HWA CHONG INSTITUTION
JC2 Preliminary Examination
Higher 3

CANDIDATE NAME	

CENTRE NUMBER						**INDEX NUMBER**				

PHYSICS

9814/01

Paper 1

21 September 2021

3 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, glue or correction fluid.

The use of an approved scientific calculator is expected, where appropriate.

Section A
Answer **all** questions.
You are advised to spend about 1 hour and 50 minutes on Section A.

Section B
Answer **two** questions only.
You are advised to spend about 35 minutes on each question in Section B.

The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use		
Section A		
1		**7**
2		**7**
3		**12**
4		**9**
5		**9**
6		**16**
Section B		
7		**20**
8		**20**
9		**20**
Deductions		
Total		**100**

Data

speed of light in free space	c	=	3.00×10^{8} m s^{-1}
permeability of free space	μ_0	=	$4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space	ε_0	=	8.85×10^{-12} F m^{-1}
			$(1/(36\pi)) \times 10^{-9}$ F m^{-1}
elementary charge	e	=	1.60×10^{-19} C
the Planck constant	h	=	6.63×10^{-34} J s
unified atomic mass constant	u	=	1.66×10^{-27} kg
rest mass of electron	m_e	=	9.11×10^{-31} kg
rest mass of proton	m_p	=	1.67×10^{-27} kg
molar gas constant	R	=	8.31 J K^{-1} mol^{-1}
the Avogadro constant	N_A	=	6.02×10^{23} mol^{-1}
the Boltzmann constant	k	=	1.38×10^{-23} J K^{-1}
gravitational constant	G	=	6.67×10^{-11} N m^{2} kg^{-2}
acceleration of free fall	g	=	9.81 m s^{-2}

Formulae

uniformly accelerated motion	s	=	$ut + \frac{1}{2}at^2$
	v^2	=	$u^2 + 2as$
moment of inertia of rod through one end	I	=	$\frac{1}{3}ML^2$
moment of inertia of hollow cylinder through axis	I	=	$\frac{1}{2}M\left(r_1^2 + r_2^2\right)$
moment of inertia of solid sphere through centre	I	=	$\frac{2}{5}MR^2$
moment of inertia of hollow sphere through centre	I	=	$\frac{2}{3}MR^2$
work done on/by a gas	W	=	$p\Delta V$
hydrostatic pressure	p	=	ρgh
gravitational potential	ϕ	=	$-Gm/r$
Kepler's third law of planetary motion	T^2	=	$\frac{4\pi^2 a^3}{GM}$
temperature	T / K	=	T / °C + 273.15

pressure of an ideal gas	p	$=$	$\frac{1}{3}\frac{Nm}{V}\langle c^2\rangle$
mean translational kinetic energy of an ideal gas molecule	E	$=$	$\frac{3}{2}kT$
displacement of particle in s.h.m.	x	$=$	$x_0 \sin \omega t$
velocity of particle in s.h.m.	v	$=$	$v_0 \cos \omega t$
		$=$	$\pm\omega\sqrt{\left(x_0^2 - x^2\right)}$
electric current	I	$=$	$Anvq$
resistors in series	R	$=$	$R_1 + R_2 + \ldots$
resistors in parallel	$1/R$	$=$	$1/R_1 + 1/R_2 + \ldots$
capacitors in series	$1/C$	$=$	$1/C_1 + 1/C_2 + \ldots$
capacitors in parallel	C	$=$	$C_1 + C_2 + \ldots$
energy in a capacitor	U	$=$	$\frac{1}{2}CV^2$
electric potential	V	$=$	$\frac{Q}{4\pi\varepsilon_0 r}$
electric field strength due to a long straight wire	E	$=$	$\frac{\lambda}{2\pi\varepsilon_0 r}$
electric field strength due to a large sheet	E	$=$	$\frac{\sigma}{2\varepsilon_0}$
alternating current/voltage	x	$=$	$x_0 \sin \omega t$
magnetic flux density due to a long straight wire	B	$=$	$\frac{\mu_0 I}{2\pi d}$
magnetic flux density due to a flat circular coil	B	$=$	$\frac{\mu_0 N I}{2r}$
magnetic flux density due to a long solenoid	B	$=$	$\mu_0 n I$
energy in an inductor	U	$=$	$\frac{1}{2}LI^2$
RL series circuits	τ	$=$	$\frac{L}{R}$
RLC series circuits (underdamped)	ω	$=$	$\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$
radioactive decay	x	$=$	$x_0 \exp(-\lambda t)$
decay constant	λ	$=$	$\frac{\ln 2}{t_{1/2}}$

Section A

Answer **all** questions in this section.

You are advised to spend about 1 hour 50 minutes on this section.

1 An ideal gas confined in a cylinder is put through a closed cycle. Initially the gas is at state **A** where it has pressure P_o, volume V_o, and temperature T_o. First, its pressure is tripled under constant volume. It then expands at constant temperature to its original pressure and finally it is compressed at constant pressure to its original volume.

The *P*-*V* diagram of the cyclic process is shown in Fig. 1.

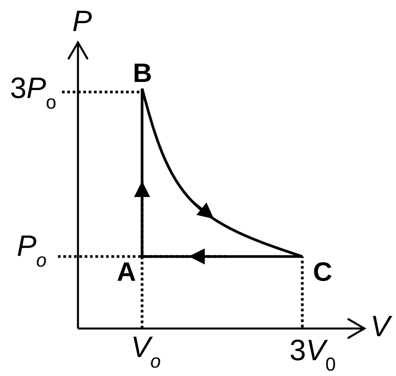


Fig. 1

(a) Determine an expression for the temperature of the gas at state **C**, in terms of T_0.

