Name: ____ Class: __________ Date: ____________

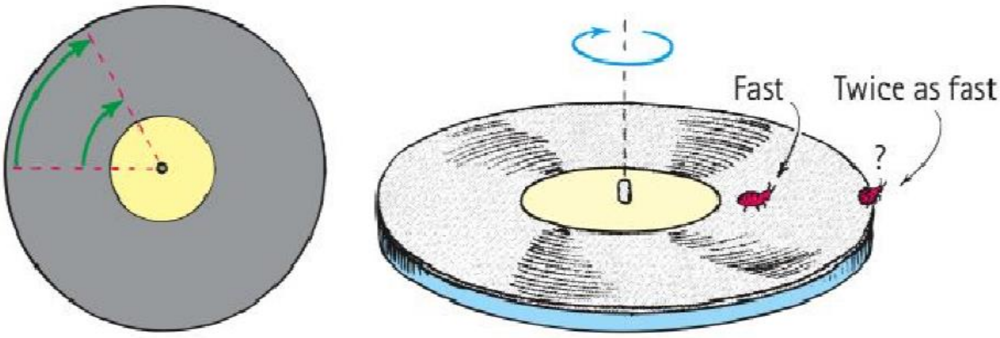
Sec 4 Physics

Topic 25 Rotational Motion and Torque

Essential questions
1. How does a force cause a turning effect on objects?
2. How can we use moments to help us understand the stability of objects?

25.1 CIRCULAR MOTION

Observe the following picture. Two bugs are standing on a spinning disc with one bug at half the disc's radius and one at its full radius.



Does the bug at the edge of the disc travel twice as fast? Do they have the same rate of spin?

Rotational speed (sometimes called angular speed) involves the number of rotations per unit time. All parts of the rotating disc above turn about the axis of rotation in the same amount of time. All parts have the same rate of rotation, or same number of rotations per unit time or same ***angular speed***. It is common to express the rate of rotation in revolutions per minute (RPM). The SI unit is radians per second (rad s^{-1}).

Tangential speed, unlike rotational speed, depends on the radial distance from the centre of rotation. Tangential speed and rotational speed are related. Have you ever ridden on a big, round, rotating platform in an amusement park? The faster it turns, the faster your tangential speed. In fact, the tangential speed is directly proportional to rotational speed at any fixed distance from the axis of rotation.

$$v = r\omega$$

where ω (Greek letter omega) is the angular speed and v is tangential speed.

Table of types of speed / velocity

Type	Definition	SI units
Linear	Speed in a single direction	ms^{-1}
Tangential	Speed parallel to the tangent of a circle / trajectory.	ms^{-1}
Radial (Centripetal)	Speed directed toward or away along a radial direction.	ms^{-1}
Angular (Rotational)	Rate of rotation.	rad s^{-1}

When tangential speed undergoes change, we speak of a *tangential acceleration.* Any change in tangential speed indicates an acceleration parallel to the tangential motion. You should recall in the Topic 2-D Kinematics, we have already discussed another acceleration – one that is directed toward the centre of curvature – centripetal acceleration. To prevent "information overload", we will not discuss tangential acceleration here.

Check your understanding

An object is rotating at 2000 rpm, how fast is it rotating in rad s^{-1}?

$$\begin{aligned} 2000 \text{ rpm} &= \frac{(2000 \text{ rev})(2\pi \text{ rad/rev})}{(60 \text{ s/min})} \\ &= 209 \text{ rad s}^{-1} \end{aligned}$$

What is the theoretical maximum rate of rotation for an object of radius 1 m?

The maximum speed limited by the speed of light and hence, the tangential speed at the outermost radius cannot exceed this limit.

$$\begin{aligned} v &= r\omega \\ 3.00 \times 10^8 &= 1(\omega) \\ \omega &= 3.00 \times 10^8 \text{ rad s}^{-1} \approx 3\ 000\ 000\ 000 \text{ rpm} \end{aligned}$$

When an object is rotating, does it exhibit radial motion? If yes, give an example.

In uniform circular motion, there is no radial velocity (though there is a radial/centripetal acceleration).

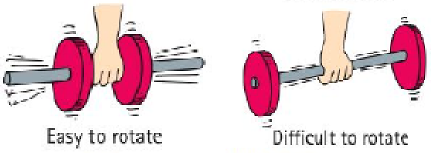
For an object to exhibit radial velocity, the object must be either spiralling inward to or outward from the centre of the circular motion.

