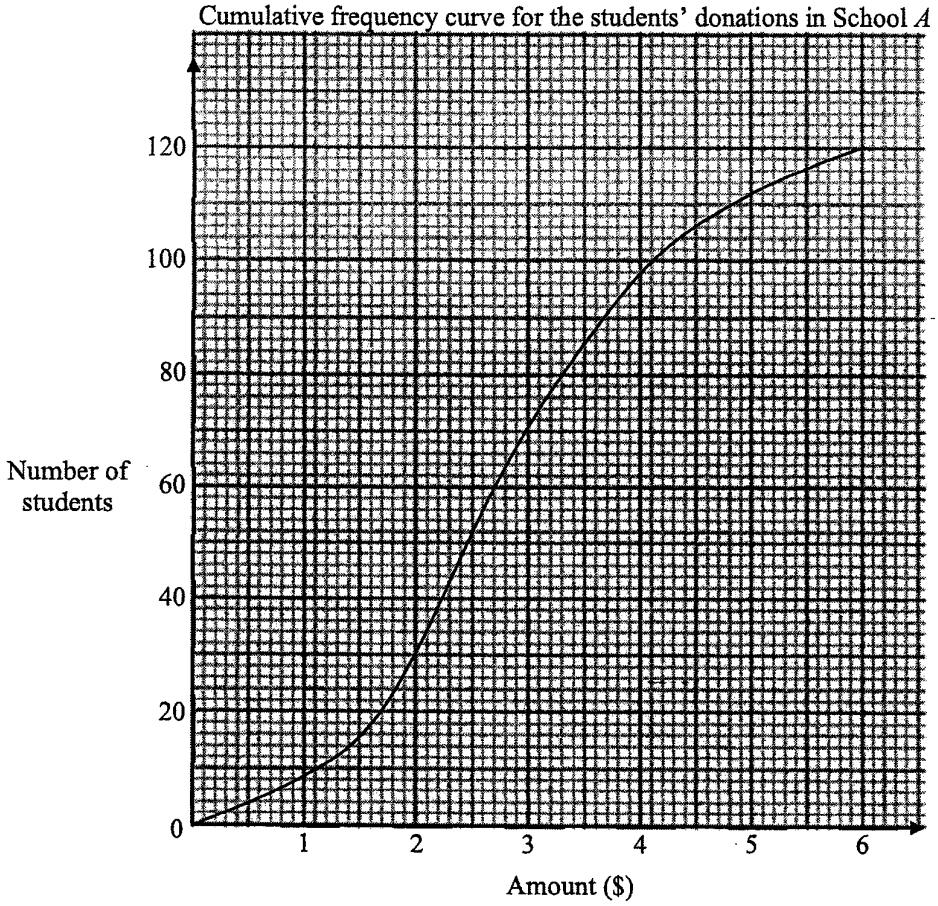
(a) Find the total surface area of the pyramid.

Answer ..... cm^2 [3]

(b) The pyramid is melted and recast into spheres.
The radius of each sphere is 2 mm.
Find the maximum number of spheres that can be recast from the pyramid.

Answer ..... spheres [4]

21 The donations by a group of students in School *A* for the victims of a recent volcanic eruption were recorded.
The cumulative frequency curve below shows the distribution of the donations.



Use your graph to estimate

(a) the median of the donations,

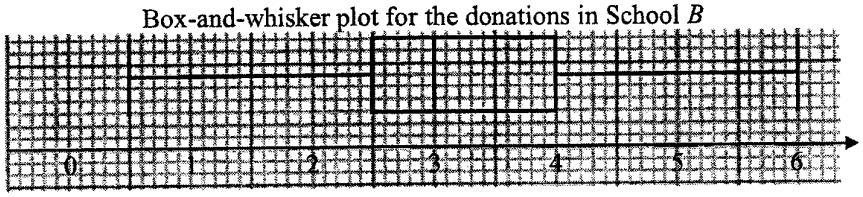
Answer $ ..... [1]

(b) the interquartile range.

Answer $ ..... [2]

(c) The box-and-whisker plot below shows the distribution of donations collected from students in School *B*.

Box-and-whisker plot for the donations in School *B*



Amount ($)

Which school is more generous in the donations? Justify your answer with 2 reasons.

Answer .....

.....

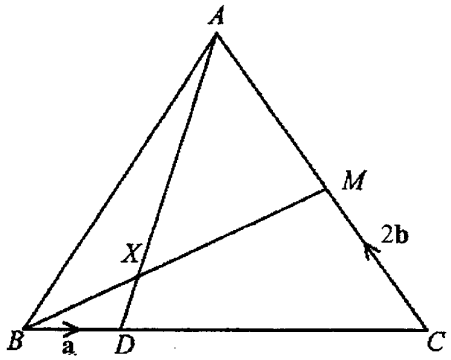
.....

.....

.....

..... [2]

22 In the diagram, $BC = 4BD$ and $DA = 5DX$.
M is the midpoint of AC.
$\overrightarrow{BD} = \mathbf{a}$ and $\overrightarrow{CM} = 2\mathbf{b}$.



(a) Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{DC}$,

Answer $\overrightarrow{DC}$ = ... [1]

(ii) $\overrightarrow{DA}$,

Answer $\overrightarrow{DA}$ = ... [1]

(iii) $\overrightarrow{DX}$.

Answer $\overrightarrow{DX}$ = ... [1]

(b) Show that $\overrightarrow{BX} = \frac{4}{5}(2\mathbf{a} + \mathbf{b})$.

Answer

[1]

(c) Express $\overrightarrow{BM}$ in terms of **a** and **b**, as simply as possible.

Answer ..... [1]

(d) Find

(i) $\frac{BX}{BM}$,

Answer ..... [1]

(ii) $\frac{\text{area of triangle } ABX}{\text{area of triangle } AMX}$,

Answer ..... [1]

(iii) $\frac{\text{area of triangle } ABX}{\text{area of triangle } ABC}$.

Answer ..... [1]

End of Paper

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 **(a)** Express as a single fraction in its simplest form $\frac{9x-2}{x^2-4x+4}+\frac{2}{x-2}$.

Answer ..... [3]

(b) Solve these simultaneous equations.

$$5x-2y=18$$
$$8x+3y=4$$

Answer $x=$ .....