[1]

(b) Show that the net work done by the gas in one complete cycle is $nRT_0(3\ln 3 - 2)$.

[3]

(c) Explain for each process below whether heat is supplied to the gas.

A to **B**: ……………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

B to **C**: ……………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

C to **A**: ……………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

…………………………………………………………………………………………………… [3]

[Total: 7]

2 **(a)** A block of mass *m* is connected to two springs which have spring constants of k_1 and k_2, respectively, as shown in Fig. 2.1.

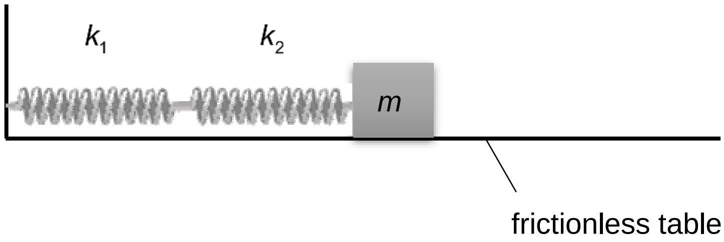


Fig. 2.1

The block moves on a frictionless table after it is displaced from equilibrium and released. It performs simple harmonic motion. Show that the period of the block is

$$T = 2\pi\sqrt{\frac{m\left(k_1 + k_2\right)}{k_1 k_2}}$$

[4]

(b) Three objects of uniform density – a hollow cylinder (**A**), a solid cylinder (**B**) and a solid sphere (**C**) – are placed at the top of an incline as shown in Fig. 2.2.

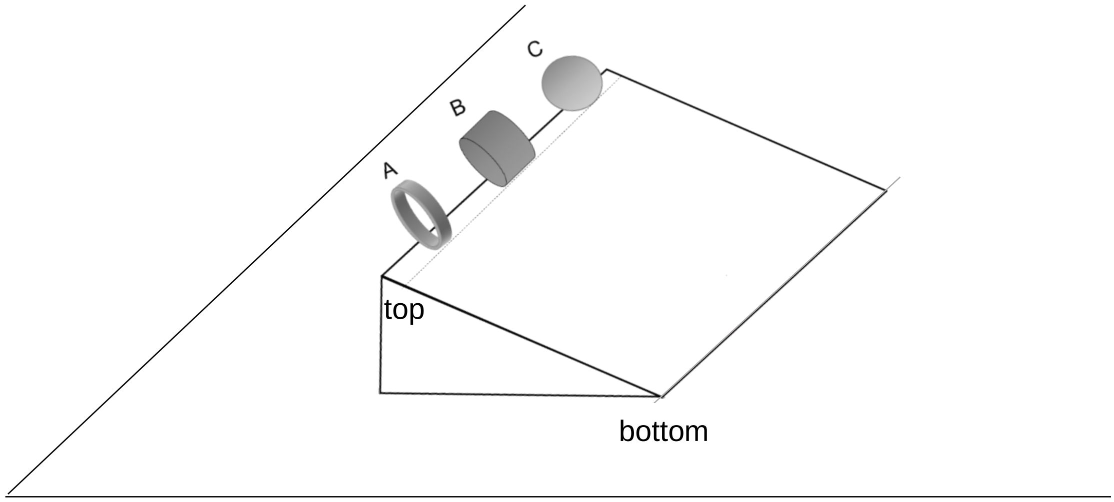


Fig. 2.2

They are released at the same elevation and roll without slipping.

Considering the fact that the outcome is independent of the masses and the radii of the objects, explain which object reaches to the bottom first.

....

....

....

....

....

....

....

.... [3]

[Total: 7]

3 (a) Our Sun is one of the approximately 10^{11} stars in our host galaxy, the Milky Way. There is a supermassive black hole at the centre of the Milky Way and several stars orbit close to this black hole. The orbital period of star S0-2 has a period of 14.53 years and a semi-major axis of 919 AU.
Calculate the mass of the supermassive black hole in solar masses.

1 AU is the mean Earth-Sun separation, 1.50×10^{11} m
1 solar mass is the mass of the Sun, 1.99×10^{30} kg.

mass = ... solar masses [2]

(b) Consider a satellite of mass m, moving at a velocity v, at a distance r from another mass M, where M is much larger than m.

(i) Write down an equation for the total energy E_{total} of the system.

[1]

(ii) Given that $E_{total} = (KE)_r + U_{eff}$, where $(KE)_r = \frac{1}{2}mv_r^2$ is the effective radial kinetic energy, determine the effective radial potential U_{eff} as a function of M, m, r, the gravitational constant G and L, the angular momentum of m relative to M.

[3]

(c) In 1969, the Apollo 11 was the first mission that brought astronauts to the Moon and safely back to Earth.
In this question, you may use that the mass of the Moon is 7.36×10^{22} kg and its radius is 1.74×10^{6} m.

(i) The trajectory taken from Earth brought Apollo 11 to a height of about 1.6×10^{5} m above the Moon's surface, moving at approximately 2500 m s^{-1}.

Show that, without a course correction, Apollo 11 would have simply flown by the Moon and escaped its gravity.

[2]

(ii) The corrective action taken by the crew of the Apollo 11 was a rocket thrust that slowed the spacecraft, while also causing the craft to descend toward the Moon. This brought the spacecraft into a bound elliptical orbit with a periapsis (the point in the orbit closest to the Moon) of 1.1×10^{5} m above the Moon's surface, where its speed was 1670 m s^{-1}.