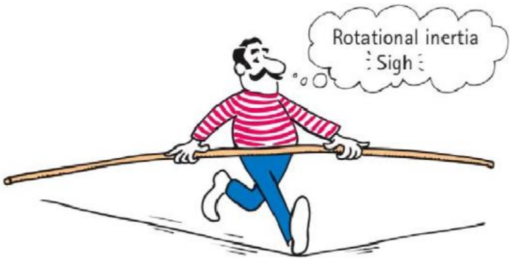
25.2 ROTATIONAL INERTIA

Just as an object at rest tends to stay at rest and an object in motion tends to remain moving unchangingly in a straight line, *an object rotating about an axis tends to remain rotating about the same axis unless interfered with by some external influence*. The property of an object to resist changes to resist its state of motion is called rotational inertia or more commonly, the **moment of inertia**.

Translational motion ⇨	Inertia
Rotational motion ⇨	Moment of inertia

Unlike inertia which depends solely on mass, the moment of inertia also depends on how the mass is distributed about the axis of rotation. When the mass is distributed further from the axis of rotation, the greater is its moment of inertia and consequently harder to rotate.





(Above): Moment of inertia depends on the distribution of mass relative to the axis of rotation.

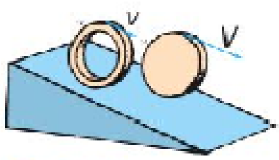
(left): The tendency of the pole to resist rotation aids the acrobat.

(Far right): Short legs have less moment of inertia than long legs. An animal with short legs has a quicker stride than people with long legs.

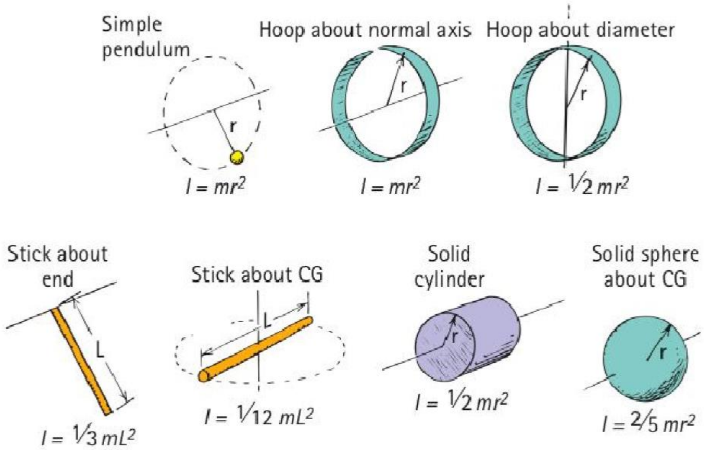
(Right): You bend your legs when you run to reduce moment of inertia.

Check your understanding

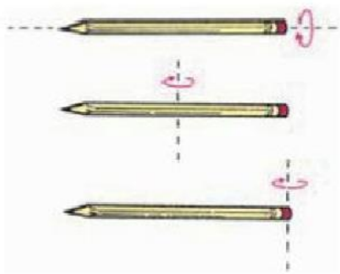
Which should roll faster, a solid cylinder or a loop?



Ans: A solid cylinder rolls down an incline faster than a loop. A loop has greater rotational inertia relative to its mass than a cylinder does.



Rotational inertia of various objects, each of mass *m*, about indicated axes.



Example 1

A pencil (above) has different moments of inertia about different rotational axes. On which axis would it be hardest to rotate? Which is the easiest?

25.3 TORQUE (MOMENT OF A FORCE)

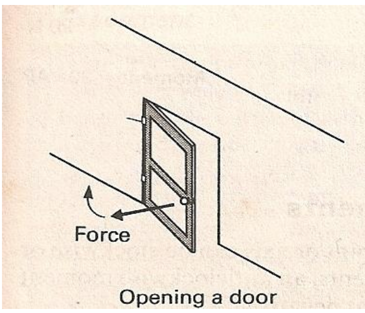
A force has the ability to

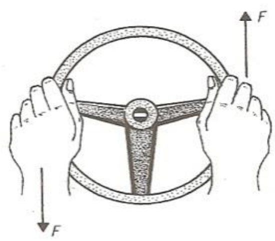
1 cause an object to accelerate.