$y=$ ..... [3]

(c) Simplify $\dfrac{9x^2-4}{3x^2-7x-6} \div (2-3x)$.

Answer ..... [3]

(d) Given that $8^{1-2x} = 32^{3-x} \times \left(\dfrac{1}{2}\right)^0$, find the value of x.

Answer $x =$ ..... [3]

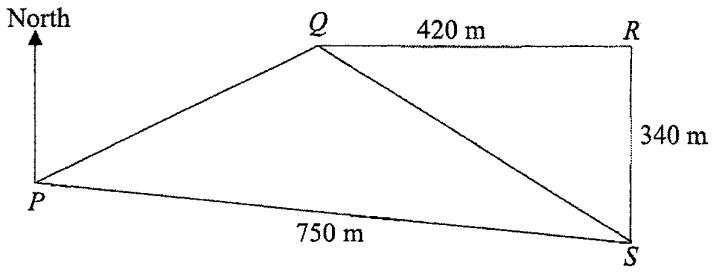
(e) **(i)** Solve the inequalities $1-(5-2x)<\frac{1}{2}(3x+1)\leq\frac{4x+2}{5}$.

Answer ..... [3]

(ii) Hence write down the largest value of x which satisfies the inequalities.

Answer $x=$ ..... [1]

2 Four points P, Q, R and S lie on level ground.
P is 750 m and on a bearing of 280° from S.
Q is 420 m due west of R and S is 340 m due south of R.



(a) Find

(i) the distance QS,

Answer ..... m [1]

(ii) the distance PQ.

Answer ..... m [4]

(b) Given that angle PQS is obtuse, find the bearing of P from Q.

Answer ..... ° [4]

3 Nancy makes T-shirts.
The matrix **M** shows the number of T-shirts of different sizes she makes in one week.

$$\mathbf{M} = \begin{array}{c} \begin{matrix} \text{small} & \text{medium} & \text{large} \end{matrix} \\ \begin{pmatrix} 0 & 3 & 4 \\ 10 & 15 & 1 \end{pmatrix} \end{array} \begin{matrix} \\ \text{Men} \\ \text{Women} \end{matrix}$$

(a) Nancy sells all of these T-shirts to a shop.
She charges \$6 for each small-sized T-shirt, \$8 for each medium-sized T-shirt and \$10 for each large-sized T-shirt.
Represent these amounts in a 3×1 column matrix **N**.

Answer **N** = [1]

(b) **(i)** Evaluate the matrix $\mathbf{P} = \mathbf{MN}$.

Answer **P** = [1]

(ii) State what the elements of **P** represent.

Answer .....

.....

..... [1]

(c) The shopkeeper sells all sizes of men's T-shirts at \$10 each.
He sells all sizes of women's T-shirts at \$11.50 each.

(i) Evaluate $\begin{pmatrix}10 & 11.5\end{pmatrix}\begin{pmatrix}0 & 3 & 4\\ 10 & 15 & 1\end{pmatrix}$.

Answer ..... [1]

(ii) Using matrix multiplication, find the total amount of money that the shopkeeper receives.

Answer \$ ..... [2]

4 **(a)** The table shows some values for $y = \frac{x^3}{4} - x + 1$.

x	-3	-2	-1	0	1	2	3
y	-2.75	1	1.75	1	0.25	1	b

Find the value of b.

Answer $b =$ ..... [1]

(b) Using a scale of 2 cm to represent 1 unit for both the axes, draw the graph of $y = \frac{x^3}{4} - x + 1$ for $-3 \leq x \leq 3$. [3]

(c) On the same grid, draw the graph of $y = \frac{1}{3}x + 1$. [2]

(d) Use your graph to find the value(s) of x where $y = \frac{1}{3}x + 1$ crosses $y = \frac{x^3}{4} - x + 1$.

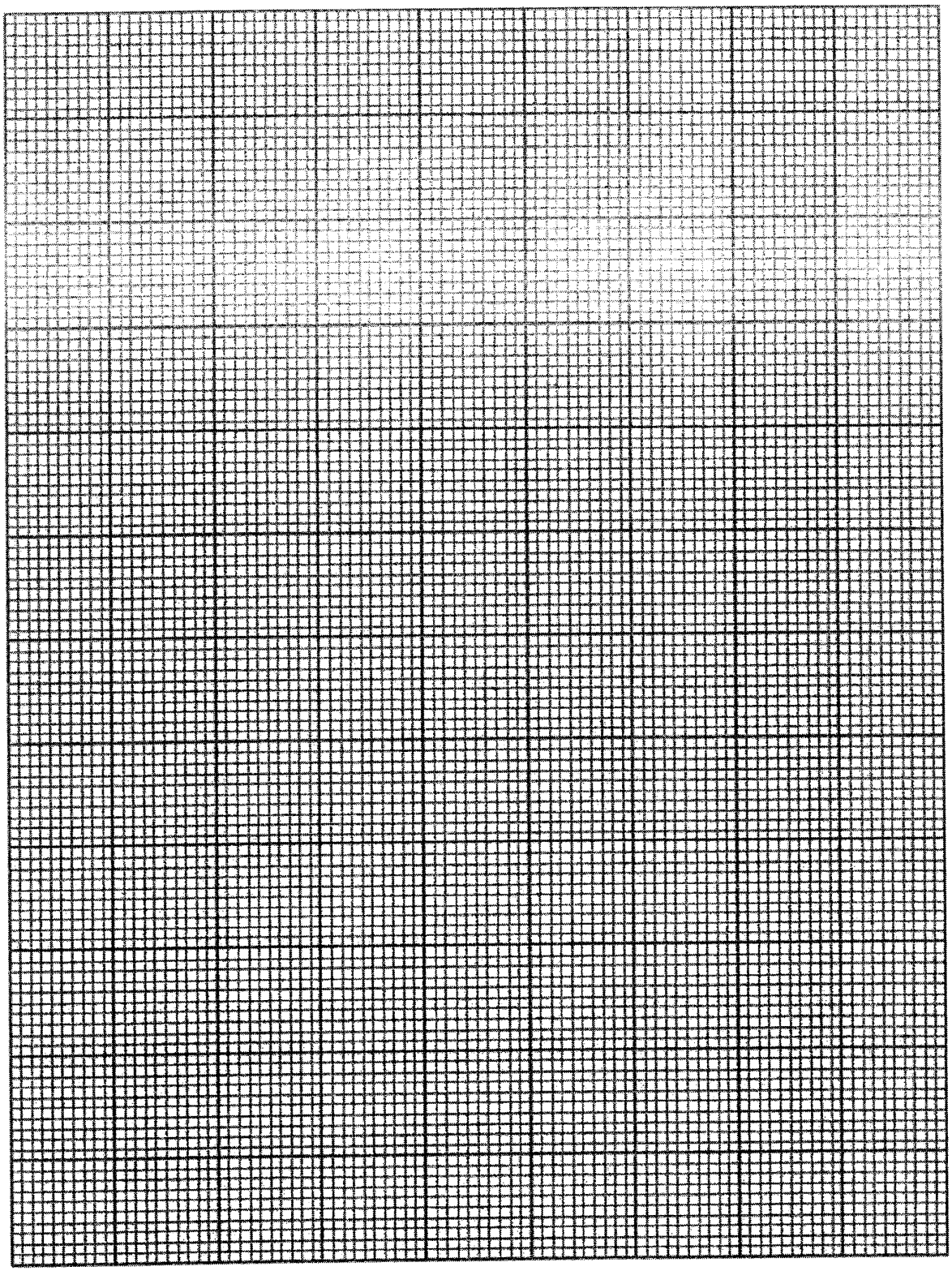
Answer ..... [2]