For apoapsis (the point in the orbit farthest from the Moon), calculate the speed of the Apollo 11.

speed = …………………….. m s^{-1} [4]

[Total: 12]

4 Two identical radio transmitters, A and B, both powered by 60 V voltages, are separated by a distance of 2.5 km. They each transmit a vertically polarized radio wave of frequency 1500 kHz.

The two waves emitted are in phase.

A radio receiver is moved along a line parallel to, and 22 km from, the line joining the two transmitters, as shown in Fig. 4.1.

Fig. 4.2 shows the variation in intensity of the resultant wave detected by the receiver with distance moved by the receiver, when the receiver is about equidistant from A and B. The intensity is shown to fluctuate regularly between 0 and 4*L*.

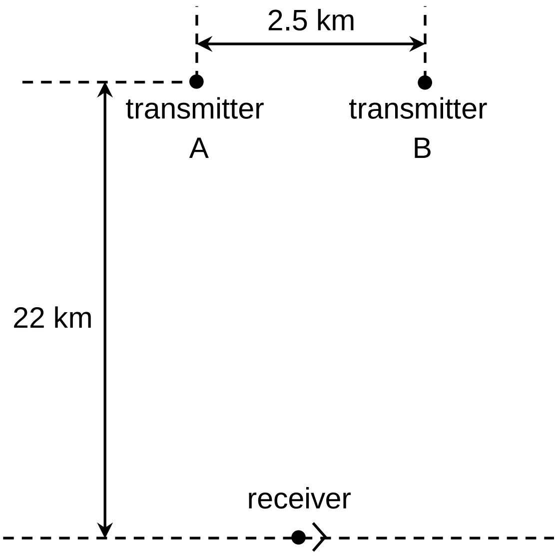


Fig. 4.1

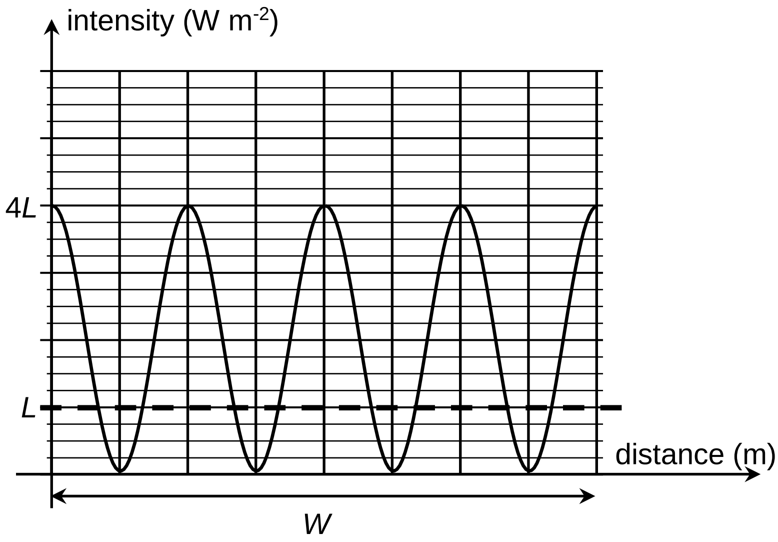


Fig. 4.2

(a) Calculate the distance *W* indicated in Fig. 4.2.

distance = …………………………… m [3]

(b) Suggest the physical meaning of the horizontal dashed line drawn in Fig. 4.2.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

…………………………………………………………………………………………………… [1]

(c) The operating voltages of transmitters A and B are now changed to 90 V and 30 V respectively. Sketch in Fig. 4.3 the new intensity variation. [3]

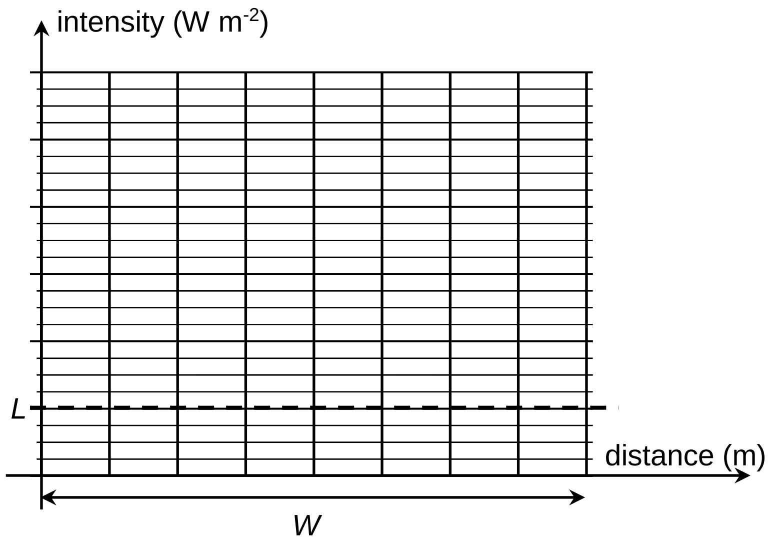


Fig. 4.3

(d) The operating voltages of transmitters A and B are now reverted to 60 V. However, the antenna of A is rotated by 90° so that it is now transmitting a horizontally polarized radio wave.