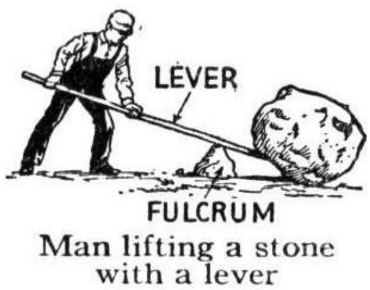
2 cause an object to turn. (turn faster or slower or even change direction of turning)

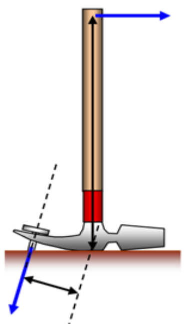
3 change the shape of an object. (deformation)

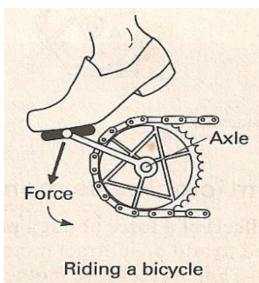
In this chapter, we shall focus our attention on one particular effect, that is, the turning effect of a force. Some examples of turning effects are shown in the pictures below.

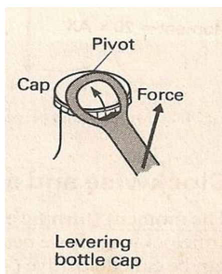












Turning effects in engineering

(Left) Tower Bridge, London. The two halves of the roadway, each about 50m long and 1000 tons, are raised by the turning effect or *moment* of forces produced by hydraulic machinery.

Strictly speaking, it is not a force that changes the state of rotational motion but a ***torque*** (or moment).

Definition:

The torque (or moment of a force) is the product of the force and the perpendicular distance of the pivot from the line of action of the force.

Mathematically,

$$\tau = d_{\perp} \times F$$

- Torque, denoted by Greek letter τ (pronounced *tau*), is a vector quantity.
- The SI unit for torque is Nm (Newton-meter).

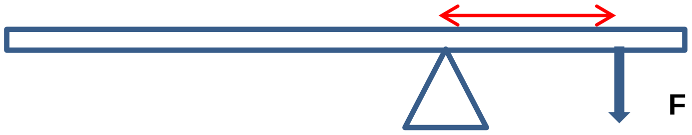
A torque is the rotational counterpart of a force. Force tends to change the translational motion of objects; torque twists or changes the state of rotation of things. If you want to make a stationary object move or a moving object to change speed, apply force. If you want to make a stationary object rotate or a rotating object change rotational speed, apply torque.

Perpendicular distance

The first step in the calculation of moments is to identify the perpendicular distance (which is also the shortest distance) from the force to the pivot (point of rotation). Let's look at some examples and see if you can identify the perpendicular distances.

Example 2

Draw a line to represent the perpendicular distance from the pivot to the line of action of the force.

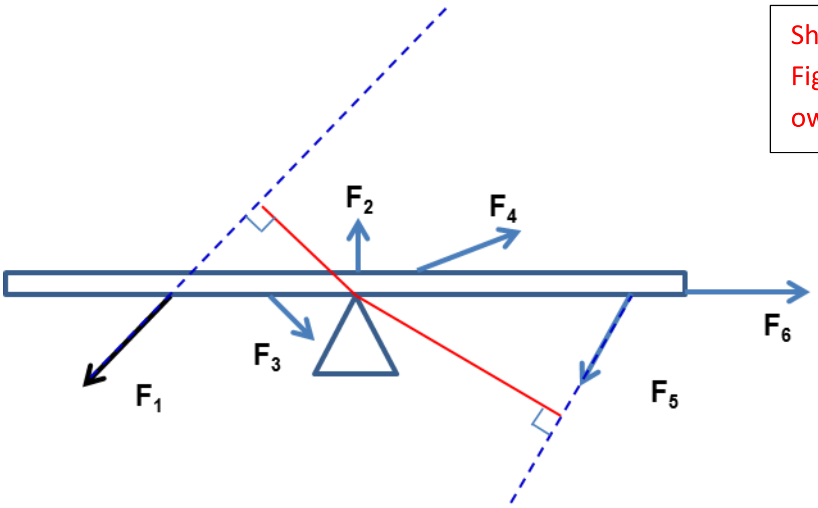


Check your understanding

(i) Draw the perpendicular distance from the pivot to the line of action of the force for each of the six forces.

(ii) Which of the forces would **not** result in a torque about the pivot?

Showing only F_1 and F_5. Figure the rest on your own.



(iii) When the line of action of the force intersects the pivot point, then there is no torque about the pivot.