(e) The value(s) of x where $y = \frac{1}{3}x + 1$ crosses $y = \frac{x^3}{4} - x + 1$ are the solutions of the equation $Ax^3 = Bx$.
Find the value of A and of B, where A and B are integers.

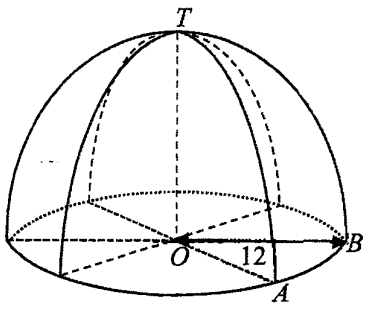
Answer $A =$ .....

$B =$ ..... [2]

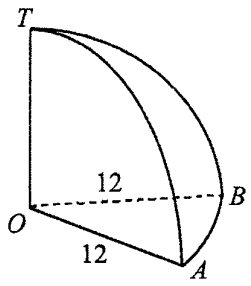
10



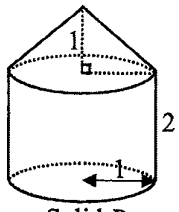
5 Solid P is a metal hemisphere of radius 12 cm and centre O which stands on a horizontal table. It is sliced into 6 equal pieces by cutting vertically downwards through radius OT, as indicated in the diagram. Solid Q is one of these 6 slices. Solid R is made up of a metal cylinder of base radius 1 cm and height 2 cm, surmounted by a cone of base radius 1 cm and height 1 cm.



Solid P



Solid Q



Solid R

Assuming there is no wastage of material, calculate

(a) **(i)** the volume of solid Q,

Answer ..... cm^3 [3]

(ii) the total **curved** surface area of solid R.

Answer ..... cm^2 [2]

(b) Solid Q is then melted and made into many pieces of solid R.

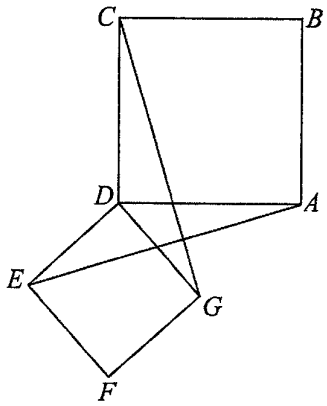
(i) Calculate the volume of solid R.

Answer ..... cm^3 [2]

(ii) Hence, calculate the number of **complete** solid R that can be obtained.

Answer ..... solid R [2]

6 (a)



In the diagram, $ABCD$ and $DEFG$ are squares.

(i) Show that angle ADE = angle CDG.

Answer

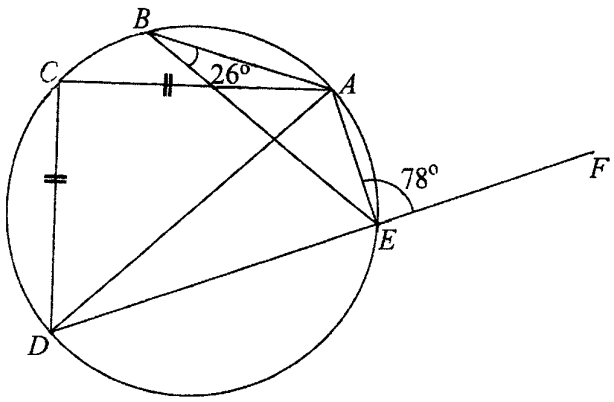
[1]

(ii) Show that triangle ADE is congruent to triangle CDG.
Give a reason for each statement you make.

Answer

[2]

(b)



In the diagram, $CA = CD$, angle $ABE = 26°$ and angle $AEF = 78°$.
DEF is a straight line.

Find, giving reason(s) for each answer,

(i) angle ADE,

Answer Angle ADE = ... ° [1]

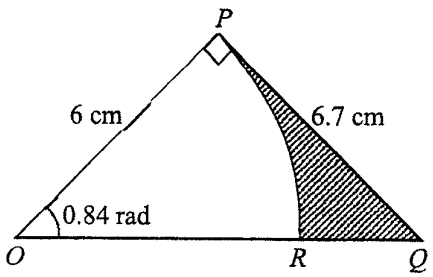
(ii) angle CDA,

Answer Angle CDA = ... ° [3]

(iii) angle CAE.

Answer Angle CAE = ... ° [1]

7



The diagram shows a right-angled triangle OPQ.
OPR is a sector of a circle with centre O and of radius 6 cm.
It is given that angle $POQ = 0.84$ radian and $PQ = 6.7$ cm.

(a) Express 0.84 radian in degrees.