Sketch in Fig. 4.4 the new intensity variation. [1]

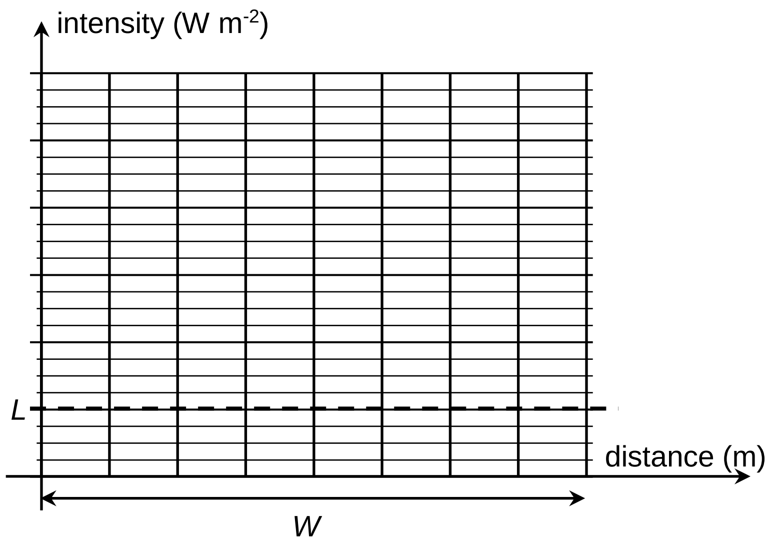


Fig. 4.4

(e) Using the graphs drawn in Fig. 4.2 and 4.4, comment on the principle of conservation of energy.

………

………

……… [1]

[Total: 9]

5 Compton scattering is the scattering of a photon after an interaction with a charged particle.

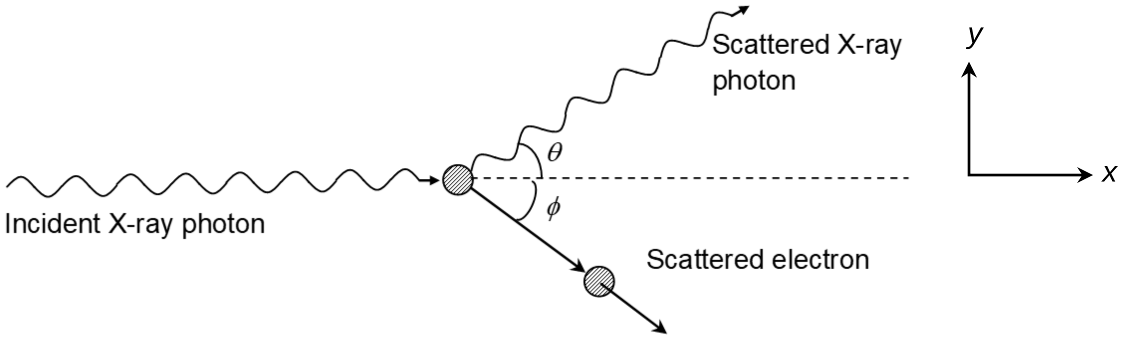


Fig. 5.1

Fig. 5.1 above shows an X-ray photon, initially moving in the x direction, colliding with a stationary electron. After the collision, the X-ray photon is scattered through an angle θ while the electron is scattered through an angle ϕ. The wavelength of the incident X-ray is λ_0 and that of the scattered X-ray is λ. The scattered electron has momentum p.

(a) In terms of p, λ_0 , λ , θ , ϕ and h (the Planck's constant), write down the equations for the principle of conservation of momentum applied to the collision in the

1. y direction

2. x direction

[2]

(b) Explain why the photon has a longer wavelength after the collision.

....

....

.... [1]

Based on momentum and energy conservation laws, the amount of lengthening of the wavelength at different scattering angle θ is predicted by the Compton shift formula, $\Delta\lambda = \dfrac{h}{mc}(1-\cos\theta)$, where m is the mass of the electron.

(c) For an incident X-ray of wavelength 6.91×10^{-11} m, calculate

(i) the maximum wavelength of the scattered X-ray photon,

wavelength = ... m [2]

(ii) the maximum energy of the scattered electron, and

energy = ... J [2]

(iii) the maximum impulse experienced by the electron during the collision.

impulse = ... N s [2]

[Total: 9]

6 The Earth reference frame or any reference frame moving at constant velocity relative to Earth is called an inertial reference frame. The change in a system's momentum is the same in any inertial reference frame. Unlike changes in momentum, changes in kinetic energy are not the same in two inertial reference frames moving relative to each other. The observation that changes in kinetic energy depend on the reference frame is disturbing. What does this result imply about conservation of energy?

The laws of momentum and energy conservation are of central importance; however, they should not be applied automatically. For them to hold at all requires that the system be closed, that is, its objects should be considered as interacting pairs. This constraint is not often appreciated and may be seen as too ''academic,'' or of little ''practical'' value.

Consider the simplest example of an interaction involving an object with near infinite mass, one-dimensional elastic collision of a ball (mass *m*) against a solid wall (mass *M*) as shown in Fig. 6.1. This common example is usually considered in the wall's rest frame, S^0.

Note that the momentum of the ball is not conserved, but its energy seemingly is (the collision is elastic).

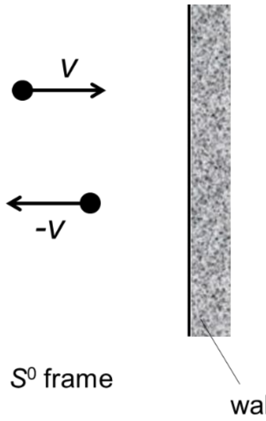


Fig. 6.1

The lack of symmetry between energy and momentum may puzzle the novice learner. This is because the ball does not comprise a closed system. So, in fact, neither the momentum nor the energy of the ball is conserved. Nevertheless, while one can neglect the energy transferred to the wall (elastic collision), one cannot neglect the transferred momentum. Consequently, some learners perceive the idea of energy conservation of the ball.