<u>Clockwise and anti-clockwise moments</u>

There are two ways to turn an object:

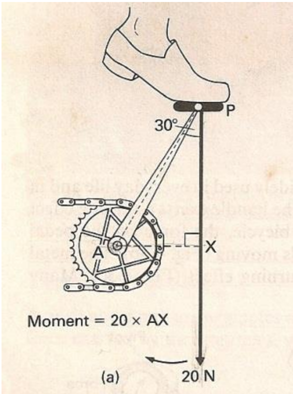
(i) a clockwise rotation;

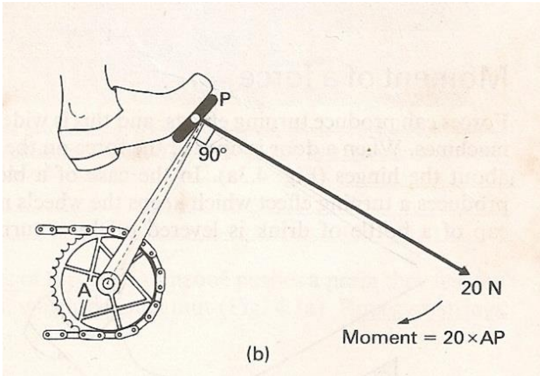
(ii) an anti-clockwise rotation. (anti-clockwise is also called counter-clockwise)

Important: *The direction of the torque is neither clockwise nor anti-clockwise. The discussion of the torque's direction requires advanced mathematics that is beyond the scope of the syllabus*.

Example 3

The two pictures below show two different ways of pedalling a bicycle. The applied force is 20 N and the length between the axle **A** and the pedal **P** is 30 cm. Calculate the moments for each of the situation. Which way is easier to pedal?





Working:

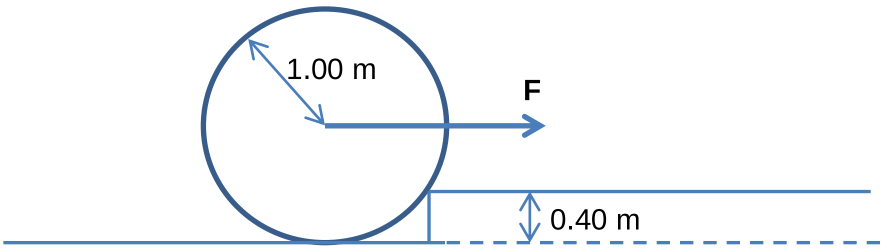
(a) $\tau = (20)(0.30\sin 30^\circ)$
$= 3.0$ Nm

(b) $\tau = (20)(0.30)$
$= 6.0$ Nm

*Example 4

What is the least horizontal force **F** such that the circular disk can be raised over the step?
Hint: The moment created by **F** *must overcome the moment caused by the disk's weight.*

Data: Radius of disk = 1.00 m
Height of step = 0.40 m
Mass of disk = 15.0 kg



Working:

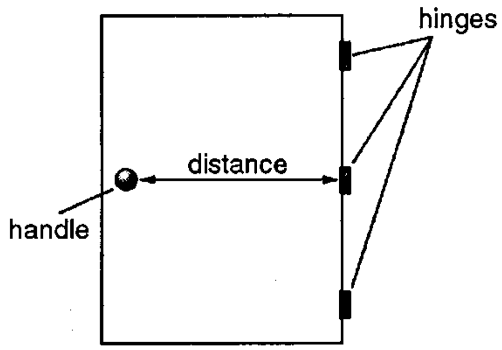
Moment of F = Moment of Weight

$F(0.60) = 15(9.81)\sqrt{1.00^2 - 0.60^2}$

$F = 196$ N

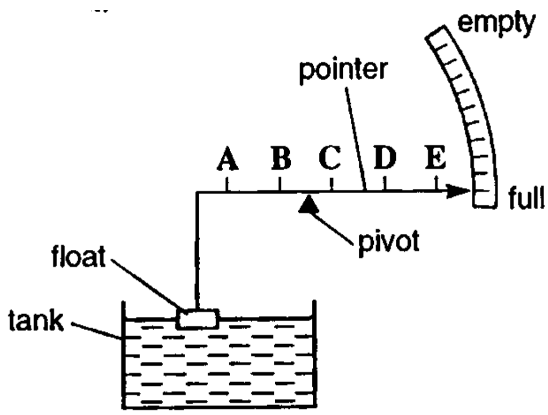
Check your understanding

A door requires a minimum moment of 32.5 N m in order to open it. What is the minimum distance of the handle from the hinges, if the door is to be pulled open with a force at the handle not greater than 50 N?



(a) 0.33 m
(b) 0.65 m
(c) 0.77 m
(d) 1.54 m

The diagram shows a gauge which measures the amount of liquid in a tank.



At which position should the pivot be placed so that the pointer moves the greatest distance as the tank is emptied?

