_Subject TG	Class	Index Number	



ST ANDREW'S SECONDARY SCHOOL PRELIMINARY EXAMINATION 2023 SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

ST ANDREW'S SCHOOL ST ANDREW'S S

MATHEMATICS

Paper 1

MONDAY

7 AUGUST 2023

2 hours 15 minutes

4052/01

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, subject TG, class and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer all questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.

Mathematical Formulae

Compound interest

Total amount =
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone =
$$\pi rl$$

Surface area of a sphere = $4\pi r^2$
Volume of a cone = $\frac{1}{3}\pi r^2 h$
Volume of a sphere = $\frac{4}{3}\pi r^3$
Area of triangle $ABC = \frac{1}{2}ab \sin C$
Arc length = $r\theta$, where θ is in radians
Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$Mean = \frac{\sum fx}{\sum f}$$

Standard deviation =
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 The number 899 899 when corrected to *m* significant figures is 900 000.State the largest possible value of *m*.

2 (a) A range of values for x is represented on the number line below.



Write down the inequalities that represent this range of values for *x*.

(b) Solve the inequality 14 + x < 12 + 3x.

3 Expand and simplify 3[u-2(w-5u)].

4 The graph shows the total number of wild elephants in a national park in Thailand at the end of each of the given years.



(a) State one misleading feature of the graph.



5 Solve $3^{2x} + 3^{2x} + 3^{2x} = 81^{x-1}$.

6 Write as a single fraction in its simplest form $\frac{1}{(2\pi)^2}$

 $\frac{5x}{(2x+3)^2} - \frac{2}{2x+3}.$

7 (a) Factorise completely $3x^2 - 4x - 4$.

(b) Hence, factorise completely $3(2y-3)^2 - 4(2y-3) - 4$. Write your answer as simply as possible.

- 8 The Earth is 1.5×10^8 kilometres from the Sun.
 - (a) A terametre is 10^{12} metres.

Find the distance of the Earth from the Sun in terametres.

Answer terametres [1]

(b) Mercury is 5.81×10^7 kilometres from the Sun.

How much nearer is the Sun to Mercury than to the Earth? Give your answer in standard form.

Answer km [2]

9 The scale of a map is 1 : 500 000.

(a) Two towns are 23 km apart.

Calculate the distance between the two towns, in centimetres, on the map.

Answer cm [1]

(b) A lake is represented by an area of 250 cm^2 on the map.

Calculate the actual area of the lake in square kilometres.

Answer km² [2]

10 The probability that Danny follows a study timetable each day is 0.6.

Find the probability that Danny

(a) follows the timetable for 2 consecutive days,

(b) follows the timetable only for 1 day out of 2 consecutive days.

11 (a) Write down all the possible subsets of $\{0, 3\}$.

(b) The Venn diagram shows the sets P, Q and R.



(i) Find n($P' \cup R$).

(ii) $k \in P$

Write down all the possible values of *k*.

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Answer k = ......[1]
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12 (a) Express 198 as the product of its prime factors.

(b) $R = 2^5 \times 3 \times 7^2$ $S = 2^3 \times 3^4 \times 5$

Find

(i) the largest integer which is a factor of both R and S,

(ii) the smallest possible integer value of p such that $R \times p$ is a multiple of S.

Answer p =[1]

13 The diagram shows the graph of the quadratic function y = -(x-p)(x-q), where p < q.



(a) Find the values of p, q and k.

(b) Find the equation of the line of symmetry of the graph.

14 The mean height of a group of students is 171 cm. In the group, there are 4 more females than males.

> The mean height of the females is 162 cm. The mean height of the males is 183 cm.

Calculate the total number of students in the group.

15 (a) The cost, c, of a circular pizza is directly proportional to the square of its diameter, d cm. A pizza with diameter 30 cm costs \$8.

Find a formula for *c* in terms of *d*.

(b) 20 workers take 4.5 hours to erect 30 metres of fence.