In fact, as regards the wall-ball system, one can say that momentum was redistributed by the collision, but the energy was not. It remained with the ball. Why? Here the infinite mass played its role. In the S^0 frame, one may account for the energy and momentum in the collision:

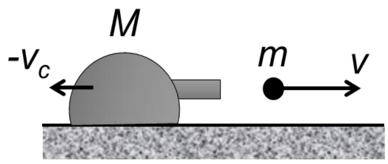
$$\frac{1}{2}m(v_{\text{ball}}^{\text{i}})^2 = \frac{1}{2}m(v_{\text{ball}}^{\text{f}})^2 + \frac{1}{2}M(v_{\text{wall}}^{\text{f}})^2 \qquad (1)$$

and

$$m(v_{\text{ball}}^{\text{i}}) = m(v_{\text{ball}}^{\text{f}}) + M(v_{\text{wall}}^{\text{f}}) \qquad (2)$$

where *m* and *M* are the masses of the ball and wall, respectively, and *i* indicated initial and *f* the final velocities of the ball and wall v_{ball} and v_{wall}, respectively.

The treatment of an explosion (an inelastic collision reversed in time) introduces internal, non-mechanical energy into our discussion. Suppose a heavy cannon of mass M recoils at speed v_c, after shooting a ball of mass m at speed v, as shown in Fig. 6.2.



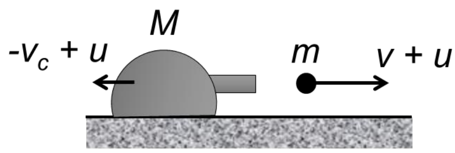
S^0 frame

Fig. 6.2

An observer on the ground, in the S^0 frame, accounts for the energy balance, assuming the recoil speed is negligible:

$$\frac{mv^2}{2} = \Delta E_{\text{int}} \qquad (3)$$

where ΔE_{int} is the change of the internal energy of the explosives converted into kinetic energy of the ball m. The cannon's share of the kinetic energy is neglected in Equation (3) apparently on the grounds of the near-infinite mass of the cannon. It is not crucial for the observer to be aware of the interaction between the cannon and the ball.



S frame

($\xleftarrow{u}$)

Fig. 6.3

This is not the case for an arbitrary observer S, moving at velocity u towards left as shown in Fig. 6.3.

By neglecting the change in velocity of the cannon, an observer in the S frame would be making an error in the energy balance:

$$\frac{(m+M)u^2}{2} + \Delta E_{\text{int}} = \frac{m(v+u)^2}{2} + \frac{Mu^2}{2} \qquad (4)$$

Equations (3) and (4) cannot be satisfied by the same amount of internal energy ΔE_{int}. This already presents a paradox: the change of the internal energy must be invariant or, in other words, the amount of explosives used cannot be greater for an observer driving past the cannon. The paradox is resolved when we discard the tacit assumption that the change in the cannon's kinetic energy is negligible.

(a) Using the reading passage, describe what is meant by a *closed system* of objects.

………..

…………………………………………..………………………………………………………………… [1]

(b) **(i)** Explain why the momentum of the ball in Fig. 6.1 is not conserved.

[2]

(ii) Determine the final velocities of the ball and the wall in the S^0 frame in Fig. 6.1 in terms of m, M and v.

final velocity of ball = ……………………

final velocity of wall = …………………… [4]

(iii) Hence, explain why the final velocity of the ball is approximately $-v$ and that of the wall is zero after the collision.

……………………………………………………………………………………………….......

………………………………………………………………………………………………......

…………………………………………………………………………………………………… [1]

(c) **(i)** Rewrite the energy balance equations (3) and (4) in the passage for the S^0 and S frames, without neglecting the motion of the cannon.

[2]

(ii) Show that change in kinetic energy of the **cannon** depends on the reference frame, i.e., is *not invariant*.

[3]

(iii) Use your results in **(c)(i)** and **(c)(ii)** to show that ΔE_{int} is independent of the reference frame, i.e., *invariant* for the system consisting of the **cannon** and the **ball**.

[3]

[Total: 16]

Section B

Answer **two** questions from this section.

You are advised to spend about 35 minutes on each question.

7 **(a)** Electric charge is uniformly distributed on a straight wire of length 2*a* parallel to the *y*-axis, as shown in Fig. 7.1.

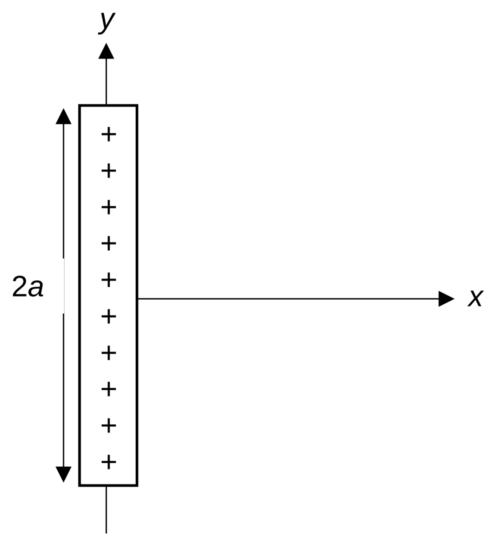


Fig. 7.1

The total amount of charge on the wire is *Q*.

(i) State an expression for the linear charge density λ in terms of *Q* and *a*.