25.4 PRINCIPLE OF MOMENTS

The principle of moments states that when an object is in rotational equilibrium, then the sum of moments about *any axis* must be zero.

This means that the total clockwise moments about an axis is equal to the total anti-clockwise moments about the same axis.

Mathematically, it reads

$$\sum \tau = 0$$

(compare this to the case where an object is in translational equilibrium, $\sum F = 0$)

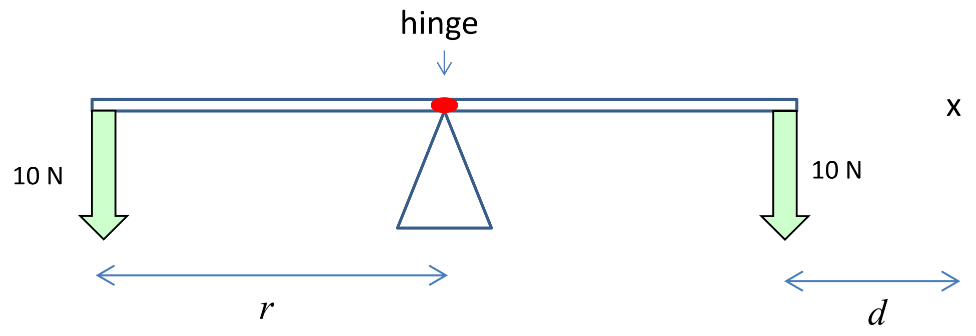
During problem solving, the principle is usually written as

sum of clockwise moments = sum of anti-clockwise moments

Although the principle is valid for any axis, we should choose the most convenient one, usually the pivot, for calculation.

Example 5

A simple see-saw consists of a plank hinged at its centre (see picture below). When two forces of 10 N act downwards on both edges, the see-saw is in static equilibrium.



(a) Taking moments about the hinge of the see-saw, explain why it is in equilibrium.
(b) According to the principle of moments, the resultant torque about any axis will be zero. Let us choose an axis (perpendicular to the plane of the paper) at the point indicated X. Show that the resultant torque about X is zero as well.

(a) Taking moments about the hinge,

Clockwise moment $= (10)(r) = 10r$

Anti-clockwise moment $= 10r$

Since the resultant torque is zero, by the principle of moments, the system is in rotational equilibrium.

(b) Taking moment about the axis through point X,

Clockwise moment (provided by the normal contact force at the hinge) $= (F)(r+d)$

Anti-clockwise moment $= (10)(2r+d) + 10d = 20(r+d)$

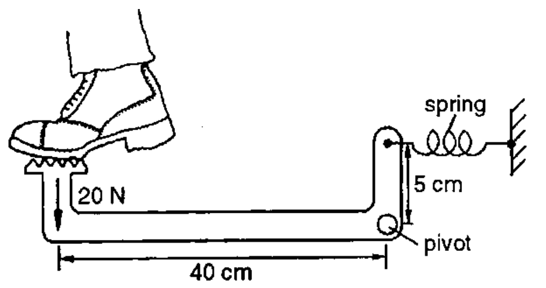
In this particular problem, you need to recognize that in order to balance the two torques generated by the 10 N forces, the hinge must exert an equal clockwise torque. From the equations, we can deduce that the normal contact force from the hinge must be 20 N.

Check your understanding

A driver's foot presses on a pedal in a car with a force of 20 N as shown in the diagram.

With what force is the spring pulled?

(a) 2.5 N
(b) 10 N
(c) 100 N
(d) 160 N
(e) 800 N

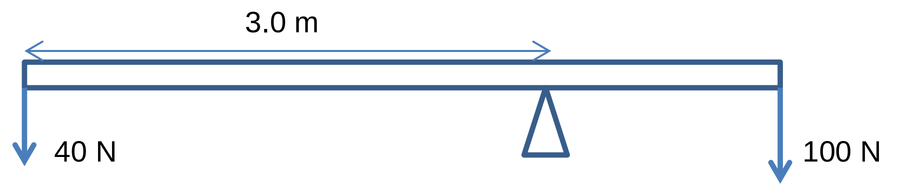


Example 6

Consider the uniform plank to be in static equilibrium,

(a) Find the length of the plank, assuming the plank is weightless.

(b) Using the answer in part (a), what would be applied force on the left become if the mass of the plank is 15.0 kg and the right-hand side force remains as 100 N?



Working:

(a) $\sum$anti-clockwise moments $=\sum$clockwise moments

$$(3.0)(40)=(x)(100)$$
$$x=1.2\text{ m}$$

Hence length of plank is 4.2 m.