Answer ..... ° [1]

(b) Find the perimeter of the shaded region.

Answer ..... cm [4]

8

Mass (x kg)	Frequency
$1 \leq x < 2$	2
$2 \leq x < 3$	a
$3 \leq x < 4$	8
$4 \leq x < 5$	7

The masses of some durians from shop A are recorded in the table above.

(a) Given that the estimated mean mass of the durians is 3.5 kg, find the value of a.

Answer $a =$ ..... [2]

(b) **(i)** Calculate an estimate of the standard deviation.

Answer ..... kg [1]

(ii) Explain why your answer to part **(b)(i)** is only an **estimate** of the standard deviation.

Answer .....

..... [1]

(c) The same number of durians from shop B were weighed and the masses have the following mean and standard deviation.

Mean (kg)	4.4
Standard Deviation (kg)	1.2

Make two comparisons between the masses of durians from shop A and shop B.

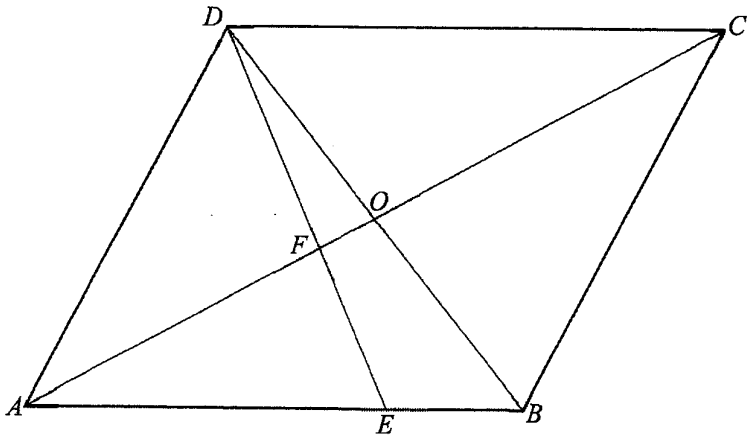
1 .....

.....

2 .....

..... [2]

9



$ABCD$ is a parallelogram whose diagonals, AC and BD, intersect at O.
E is a point on AB such that $AE = 2EB$. DE intersects AC at F.

It is given that O is the point $(0, 0)$. D is the point $(-4, 5)$ and A is the point $(-7, -5)$.

(a) **(i)** Express $\overrightarrow{AD}$ as a column vector.

Answer $\overrightarrow{AD} = \begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [2]

(ii) Find $\left|\overrightarrow{AD}\right|$.

Answer ..... units [2]

(b) Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, express the following vectors in terms of $\mathbf{a}$ and/or $\mathbf{b}$, giving each of your answers in its simplest form.

(i) $\overrightarrow{AC}$

Answer $\overrightarrow{AC} =$ ..... [1]

(ii) $\overrightarrow{CD}$

Answer $\overrightarrow{CD} =$ ..... [1]

(c) Show that $\overrightarrow{DE} = \frac{1}{3}(5\mathbf{b} + \mathbf{a})$.

Answer

[2]

(d) It is given that $\overrightarrow{FA} = \frac{4}{5}\overrightarrow{OA}$.

By finding $\overrightarrow{DF}$, show that D, E and F lie on the same straight line.

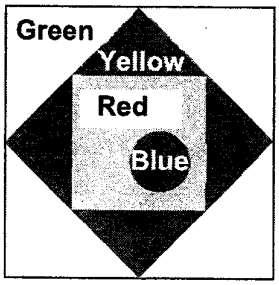
Answer

[3]

(e) Find the numerical value of $\frac{\text{area of triangle } AEF}{\text{area of triangle } CDF}$.

Answer ..... [1]

10



At a game stall, a target board, 1 metre by 1 metre, has four colours as shown. It is made up of three squares and a circle of radius 0.2 metre.
It is assumed that all darts hit the board.

A dart is thrown at the board.

(a) **(i)** Find the probability of hitting the green area.

Answer ..... [1]

(ii) Find, in terms of π, the probability of hitting the blue area.

Answer ..... [1]

(iii) Given that the probability of the dart hitting the yellow area is $\frac{1}{4}$, show that the probability of a dart hitting the red area is $\frac{25-4\pi}{100}$.

Answer

[2]

(b) If two darts are thrown simultaneously, find the probability

(i) of both darts hitting the green area,

Answer ..... [1]

(ii) of both darts separately hitting green and yellow areas,

Answer ..... [2]

(iii) that at least one of the darts hit the yellow area.

Answer ..... [2]

11 Glen and Jane are on a holiday in Germany.
They are planning a trip from Berlin to Munich.
They need to be in Munich latest by 4 p.m. and can choose to travel by train or bus.
They plan to keep their travelling time and cost to the minimum.

Information that Glen and Jane need is shown in the **Travel Information** table below.

Travel Information

Part of train timetable

Berlin	(*Depart*)	09 27	11 22	12 44	14 53
Train Fare	(*Per Pax*)	SGD 68	SGD 88	SGD 78	SGD 58

* Distance from Berlin to Munich is 584.6 km.
* Average train speed is 94.8 km/h.

Part of bus timetable

Depart from Berlin	07 40	07 50	08 15
via	Wunsiedel	Bayreuth	Dresden
Arrive in Munich	16 25	15 30	16 15

Expected average speed of bus on this journey is 45 miles per hour.

For each traveller, the bus fare is charged based on the following:
- SGD 8 online booking surcharge
- SGD 0.05 per kilometre for the journey

* 1 mile is equivalent to 1.6093 kilometres.

(a) Find the distance, in miles, between Berlin and Munich.

Answer ..... miles [1]

(b) Calculate the time taken, in hours and minutes, for the train journey.

Answer ..... hours ..... minutes [2]

(c) Showing clearly all the calculations for the travelling costs and travelling time, recommend the mode of transport from Berlin to Munich for the couple.

Give one advantage and one disadvantage for your recommended choice of transport.

Answer

I would recommend the couple to travel by .....

Advantage: .....