.... [1]

(ii) Use your answer in **a(i)** and apply Gauss's law with an appropriately chosen Gaussian surface to show that an approximation for the electric field at the position (x, 0, 0), where $x << a$, is given by

$$E_x = \frac{\lambda}{2\pi\varepsilon_0 x}$$

You may wish to draw a diagram to help your answer.

[3]

(iii) The electric field can be determined more accurately than determined in **(a)(ii)** by superimposing the point charge fields of infinitesimal charge elements. This can be done by summing the fields of charged segments of length d*y*, as shown in Fig. 7.2.

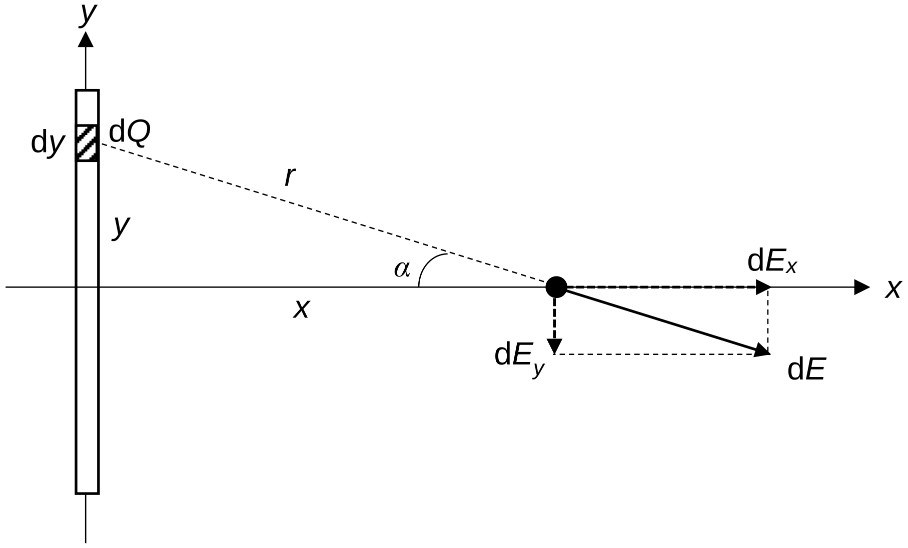


Fig. 7.2

1. Show that the electric field at position (*x*, 0, 0) is given by:

$$E_x = \frac{\lambda a}{2\pi\varepsilon_o x\sqrt{a^2 + x^2}}$$

(Hint: $\frac{dy}{d\alpha} = x\sec^2\alpha$)

[5]

2. State the condition such that the expression in **(a)(iii)1** becomes the same as **(a)(ii)**. Show the relevant working clearly.

[2]

(b) A dipole of charge $\pm q$ is a distance r from an infinitely-long wire of **negative** charge of linear charge density λ.

(i) The dipole moment $\vec{p}$ is parallel to the line of charge. Using the result in **(a)(ii)**, express the magnitude of the torque on the dipole in terms of r, p, and λ.

………………………………………………………………………………………………… [1]

(ii) The dipole moment is now pointing directly at the line of charge (perpendicular to it). Is there a net force on the dipole, and if so, is it toward or away from the line of charge?

…………………………………………………………………………………………………

…………………………………………………………………………………………………

………………………………………………………………………………………………… [1]

(iii) Suppose we release the dipole of length d at position **A** from rest as shown in Fig. 7.3. It rotates to position **B**.

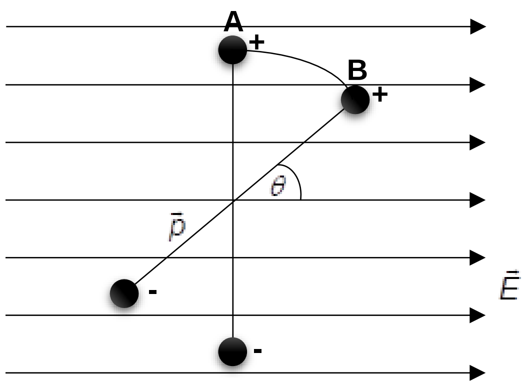


Fig. 7.3

Express the change in potential energy in terms q, E, d and θ.

………………………………………………………………………………………………… [1]

(c) Long, straight conductors with square cross sections and each carrying current I are laid side by side to form an infinite current sheet as shown in Fig. 7.4. The conductors lie in the *xy*-plane, are parallel to the *y*-axis, and carry current in the –*y* direction. There are *n* conductors per unit length measured along the *x*-axis.

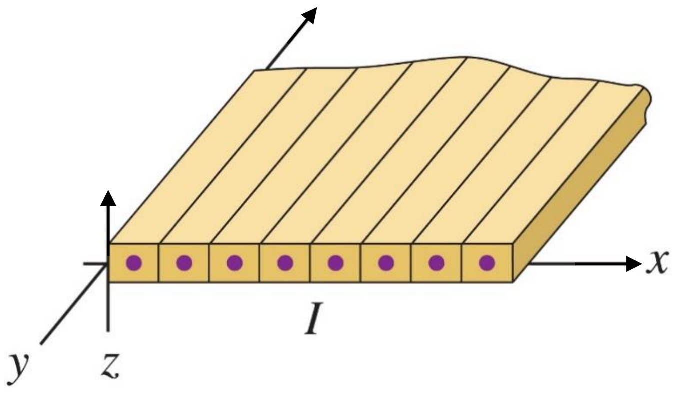


Fig. 7.4

(i) Determine the direction of the magnetic field above and below the current sheet.

direction (above) = ………………………………

direction (below) = …………………………… [1]

(ii) Use Ampere's law to find an expression for the magnetic field strength a distance *a* above and below the current sheet.
You may wish to draw a diagram to help your answer.