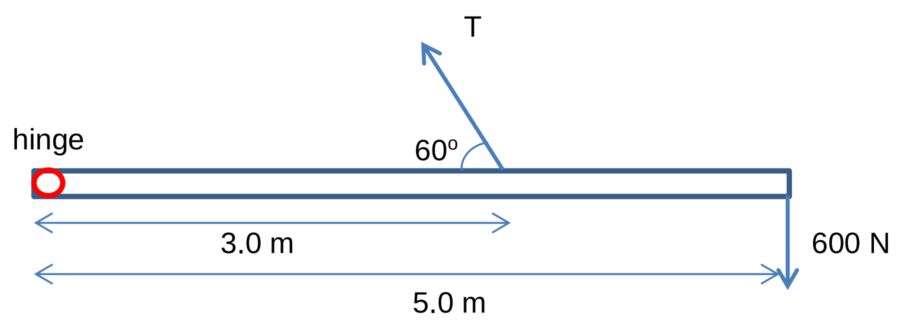
(b) $\sum$anti-clockwise moments $=\sum$clockwise moments

$$(0.9)(15.0)(9.81)+(3.0)(F_{new})=(1.2)(100)$$
$$F_{new}=\ -4.1\text{ N}$$

The applied force on the left will become 4.1 N point upwards.

Example 7

The board in the diagram is hinged at one end and supported by a vertical rope 3.0 m from the hinge. A boy of weight 600 N stands on the other end of the board, which is 5 m from the hinge. Neglect the weight of the board, calculate the tension T in the rope.



Working:

Clockwise moment = Anti-clockwise moment

$$(600)(5.0)=(T\sin 60^\circ)(3.0)$$
$$T=1154\text{ N}$$
$$=1.2\times10^{3}\text{ N} \qquad \text{(2sf)}$$

25.5 STATIC EQUILIBRIUM

In the earlier chapter, we have discussed the meaning of static equilibrium, that is, an object that is not moving. With the knowledge of rotational equilibrium, we can refine the condition for static equilibrium.

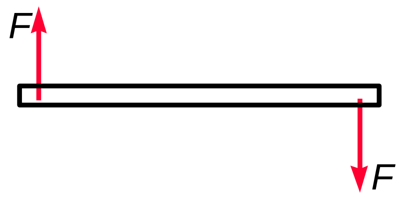
For an object to be in static equilibrium,

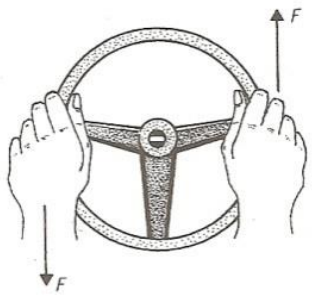
Sum of forces acting on the object = zero $\left(\sum F = 0 \text{ or resultant force} = \text{zero}\right)$

Sum of torque acting on the object about any axis = zero $\left(\sum \tau = 0 \text{ or resultant torque} = \text{zero}\right)$

25.6 COUPLE

A couple consists of a pair of equal forces acting in opposite directions and whose lines of action do not intersect with each other. An example of a couple would be the pair of forces acting on the steering wheel of a car. One important feature of a couple is that it only produces a resultant torque but not a resultant force and hence causes rotational motion only.

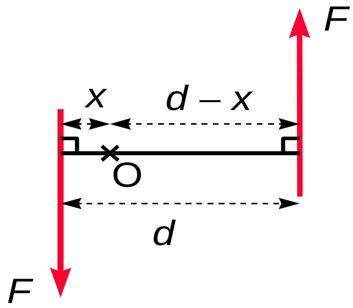




Definition:

The moment (torque) of a couple is the product of the magnitude of one of the force and the perpendicular distance between the two forces.

Proof: Consider taking moments about point O:



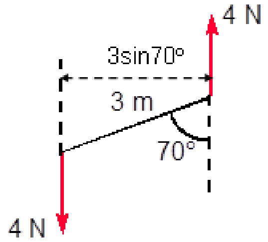
Torque of a couple $= F(x) + F(d - x)$

$= Fd$

Note: This magnitude is *independent* of the position of the point **O** chosen.

Example 8

What is the moment of the following couple?