Disadvantage: ..... [7]

End of Paper

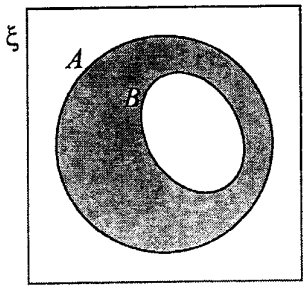
Answer key St. Gabriel

1a	$\frac{11x-6}{(x-2)^2}$
1b	$x=2,\ y=-4$
1c	$\frac{-1}{x-3}$ or $\frac{1}{3-x}$
1d	$x=-12$
1e	$x \le -\frac{1}{7}$
1f	$x=-\frac{1}{7}$
2ai	540 m
2aii	381 m
2b	236.6°
3a	$\begin{pmatrix}6\\8\\10\end{pmatrix}$
3bi	$\begin{pmatrix}64\\190\end{pmatrix}$
3bii	The elements represent the total amount collected from the sales of men's and womens' t-shirts respectively.
3ci	$\begin{pmatrix}115 & 202.5 & 51.5\end{pmatrix}$
3cii	$\begin{pmatrix}115 & 202.5 & 51.5\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = (369)$
4a	b = 4.75
4d	$-2.4 \le x_1 \le -2.2,\ x_2 = 0,\ 2.2 \le x_3 \le 2.4$
4e	$A=3,\ B=16$
5ai	$603\ \text{cm}^3$
5aii	$17.0\ \text{cm}^2$
5bi	$7.33\ \text{cm}^3$
5bii	82
6ai	$\angle ADE = 90° + \angle ADG$ $\angle CDG = 90° + \angle ADG$ $\therefore \angle ADE = \angle CDG$
6aii	$AD = CD$ (sides of square $ABCD$) $\angle ADE = \angle CDG$ [from 6(a)(i)] $DE = DG$ (sides of square $DEFG$) $\therefore \Delta ADE \equiv \Delta CDG$ (SAS)
6bi	$\angle ADE = 26°$ ($\angle$s in same segment)
6bii	51°
6biii	103°

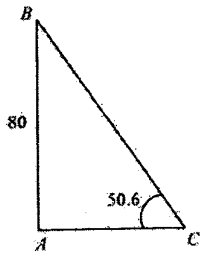
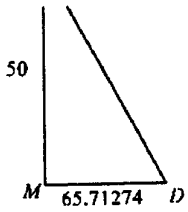
7a	48.1°
7b	14.7 cm
8a	$a = 3$
8bi	0.949 kg
8bii	The **mid-values** of the masses are used instead as the **exact** masses were not known.
8c	The masses of durians from shop B are **heavier (or have higher mass)** than shop A because the mean mass of durians from shop B (4.4kg) is **greater** than shop A (3.5kg). The masses of durians from shop B have a **greater** spread than shop A because the standard deviation of the masses of durians from shop B (1.2kg) is **greater** than shop A (0.949kg).
9ai	$\begin{pmatrix} 3 \\ 10 \end{pmatrix}$
9aii	10.4 units
9bi	$-2\mathbf{a}$
9bii	$\mathbf{a}-\mathbf{b}$
9d	$\overrightarrow{DF} = \frac{1}{5}(5\mathbf{b}+\mathbf{a})$ $\overrightarrow{DE} = \frac{1}{3}(5\mathbf{b}+\mathbf{a})$ Since $\overrightarrow{DF} = k\overrightarrow{DE}$, $\overrightarrow{DF}$ is parallel to $\overrightarrow{DE}$ and **D is a common point**, hence D, E, and F lie on the same straight line.
9e	$\frac{4}{9}$
10ai	$\frac{1}{2}$
10aii	$\frac{\pi}{25}$ or 0.04π
10bi	$\frac{1}{4}$
10bii	$\frac{1}{4}$
10biii	$\frac{7}{16}$
11a	363 miles
11b	6 hrs 10 mins
11c	Recommend Bus Advantage: Lower cost by bus ($71.52) than train ($136) Disadvantage: Longer travelling time by bus (7 hours 40 min) than train (6 hours 10 min) OR Recommend Train Advantage: Shorter travelling time by train (6 hours 10 min) than bus (7 hours 40 min) Disadvantage: Higher cost by train ($136) than bus ($71.52)

2022 Sec 4E5N/4NA(OOS)
EM PRELIM P1
Marking Scheme with Marker's Report

Solutions:			
1	$\sqrt{-\frac{35}{27}-\left(\frac{-11^2}{81}\right)}=\frac{4}{9}$		
2	Different scales used for the vertical axes.		
3a	Diagram 2		
3b	Height of cylindrical part of the container / Depth of water when the cylindrical part of the container is fully filled up		
4a	$4+7x-x^2=-(x^2-7x-4)$ $=-\left[x^2-7x+\left(\frac{-7}{2}\right)^2-\left(\frac{-7}{2}\right)^2-4\right]$ $=-\left[\left(x-\frac{7}{2}\right)^2-\frac{65}{4}\right]$ $=-\left(x-\frac{7}{2}\right)^2+\frac{65}{4}$		
4b	Max value $=\frac{65}{4}$		
5a	$\frac{10+n}{16+n}$		
5b	$\frac{10+n}{16+2n}=\frac{9}{16}$ $160+16n=144+18n$ $2n=16$ $n=8$ Total number of cubes = 32		
6	$\angle AOC=360°-216°$ ($\angle$ at a point) $=144°$ $\angle BOC=\angle OBA$ $=90°$ (tan $\perp$ and alt. $\angle$s. $OC \parallel AB$) $\angle AOB=144°-90$ $=54°$		
7	$\frac{3-2x}{4}=6-\frac{x+5}{7}$ $\frac{3-2x}{4}=\frac{42-x-5}{7}$ $21-14x=148-4x$ $10x=-127$ $x=-12.7$		