[3]

(iii) A second infinite current sheet is placed at a distance d below the first current sheet in Fig. 7.4 and is parallel to it. The second sheet carries current into the plane of the page. Each sheet has n conductors per unit length. The arrangement is shown below in Fig. 7.5.

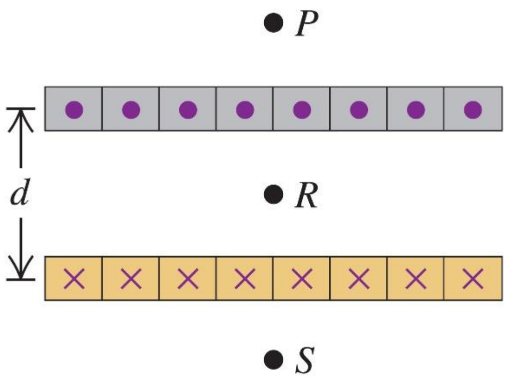


Fig. 7.5

Calculate the magnitude of the net magnetic field at points P (above the upper sheet), R (midway between the two sheets) and S (below the lower sheet) respectively.

Point P: ……………………………………………….

Point R: ……………………………………………….

Point S: ……………………………………………… [2]

[Total: 20]

8 **(a)** A uniform, thin rod of length L and mass M is shown in Fig. 8.1.

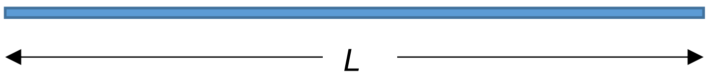


Fig 8.1

Determine the moment of inertia of this rod about its centre of mass in terms of M and L.

moment of inertia = ………………… [2]

(b) A uniform, thin rod AB is at rest and is pivoted at a distance of one quarter of its length from its top end A as shown in Fig. 8.2. Its total length is 2.00 m and its mass is 0.300 kg.

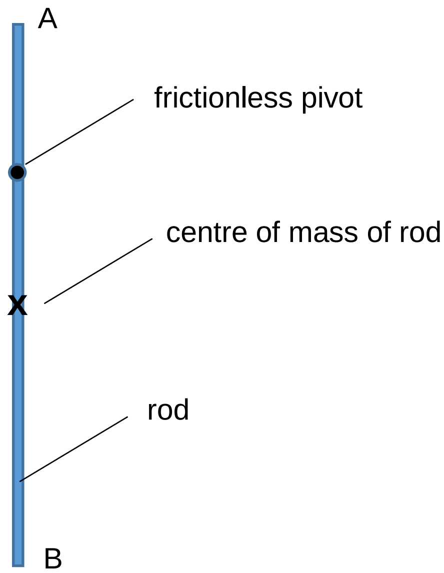


Fig. 8.2

(i) Show that the moment of inertia of the rod about the pivot is 0.175 kg m^2.

[2]

A sticky gum which has a mass of 100 g, is moving horizontally at a speed of 40.0 m s^{-1} towards the centre of mass of this rod as shown in Fig. 8.3. The gum hits the rod AB and sticks to it during the subsequent motion.

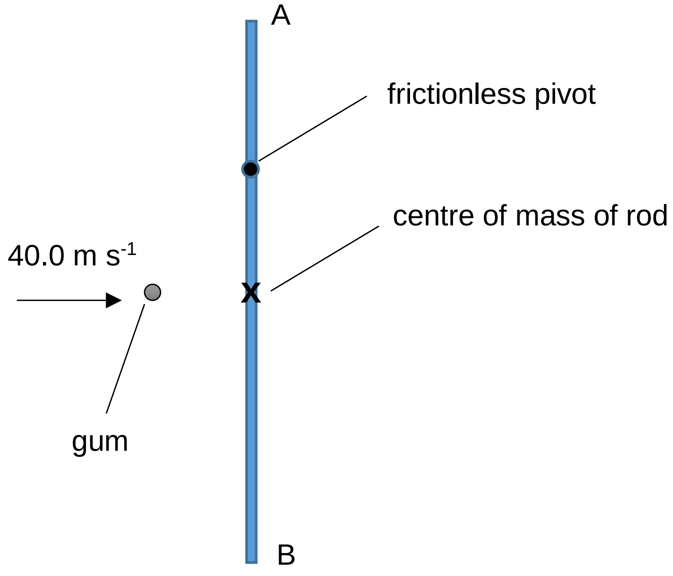


Fig. 8.3

(ii) By using the conservation of angular momentum, show that the angular speed of the gum immediately after the impact is 10.0 rad s^{-1}.

[2]

(iii) After a brief moment, the moving combined bodies rotated 60^o about the frictionless pivot from its original position as shown in Fig. 8.4. Without any frictional force on bodies during motion,

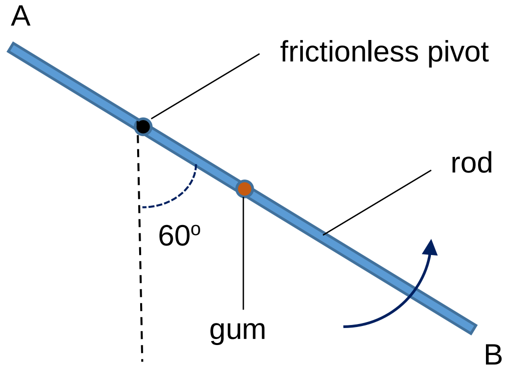


Fig. 8.4

1. show that the angular speed of the combined bodies is 9.50 rad s^{-1} at this moment,

[3]

2. determine the magnitude of the angular acceleration of the combined bodies at this moment,

angular acceleration = ……………… rad s^{-2} [2]

3. determine the magnitude of the resultant acceleration of the centre of mass of the combined bodies,

resultant acceleration = ………….. m s^{-2} [3]

4. At the instant shown in Fig. 8.4, the pin securing the combined bodies to the pivot drops off, resulting in the release of the combined bodies. It flies off and air resistance can be neglected in its subsequent motion.