Working:

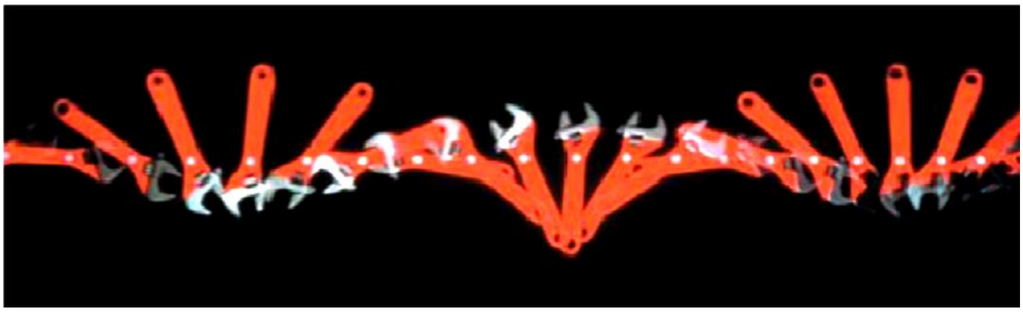
Moment of couple = 4 × 3sin70°

= 11.3 Nm

25.7 CENTRE OF GRAVITY

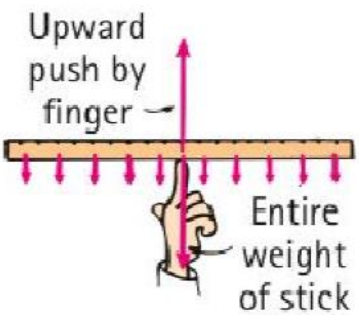
The centre of gravity is the point where all the weight seems to act. It is the average position of the weight distribution.

We have already learnt something about the centre of gravity when we analysed forces using free body diagrams. The dot in the dot diagram is actually the centre of gravity of the object. In the picture below, a wench is sliding and spinning across a smooth horizontal table. As there is no resultant force, the centre of gravity (white dot) moves equal distance in a straight line at equal time intervals.

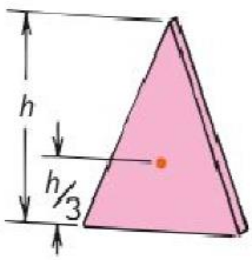


Locating the centre of gravity (CG)

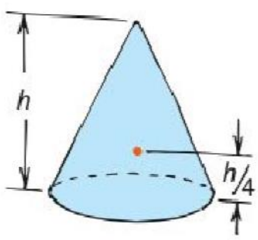
The location of the centre of gravity for a regularly shaped object is its geometric centre. For the circle, square, rectangle, hexagon, etc (2 dimensional shapes) or sphere, cube, cuboid, etc (regular 3D shapes), the CGs are easy to find. For the triangle and the cone, the heights of their centres of gravity are $\frac{h}{3}$ and $\frac{h}{4}$ respectively.



the CG must lie along the line of action of the normal contact force.



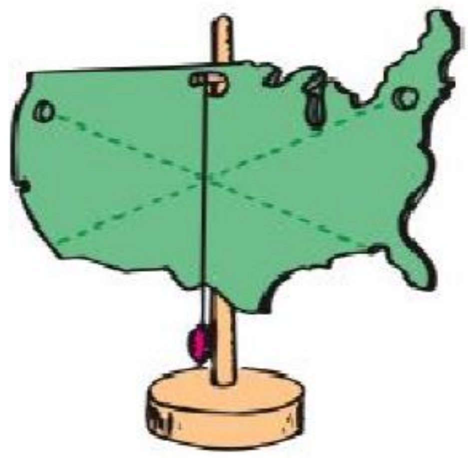
CG of the triangular mass



CG of the conical mass

<u>Locating CG of irregular objects (*plumb line method*)</u>

The centre of gravity of any freely suspended object lies directly beneath (or at) the point of suspension. If a vertical line is drawn through the point of suspension, the centre of gravity lies somewhere along that line. To determine exactly where it lies along the line, we have to suspend the object from some other point and draw a second vertical line through that point of suspension. The centre of gravity lies where the two lines intersect.

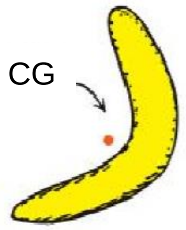


Check your understanding

1. Where is the centre of gravity of a donut?

2. Can a rigid object have more than one centre of gravity?

The centre of gravity can lie outside the object itself. Here are some examples of CGs outside the objects.

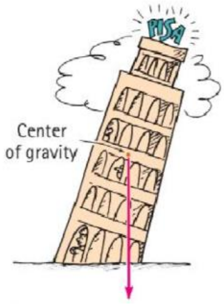


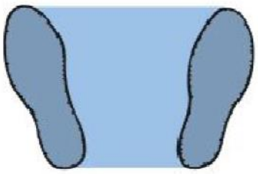
A boomerang

An athlete executes a “Fosbury flop” to clear the bar. Her CG lies beneath the bar throughout the whole process.