Assuming that all workers work at the same rate, how many hours will it take 100 workers to erect 90 metres of fencing?

Answer hours [2]

16 The diagram shows a solid cone and a solid hemisphere.



The cone has base radius x cm and height h cm.

The hemisphere has base radius x cm.

The total surface areas of the cone and the hemisphere are equal.

The volume of the cone is 90 cm^3 .

Find the value of *x*.

Answer x =[4]

17 The diagram shows a segment *ABC* of a circle with a line segment *AC*.



- (a) Using ruler and compasses only,
 - (i) construct two perpendicular bisectors on the diagram to find the centre of the circle. Mark and label *O* as the centre of the circle. [2]
 - (ii) construct the bisector of angle *ACB*. [1]
- (b) Point Q is inside the segment ABC such that it is nearer to A than to B and nearer to BC than to AC.

On the diagram, mark and label a possible position of Q.

[1]

18 Two machines, A and B, produce ice cream 24 hours a day.Machine A produces 4 kg vanilla and 2 kg chocolate ice cream in an hour.Machine B produces 3 kg vanilla and 5 kg chocolate ice cream in an hour.This information can be represented by the matrix P.

	V	anilla	Chocol	ate	e
р	(4	2)	Machine A
P =		3	5)	Machine B

(a) Evaluate the matrix $\mathbf{Q} = 24\mathbf{P}$.

Answer
$$\mathbf{Q} = [1]$$

(b) The price of vanilla ice cream is \$30 per kg. The price of chocolate ice cream is \$32 per kg.

Represent the prices by a 2×1 matrix **M**.

Answer $\mathbf{M} = [1]$

(c) Evaluate the matrix S = QM.

Answer $\mathbf{S} = [1]$

(d) Explain what each element in matrix **S** represents.

......[1]

Models of trains are made to different scales.

19

(a) A model of the train is made to a scale of 1 : 75. The water tank in the model has a capacity of 8 millilitres.

Find the capacity of the water tank in the real train. Give your answer in litres.

Answer litres [2]

(b) Two other models of the same train are made to different scales. The parts used to build the two models have the same materials as the real train.

Model X is made to a scale of 1 : 120. Model Y is made to a scale of 1 : 60.

Rajesh claims that the mass for Model X is twice the mass for Model Y.

Explain, with calculations, why Rajesh's claim is wrong.

Answer

 [3]

20 The diagram shows an empty swimming pool in the shape of a prism *ABCDEFGH*. BC = 10 m, CD = 15 m, DE = 1 m and EF = CG = 2 m.Angle $CDE = \text{angle } DEF = \text{angle } DCG = 90^{\circ}.$



(a) The swimming pool is filled with water at a constant rate of 0.05 m^3 per minute.

Calculate the time taken, in hours, for the swimming pool to be completely filled with water.

Answer hours [4]

(b) Which of these diagrams represents the graph that shows how the depth (D) of the water in the pool above the line *GH* changes with time (t) as the pool is filled?



[Turn over



ABCD is a parallelogram. ABP and QDC are straight lines. Angle ADP = angle CBQ = 90°.

(a) Prove that triangle *ADP* is congruent to triangle *CBQ*. Give a reason for each statement you make.

Answer

21

(b) Explain why *BQ* is parallel to *PD*.

[2]

[3]



Answer m/s^2 [1]

(ii) his speed at t = 150.

22

(a)

Answer	 m/s [1]
11.00000	 L	~ J

Describe briefly Ben's motion during the 200 seconds. **(b)**

.....[1]

At t = 0, Ben's speed was u m/s. (c) Ali and Ben ran the same distance from t = 0 to t = 150.

Find the value of *u*.

- **23** A is the point (0, -7) and B is the point (3, -1).
 - (a) Find the equation of the line *AB*.

(b) The position vector of
$$T$$
 is $\binom{m}{7}$ and $\left|\overline{BT}\right| = 10$ units.