Solutions:			
8a			
8b	1, 9, 25, 49, 81		
8c	0		
9a	$x = 15, y = 19, z = 23$		
9b	$4n + 7$ or $11 + 4(n-1)$		
10a	$2x^{-3n} = 2\left(x^{n}\right)^{-3}$ $= \frac{2}{1000}$ $= \frac{1}{500}$ or 0.02 o.e		
10b	$\left(\frac{8m^3}{n^{-6}}\right)^{\frac{1}{3}} \div \frac{m^{-4}}{n^3} = \frac{2m}{n^{-2}} \times \frac{n^3}{m^{-4}}$ $= 2m^5n^5$		
11a	7		
11b	$y = \sqrt{9-4x}$ $y^2 = 9-4x$ $4x = 9-y^2$ $x = \frac{9-y^2}{4}$ or $x = -\frac{y^2-9}{4}$ or $x = \frac{(3+y)(3-y)}{4}$		
12	$p = \frac{360}{10} = 36$ $q = \frac{36}{2} = 18$ $r = 36 + 18 = 54$		
13a	smallest $x = 27$ corresponding $y = 15$		
13b	$y = 50 - (26 + 13)$ $= 11$		
14a	Total in US\$ $= 5000\left(1+\frac{2.5}{100}\right)^{2}$ $= 5253.125$		

Solutions:			
	= US$5253.13		
14b	In Jan 2020, US$5000 = 1.3453 × 5000 = S$6726.50 In Jan 2022, US$5253.125 = 5253.125 ÷ 0.7415 = S$7084.46 Mr Lim made a gain		
15a	$\sqrt{36}$ cm : $\sqrt{9}$ km 6cm : 3km 6cm: 300000cm 1:50000 $n = 50000$		
15b	1.64 km = 164000 cm Length of road $= \frac{164000}{50000}$ $= 3.28$ cm		
16a	$(2a-3)^2 - 4a(a-4)$ $= 4a^2 - 12a + 9 - 4a^2 + 16a$ $= 4a + 9$		
16b	$14x^2 - 7xy + 3ay - 6ax$ $= 7x(2x-y) + 3a(y-2x)$ $= 7x(2x-y) - 3a(2x-y)$ $= (7x-3a)(2x-y)$ or any equivalent form		
17a	$1188 = 2^2 \times 3^3 \times 11$		
17b	$m = 2 \times 11^2$ $= 242$		
17c	$360 = 2^3 \times 3^2 \times 5$ HCF of 360 and 1188 = $2^2 \times 3^2$ =36		
18ai	$\angle ECD = 76° - 34°$ $= 42°$ $\angle EBA = \angle DCA$ (Corresponding Angles, *BE//CD*) Or $\angle BED = \angle ECD$, Alternate Angles, *BE//CD*		
18aii	$\angle EDC = 180° - 34° - 76°$		

Solutions:			
	$= 70°$ **Angle sum of triangle**		
18b	EC is the longest side because it is opposite the largest interior angle. $\angle EDC = 70°$ $\angle CED = 68°$ $\angle ECD = 42°$ Or $\frac{EC}{\sin \angle EDC} = \frac{DC}{\sin \angle DEC} = \frac{ED}{\sin \angle ECD}$ $\frac{EC}{\sin 70^o} = \frac{DC}{\sin 68^o} = \frac{ED}{\sin 42^o}$ Since $\sin 70^o > \sin 68^o > \sin 42^o$ $EC > DC > ED$		
19	 $\frac{AB}{AC} = \tan 50.6°$ $AC = \frac{80}{\tan 50.6°}$ $=65.71274$  $\frac{50}{65.71274} = \tan x^o$ $x^o = \tan^{-1} \frac{50}{65.71274}$ $=37.267^0$ $=37.3^0$ (1d.p)		

Solutions:			
20a	$VX^2 = OX^2 + VO^2$ (Pythagoras' Theorem) $VX = \sqrt{3.5^2 + 8^2}$ $VX = \sqrt{76.25}$ or 8.7321 (5 s.f.) Total surface area of the pyramid = $4\left(\frac{1}{2} \times 7 \times \sqrt{76.25}\right) + 7^2$ $= 171.2497444$ $= 171$ cm^2 (correct to 3 s.f.)		
20b	Vol. of the Pyramid $= \frac{1}{3} \times 7^2 \times 8$ $= 130\frac{2}{3}$ cm^3 Vol. of 1 sphere $= \frac{4}{3} \times \pi \times (0.2)^3$ $= \frac{4}{375}\pi$ cm^3 or $\frac{32}{3}\pi$ mm^3 Maximum number of spheres $= 130\frac{2}{3} \div \frac{4}{375}\pi$ $= 3899.29$ (5 s.f.) $= 3899$ (round down)		
21i	\$2.70		
21ii	IQR = \$3.70-\$2.00 \$1.70		
21iii	**Acceptable Answers** School B, since the median of donations of School B (\$3.00) is higher the median of donations of School A (\$2.70). School B has a lower interquartile range of (\$1.30) compared to School A (\$1.70), donations are more consistent/less widespread with the greater donations than School A. School B has a higher upper quartile (\$4) than School A (\$3.60), so 25% of students in School B donated \$4 of more compared to less than 25% of students in School A.		

Solutions:			
22ai	$\overrightarrow{DC} = 3\mathbf{a}$		
22aii	$\overrightarrow{DA} = \overrightarrow{DC} + \overrightarrow{CA}$ $= 3\mathbf{a} + 2\overrightarrow{CM}$ $= 3\mathbf{a} + 4\mathbf{b}$		
22aii	$\overrightarrow{DX} = \frac{1}{5}\overrightarrow{DA}$ $= \frac{1}{5}(3\mathbf{a} + 4\mathbf{b})$		
22b	$\overrightarrow{BX} = \overrightarrow{BD} + \overrightarrow{DX}$ $= \mathbf{a} + \frac{1}{5}(3\mathbf{a} + 4\mathbf{b})$ $= \mathbf{a} + \frac{3}{5}\mathbf{a} + \frac{4}{5}\mathbf{b}$ $= \frac{8}{5}\mathbf{a} + \frac{4}{5}\mathbf{b}$ $= \frac{4}{5}(2\mathbf{a} + \mathbf{b})$ $\overrightarrow{BX} = \frac{4}{5}(2\mathbf{a} + \mathbf{b})$ (Shown)		
22c	$\overrightarrow{BM} = \overrightarrow{BC} + \overrightarrow{CM}$ $= 4\mathbf{a} + 2\mathbf{b}$ $= 2(2\mathbf{a} + \mathbf{b})$		
22di	$\frac{BX}{BM} = \frac{\lvert\frac{4}{5}(2\mathbf{a} + \mathbf{b})\rvert}{\lvert 2(2\mathbf{a} + \mathbf{b})\rvert} = \frac{2}{5}$		
22dii	$\frac{\text{area of } \Delta ABX}{\text{area of } \Delta AMX} = \frac{\frac{1}{2} \times BX \times \perp h}{\frac{1}{2} \times MX \times \perp h} = \frac{BX}{MX}$ $= \frac{2}{3}$		
22diii	$\frac{\text{area of } \Delta ABX}{\text{area of } \Delta ABC} = \frac{\text{area of } \Delta ABX}{\text{area of } \Delta ABM} \times \frac{\text{area of } \Delta ABM}{\text{area of } \Delta ABC} = \frac{2}{5} \times \frac{1}{2}$ $= \frac{1}{5}$		