25.8 STABILITY

The location of the centre of gravity is important for stability. If we draw a line straight down from the CG of an object of any shape and it falls inside the base of the object, then it is in **stable equilibrium**. If it falls outside of the base, then it is unstable.





Check your understanding

1. Why doesn't the Leaning Tower of Pisa topple?

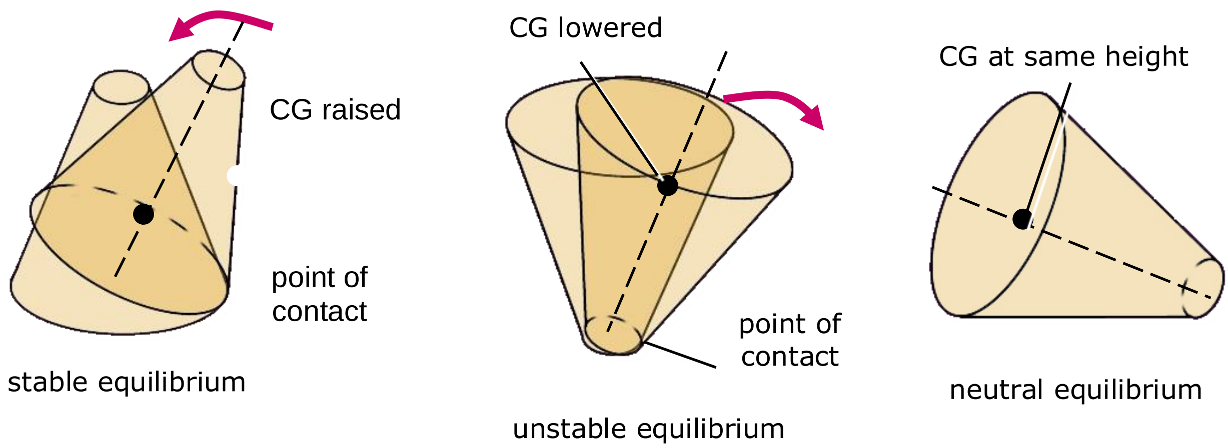
2. Why do you have to keep your feet apart when you stand in a bumpy bus ride?

<u>Three kinds of equilibriums</u>

Stable equilibrium: The object restores itself to the original state after being subjected to an external disturbance. The CG rises and falls back.

Unstable equilibrium: The object cannot restore to its original state after being subjected to an external disturbance. The CG falls and continues to fall until a new equilibrium is reached.

Neutral equilibrium: The object when subjected to a disturbance moves to a new position. The CG neither rises nor falls; it remains at the same level above the supporting surface.



<u>Improving stability of objects</u>

Two ways to improve the stability of objects:

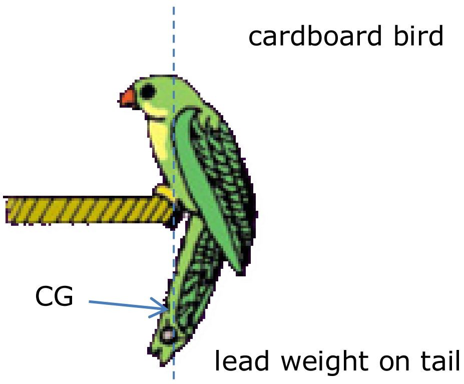
1. ____

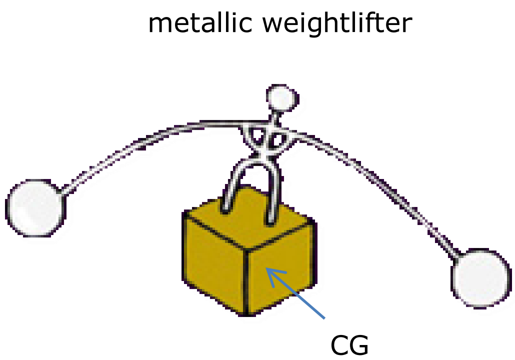
2. ____

Real-life applications include elevated chair for toddlers has a wide base and racing car has both low CG and wide base.

Interesting applications: Balancing toys and tricks

Some toy manufacturers have made use of the concept of stability to create balancing toys.





The CG of the toys lies below the support. When slightly displaced, these toys will not topple over but oscillate about the support (pivot). The physics behind the toy is very similar to a swinging pendulum.