Anglo - Chinese School

(Independent)



FINAL EXAMINATION 2022 YEAR 3 INTEGRATED PROGRAMME CORE MATHEMATICS PAPER 2

Thursday

6th October 2022

1 hour 30 minutes

ADDITIONAL MATERIALS:

Answer Paper (6 sheets) Graph Paper (1 sheet)

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so. A calculator is required for this paper. Answer all the questions on the answer sheets provided. At the end of the examination, fasten the answer sheets together. Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures. Answers in degrees are to be given to one decimal place.

INFORMATION FOR STUDENTS

The maximum mark for this paper is 80.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer all the questions on the answer sheets provided. Begin each question on a new page.

1. [Maximum mark: 6]

(a) Simplify
$$\frac{a^2 - 2ab + b^2 + b - a}{a^2 - b^2}$$
. [3]

(b) Subtract $\frac{4}{x^2-4}$ from $\frac{1}{2-x} - \frac{1}{x+2}$, expressing your answer as a single fraction in its simplest form. [3]

2. [Maximum mark: 8]

Joe bought x number of books, each at the same price, for a total cost of \$336.

(a) Write down an expression for the cost of each book in terms of x. [1]

Joe sold 20 of them for \$480, and the rest at a loss of \$4 per book.

- (b) Write down an expression for the total amount, in dollars, he received for all the books. [2]
- (c) Given that Joe made a profit of \$184 altogether, form an equation in x and show that it reduces to $x^2 94x + 1680 = 0$. [2]
- (d) Hence, solve the equation $x^2 94x + 1680 = 0$ and state the cost price of each book. [3]
- 3. [Maximum mark: 13]

(a) Evaluate
$$\log_3\left(\frac{2.15+e^2}{0.25^{-2}}\right)$$
, leaving your answer correct to 2 significant figures. [3]

(b) Given that
$$(144p^4)^{\frac{3}{2}} \div (216p^{-3})^{-\frac{1}{3}} = 2^x 3^y p^z$$
, evaluate x , y and z . [4]

(c) Simplify
$$\frac{6^{2w} + 2(3^{2w})}{4^{w+1} + 8}$$
. [3]

(d) Find the range of values of x if $7^{2x^2-5x-2} < 7$. [3]

4. [Maximum mark: 8]

(a) Solve the equation $3^{y+1} = 4^y$. [4]

(b) Solve the equation
$$\log_3(x) + \log_3(x-3) = \log_9(9x^2)$$
. [4]

5. [Maximum mark: 12]

In the figure, *A*, *B* and *C* are three points on a horizontal field. *A* is due west of *B*, the bearing of *B* from *C* is 125°, AB = 430 m and BC = 460 m.



(b)	Given that the angle of elevation of the hot air balloon from B is 5.2°, find the an	gle of
	elevation of the hot air balloon from A.	[3]

6. [Maximum mark: 9]

(a)	Find the smallest positive integer value of m given that the straight line $y =$	mx-5
	intersects the curve $y = x^2 - 2m$ at two distinct points.	[5]

(b) Prove that
$$3x^5 - 25x^3 + 60x = 0$$
, has only one real root. [4]

7. [Maximum mark: 14]

Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = 3x + \frac{60}{x} - 35$. Some corresponding values of x and y are given in the following table:

x	1.5	2	2.5	3	4	5	6	7
у	9.5	а	-3.5	-6	b	-8	-7	-5.4

(a) Find the value of a and of b.

(b) Taking 2 cm to represent 1 unit on the *x*-axis and 1 cm to represent 1 unit on the *y*-axis,

draw the graph of
$$y = 3x + \frac{60}{x} - 35$$
 for $1.5 \le x \le 7$. [4]

Use your graph to find

- (c) the least value of y, [1]
- (d) the range of values of x for which $y \le -7$, [2]

(e) the gradient of the curve at the point x = 2 by drawing a suitable straight line, [2]

(f) the solutions of the equation
$$5x + \frac{60}{x} - 40 = 0$$
 by drawing a suitable straight line. [3]

8. [Maximum mark: 10]

(a) It is given that

$$\log_a x = 12$$
$$\log_b x = 60$$

- (i) Express a in terms of b.
- (ii) If $\log_{abc} x = 6$, prove that $\log_c x = 15$. [4]
- (b) Using the two single digits a and b, explain clearly whether there exists a number, ab, such that the sum of ab and its reverse ba is a prime number. [3]

----- END OF PAPER 2 ------

[3]

[2]

Answers:

1a)
$$\frac{(a-b)-1}{(a+b)}$$

1b) $\frac{-2}{x-2}$
2a) $\frac{336}{x}$
2b) $480 + (x-20)\left(\frac{336}{x}-4\right)$
2d) $\$4.80 \text{ or }\14
3a) -0.47
 $x = 7$
3b) $y = 4$
 $z = 5$
3c) $\frac{3^{2w}}{4}$
3d) $-\frac{1}{2} < x < 3$
4a) 3.82
4b) 6
5ai) 269
5aii) 66.4
5aii) 011.4
5aiv) 56727
5b) 8.84
6a) 3
7a) $a = 1, b = -8$













Answer: -8.17

Answer: $3.33 \le x \le 6$



Answer: Gradient = -12

7f)



Answer: 2 or 6

8a) $a = b^5$