Find the two possible values of *m*.



PQRST is a regular pentagon.R, U and T are points on a circle, centre O.QR and PT are tangents to the circle.RSU is a straight line.

Prove that triangle *STU* is isosceles. Give a reason for each step of your working.

Answer

25 A group of 120 adults completed a puzzle. The cumulative frequency diagram shows the distribution of their times, in seconds, to complete the puzzle.



- (a) Use the graph to estimate
 - (i) the median,

Answer seconds [1]

(ii) the interquartile range.

(b) 40% of the adults took between 125 seconds and k seconds to complete the puzzle.

Use the cumulative frequency diagram to find the value of *k*.

(c) A group of 120 students also completed the same puzzle. Their times to complete the puzzle are summarised in the table below.

Median	187 seconds
Interquartile range	81 seconds

Make two comparisons between the times to complete the puzzle by the adults and by the students.

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Answer Key

1	3		13	(a)	p = -1, q = 4, k = 4
2	(a)	$-1.5 < x \le 4$		(b)	$x = \frac{3}{2}$
	(b)	<i>x</i> > 1	14	28	
3 4	33u (a)	- 6w The vertical axis does not start from zero.	15	(a)	$c = \frac{2d^2}{225}$
	(b)	It gives the reader a wrong impression that the number of wild elephants in the national park	16	(b) $x = 1$	2.7 3.67
5	$x = \frac{1}{2}$	falls significantly over 7 years.	18	(a)	$\begin{pmatrix} 96 & 48 \\ 72 & 120 \end{pmatrix}$
6	$\frac{x}{(2x)}$	$\frac{-6}{(-3)^2}$ or $\frac{x-6}{4x^2+12x+9}$		(b)	$\begin{pmatrix} 30\\ 32 \end{pmatrix}$
7	(2 <i>x</i>)	(3x+2)(x-2)		(c)	$\begin{pmatrix} 4416\\ 6000 \end{pmatrix}$
8	(b) (a)	(6y-7)(2y-5) 0.15		(d)	The total price of the ice-cream produced by Machine <i>A</i> and Machine <i>B</i> respectively.
	(b)	9.19×10 ⁷	19	(a)	3375
9	(a) (b)	4.6 6250		(b)	The mass for model <i>Y</i> is 8 times heavier than the mass for model <i>X</i> or mass of <i>X</i> : mass of <i>Y</i>
10	(a)	0.36	20	(9)	= 1:8.
11	(b) (a)	0.48	20	(a) (b)	$7\frac{1}{3}$ Diagram 3
	(b)	5	21	(a)	Angle ADP = angle CBQ = 90° (given) AD = CB
10	(c)	2, 3, 5, 7 $2x^2x^2x^{-11}$			(opposite sides of a parallelogram) Angle DAP = angle BCQ (opposite angles of a parallelogram)
12	(a) (b)	2×3 ×11 24			ASA Property
	(c)	135			

(b) Since triangle ADP is congruent to triangle CBQ, angle APD = angle CQBAngle ABQ = angle CQB(alternate angles) Since angle ABQ = angle ADP(corresponding angles), BQ is parallel to PD.

or

Since angle ADP = angle CBQ = 90° are opposite angles of a quadrilateral and both opposite sides are parallel, the quadrilateral formed is a rectangle. Since it is a rectangle, the other two opposite sides are parallel.

22 (a) (i) 0.02

(ii) 5

- (b) Ben was running at a constant speed or uniform speed.
- (c) u = 6.5

23 (a)
$$y = 2x - 7$$

- **(b)** m = -3 or 9
- **25** (a) (i) 195

(**ii**) 70

- **(b)** *k* = 200
- (c) On average, students completed the puzzle faster than adults as their median time of 187 seconds is less than that of adults' median time of 195 seconds.

Students' times to complete the puzzle are more widespread than adults as their interquartile range of 81 seconds is higher as compared to adults' interquartile range of 70 seconds.