2022 Sec 4E5N/4NA(OOS)
EM Prelim P2
Marking Scheme with Marker's Report

Solution:			
1a	$\frac{9x-2}{x^2-4x+4}+\frac{2}{x-2}$ $=\frac{9x-2}{(x-2)^2}+\frac{2}{x-2}$ --- factorization of denominator $=\frac{9x-2}{(x-2)^2}+\frac{2(x-2)}{(x-2)^2}$ --- correct denom. & numerator $=\frac{11x-6}{(x-2)^2}$		
1b	Any method to solve either substitution or elimination $x=2,\ y=-4$ --- A1 each		
1c	$\frac{(3x+2)(3x-2)}{(3x+2)(x-3)}\times\frac{1}{(2-3x)}$ $=\frac{-(3x+2)(2-3x)}{(3x+2)(x-3)}\times\frac{1}{(2-3x)}$ $=\frac{-1}{x-3}$ or $\frac{1}{3-x}$		
1d	$(2^3)^{1-2x}=(2^5)^{3-x}$ Comparing indices:- $3(1-2x)=5(3-x)$ $3-6x=15-5x$ $x=-12$		
1e	$1-5+2x<\frac{3}{2}x+\frac{1}{2}$; $\frac{3}{2}x+\frac{1}{2}\le\frac{4}{5}x+\frac{2}{5}$ $\frac{1}{2}x<4\frac{1}{2}$; $\frac{7}{10}x\le-\frac{1}{10}$ $x<9$; $x\le-\frac{1}{7}$ $x\le-\frac{1}{7}$		
1f	$x=-\frac{1}{7}$		

Solutions:			Marker's comments
2ai	$QS = \sqrt{420^2 + 340^2} = \sqrt{292000}$ $= 540.37$ $\approx 540\text{ m}$		
2aii	$\angle RSQ = \tan^{-1}\frac{420}{340}$ $= 51.009°$ $\angle QSP = 360° - 21.009° - 280°$ *(∠s at a point)* $= 28.991°$ $PQ = \sqrt{750^2 + \left(\sqrt{292000}\right)^2 - 2(750)(\sqrt{292000})\cos 28.991}$ $= 381.571$ $= 381\text{ m}$		
2b	$\frac{\sin \angle PQS}{750} = \frac{\sin 28.991°}{381.46}$ $\angle PQS = 72.35°(acute)$ (72.33°) *obtuse* $\angle PQS = 180° - 72.35°$ $= 107.65°$ (107.67°) *or* *Find using Cosine rule* $\angle RQS = 180° - 90° - 51.009°$ *(∠ sum of Δ)* $= 38.991°$ Bearing required $= 128.991° + 107.65°$ $= 236.6°$ (236.7°)		
3a	$\begin{pmatrix} 6 \\ 8 \\ 10 \end{pmatrix}$		
3bi	$\begin{pmatrix} 64 \\ 190 \end{pmatrix}$		
3bii	The elements represent the **total amount collected** from the sales of men's and womens' t-shirts **respectively**.		
3ci	$\begin{pmatrix} 115 & 202.5 & 51.5 \end{pmatrix}$		
3cii	$\begin{pmatrix} 115 & 202.5 & 51.5 \end{pmatrix} = (369)$ ***or*** $\begin{pmatrix} 10 & 11.5 \end{pmatrix}\begin{pmatrix} 7 \\ 26 \end{pmatrix} = (369)$ **Total amount received = $369**		

Solution:			
4a	$b = 4.75$		
4b	All points plotted correctly Correct Labelled Axis & Scale *Smooth* curve drawn with curve ruler passing through all points		
4c	Ruled straight line through (0,1) Must show gradient = $\frac{1}{3}$		
4d	$-2.4 \le x_1 \le -2.2,\ x_2 = 0,\ 2.2 \le x_3 \le 2.4$		
4e	$A=3,\ B=16$ $\frac{1}{3}x+1=\frac{x^3}{4}-x+1$ $4x+12=3x^3-12x+12$ $3x^3=16x$		
5ai	Vol of solid $P = \frac{2}{3}\times\pi\times12^3$ $= 1152\,\pi$ $= 3619.114737\ \text{cm}^3$ Vol of solid $Q = 3619.114737 \div 6$ $\approx 603\ \text{cm}^3$		
5aii	Slant height of cone $= \sqrt{2}$ cm Total Curved surface area of solid R $=(\pi\times1\times\sqrt{2})+(2\times\pi\times1\times2)$ $=17.0\ \text{cm}^2$		
5bi	Vol of solid R $=(\frac{1}{3}\times\pi\times1^2\times1)+(\pi\times1^2\times2)$ $=7.33038\ \text{cm}^3$ $=7.33\ \text{cm}^3$		
5bii	Number of complete solid R that can be obtained = 82 (nearest whole number)		
6ai	$\angle ADE = 90° + \angle ADG$ $\angle CDG = 90° + \angle ADG$		