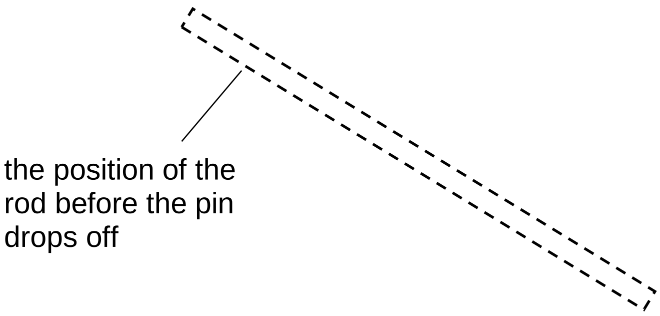


Fig. 8.5

Sketch in Fig. 8.5 the position of combined bodies with ends **A** and **B** labelled **and** angle indicated clearly, when it is back at the same **height** where the pin at the pivot dropped. Explain clearly how you arrived at your deduction.

[6]

[Total: 20]

9 (a) In the circuit of Fig. 9.1, the battery emf is 50.0 V, the resistance is 250 Ω, and the capacitance is 0.500 μF.

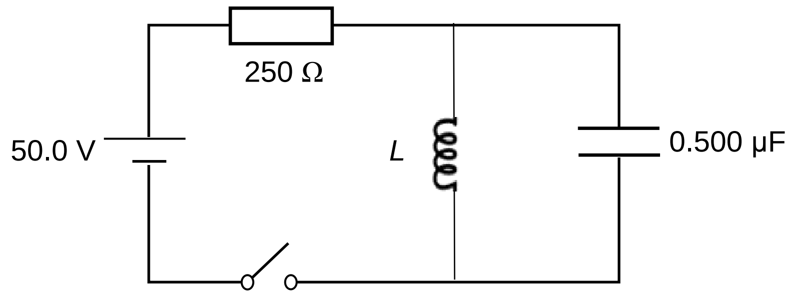


Fig. 9.1

The switch is closed for a long time.

(i) State the potential difference measured across the capacitor.

potential difference = ……………. V [1]

(ii) Calculate the current in the inductor.

current = …………….. A [1]

(b) After the switch is opened, the potential difference across the capacitor reaches a maximum value of 150 V.

(i) Show that the value of the inductance is 0.281 H. Explain your answer.

[3]

(ii) Calculate the charge on the capacitor when the current is half its maximum value.

charge = ……………………….. C [3]

(iii) Calculate the natural frequency of the electrical oscillation.

frequency = ………………. Hz [2]

(iv) Sketch, in Fig. 9.2, the graph showing the variation of the electric energy stored in the capacitor (U_C) and the magnetic energy stored in the inductor (U_L) as a function of time, for one period of oscillation.

You are to indicate numerical values on both axes. Label the graphs clearly.

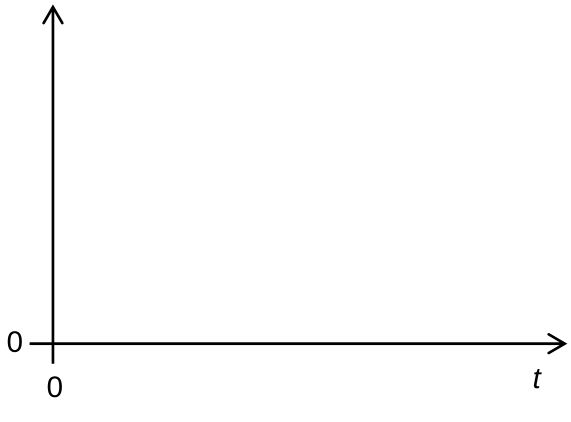


Fig. 9.2

[4]

(c) Another resistor of the same resistance *R* of 250 Ω is added to the set-up as shown in Fig. 9.3.

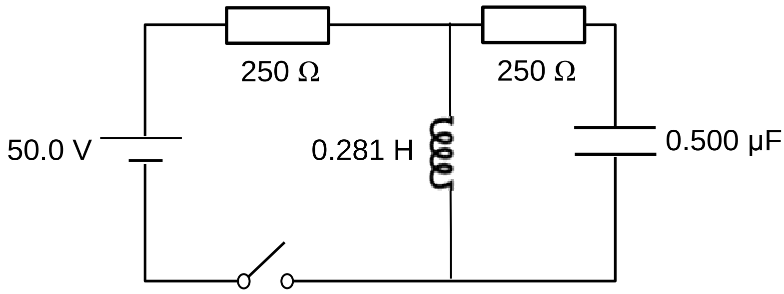


Fig. 9.3

Again, assume the switch is open after having been closed for a long time.

(i) Use energy considerations to show that after the switch is open, the current in the right-hand loop of the circuit satisfies the expression

$$\frac{d^2i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{1}{LC}i=0.$$

[2]

(ii) By direct substitution or otherwise, show that the following solution satisfies the equation in **(c)(i)**:

$$i = I_{max} e^{-\gamma t} \cos(\omega_D t)$$

where γ and ω_D are constants.

Determine γ and ω_D in terms of R, L and C.

[4]

[Total: 20]

End of Paper