Solution:			
	$\therefore \angle ADE = \angle CDG$		
6aii	$AD = CD$ (sides of square $ABCD$) $\angle ADE = \angle CDG$ [from 6(a)(i)] $DE = DG$ (sides of square $DEFG$) $\therefore \Delta ADE \equiv \Delta CDG$ (SAS)		
6bi	$\angle ADE = 26°$ ($\angle$s in same segment)		
6bii	$\angle AED = 180 - 78$ (adj. $\angle$s on a st. line) $= 102°$ $\angle ACD = 180 - 102$ ($\angle$s in opp. segments) $= 78°$ $\angle CDA = \frac{180° - 78°}{2}$ (base $\angle$ of isos. triangle) $= 51°$		
6biii	$\angle CAE = 180 - 26 - 51$ (angles in opp. segments) $= 103°$		
7a	48.1°		
7b	Arc length, $PR = 6(0.84)$ $= 5.04$ cm $OQ^2 = 6^2 + 6.7^2$ $OQ = 6^2 + 6.7^2$ $= 8.9939$ $RQ = 8.9939 - 6$ $= 2.9939$ Perimeter $= 5.04 + 2.9939 + 6.7$ $= 14.7$ cm	Answers may vary slightly due to students using either Trigo Ratios, Pythagoras' Theorem or Cosine Rule to find OQ.	
8a	$\frac{2\times1.5 + a\times2.5 + 8\times3.5 + 7\times4.5}{17 + a} = 3.5$ $2.5a + 62.5 = 59.5 + 3.5a$ $a = 3$		
8bi	0.949 kg		

Solution:			
8bii	The **mid-values** of the masses are used instead as the **exact** masses were not known. Masses were given in range. (adjusted answer key)		
8c	The masses of durians from shop *B* are **heavier (or have higher mass)** than shop *A* because the mean mass of durians from shop *B* (4.4kg) is greater than shop *A* (3.5kg). The masses of durians from shop *B* have a **greater** spread than shop *A* because the standard deviation of the masses of durians from shop *B* (1.2kg) is greater than shop *A* (0.949kg). Or The masses of durians from shop *B* are less consistent than shop *A* because the standard deviation of the masses of durians from shop *B* (1.2kg) is greater than shop *A* (0.949kg).		
9ai	$\overrightarrow{AD}$ $= \overrightarrow{OD} - \overrightarrow{OA}$ $= \begin{pmatrix} -4 \\ 5 \end{pmatrix} - \begin{pmatrix} -7 \\ -5 \end{pmatrix}$ $= \begin{pmatrix} 3 \\ 10 \end{pmatrix}$		
9aii	$\left\lvert \overrightarrow{AD} \right\rvert$ $= \sqrt{3^2 + 10^2}$ $= 10.4$ units		
9bi	$-2\mathbf{a}$		
9bii	$\mathbf{a} - \mathbf{b}$		

Solution:			
9c	$\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE}$ $= 2\mathbf{b} + \frac{1}{3}(\mathbf{a} - \mathbf{b})$ $= 2\mathbf{b} + \frac{1}{3}\mathbf{a} - \frac{1}{3}\mathbf{b}$ $= \frac{5}{3}\mathbf{b} + \frac{1}{3}\mathbf{a}$ $= \frac{1}{3}(5\mathbf{b} + \mathbf{a})$ (shown) Also accepted: $\overrightarrow{DE} = \overrightarrow{DA} + \overrightarrow{AE}$ or $= \mathbf{b} + \mathbf{a} + \frac{2}{3}(\mathbf{b} - \mathbf{a})$ $= \frac{1}{3}(5\mathbf{b} + \mathbf{a})$ (shown)		
9d	$\overrightarrow{DF} = \overrightarrow{DO} + \overrightarrow{OF}$ $= \mathbf{b} + \frac{1}{5}\mathbf{a}$ $= \frac{1}{5}(5\mathbf{b} + \mathbf{a})$ Must show same vector as 9c $\overrightarrow{DE} = \frac{1}{3}(5\mathbf{b} + \mathbf{a})$ Or $= \frac{5}{3}\left(\mathbf{b} + \frac{1}{5}\mathbf{a}\right)$ $= \frac{5}{3}\overrightarrow{DF}$ Since $\overrightarrow{DF} = k\overrightarrow{DE}$, $\overrightarrow{DF}$ is parallel to $\overrightarrow{DE}$ and ***D* is a common point**, hence *D*, *E*, and *F* lie on the same straight line.		
9e	$\frac{4}{9}$		
10ai	$\frac{1}{2}$		
10aii	$\frac{\pi}{25}$ or 0.04π		

Solution:			
10aiii	$1-\frac{1}{2}-\frac{1}{4}-\frac{\pi}{25}$ $=\frac{1}{4}-\frac{\pi}{25}$ $=\frac{25-4\pi}{100}$ (shown)	P(Red) = P(Yellow-Blue) or $=\frac{1}{4}-\frac{\pi}{25}$ $=\frac{25-4\pi}{100}$ (shown)	
10bi	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$		
10bii	$\left(\frac{1}{2}\times\frac{1}{4}\right)+\left(\frac{1}{4}\times\frac{1}{2}\right)$ $=\frac{1}{4}$		

Solution:			Marker's comments
10biii	P(at least 1 dart hit yellow area) = 1− P(both darts not hitting **yellow** area) $=1-\left(\frac{3}{4}\right)^2$ $=\frac{7}{16}$		
11a	1.6093 km → 1 mile 1 km → $\frac{1}{1.6093}$ miles 584.6 km → 363 miles		
11b	Time taken for train journey $=\frac{584.6}{94.8}$ $=\frac{37}{6}$ h = 6 hrs 10 mins		

Solution			Marker's comments
11c	**By Train** Both Glen and Jane can depart only at 09 27 from Berlin in order to reach Munich latest by 4pm. Cost of train ride $=68\times2$ $=\$136$ **By Bus** Both Glen and Jane have to take the bus via Bayreuth from Berlin in order to reach Munich latest by 4pm. Duration of bus ride = 7 hrs 40 mins Distance covered by bus in miles $=7\frac{2}{3}\times45$ $=345$ miles $\times$ 1.6093 $=555.2085$ km Cost of bus ride $=[8+(555.2085\times0.05)\times2$ $=\$71.52$ **Recommend Bus** Advantage: Lower cost by bus ($71.52) than train ($136) Disadvantage: Longer travelling time by bus (7 hours 40 min) than train (6 hours 10 min) OR **Recommend Train** Advantage: Shorter travelling time by train (6 hours 10 min) than bus (7 hours 40 min) Disadvantage: Higher cost by train ($136) than bus ($71.52)		