XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Preliminary Examinations 2022

CANDIDATE NAME	Mark Scheme (Draft 3)
CLASS	
INDEX NUMBER	

ADDITIONAL MATHEMATICS

4049/02

Paper 2

30 August 2022

2 hour 15 minutes

Secondary 4 Express/ 5 Normal Academic
Setter : Mr Johnson Chua
Vetter : Mrs Loh Si Lan
Moderator: Ms Pang Hui Chin

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the paper, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **90**.

Errors	Qn No.	Errors	Qn No.
Accuracy		Simplification	
Brackets		Units	
Geometry		*Marks Awarded*	
Presentation		*Marks Penalised*	

For Examiner's Use
90

Parent's/Guardian's Signature:

1 (a) Solve the equation $\lg(x+3)=1-\lg(x-2)$. [3]

$\lg(x+3)+\lg(x-2)=1$

$\lg[(x+3)(x-2)]=1$ —(M1)

$(x+3)(x-2)=10$ —(M1)

$x^2+x-6-10=0$

$x^2+x-16=0$

$x=\dfrac{-1\pm\sqrt{1^2-4(1)(-16)}}{2}$

$=\dfrac{-1\pm\sqrt{65}}{2}$

$=[3.53$ or $-4.53]$ —(A1)
(rej)

Alternative

$\lg(x+3)=\lg\left(\frac{10}{x-2}\right)$ —(M1)

$x+3=\dfrac{10}{x-2}$

$(x+3)(x-2)=10$ —(M1)

(b) Given that $\log_2 p=a$, $\log_q 8=b$ and $\dfrac{p}{q}=2^c$, express c in terms of a and b. [3]

$\log_q 8=b$

$\dfrac{\log_2 8}{\log_2 q}=b$

$\dfrac{3}{\log_2 q}=b$

$\log_2 q=\dfrac{3}{b}$

$q=2^{\frac{3}{b}}$ —(M1)

$\dfrac{p}{q}=\dfrac{2^a}{2^{\frac{3}{b}}}$ —(M1)

$=2^{a-\frac{3}{b}}$

$\therefore c=a-\dfrac{3}{b}$. —(A1)

accept also: $c=\dfrac{ab-3}{b}$.

Alternative #1

$\log_q 8=b$

$q^b=8$

$q=8^{\frac{1}{b}}$

$=2^{\frac{3}{b}}$ —(M1)

$\dfrac{p}{q}=\dfrac{2^a}{2^{\frac{3}{b}}}$ —(M1)

$=2^{a-\frac{3}{b}}$

$c=a-\dfrac{3}{b}$ —(A1)

Alternative #3

$\dfrac{\log_2 8}{\log_2 q}=b$.

$\dfrac{3}{b}=\log_2 q$.

$\log_2\dfrac{p}{q}=\log_2 2^c$

$\log_2 p-\log_2 q=c$.

$a-\dfrac{3}{b}=c$.

Alternative #2

$\dfrac{p}{q}=2^c$

$\log_2\dfrac{p}{q}=c$

$\log_2 p-\log_2 q=c$

$a-\log_2 q=c$ —(M1)

$\log_q 8=b$

$\dfrac{\log_2 8}{\log_2 q}=b$ —(M1)

$\dfrac{3}{\log_2 q}=b$

$\log_2 q=\dfrac{3}{b}$

$\therefore c=a-\dfrac{3}{b}$ —(A1)

2 Given that $y=\frac{4x}{\sqrt{1-4x}}$, show that $\frac{dy}{dx}=\frac{4-8x}{\sqrt{(1-4x)^3}}$. [4]

$u=4x \quad\Big|\quad v=(1-4x)^{\frac{1}{2}}$

$\frac{du}{dx}=4 \quad\Big|\quad \frac{dv}{dx}=\frac{1}{2}(1-4x)^{-\frac{1}{2}}(-4)$

$= -2(1-4x)^{-\frac{1}{2}}$

$= -\frac{2}{\sqrt{1-4x}}$

M1 for either $\frac{du}{dx}$ or $\frac{dv}{dx}$ correct.

$$\frac{dy}{dx}=\frac{4\sqrt{1-4x}-4x\left(\frac{-2}{\sqrt{1-4x}}\right)}{1-4x} \quad \text{— (M1)}$$

$$=\frac{\left[\frac{4(1-4x)+8x}{\sqrt{1-4x}}\right]}{1-4x} \quad \text{(M1)}$$

$$=\frac{4-16x+8x}{\sqrt{(1-4x)^3}}$$

$$=\frac{4-8x}{\sqrt{(1-4x)^3}} \quad \text{— (A1)}$$

Alternative: $y=4x(1-4x)^{-\frac{1}{2}}$

$u=4x \qquad v=(1-4x)^{-\frac{1}{2}}$

$\frac{du}{dx}=4 \qquad \frac{dv}{dx}=-\frac{1}{2}(1-4x)^{-\frac{3}{2}}(-4)$

$= 2(1-4x)^{-\frac{3}{2}}$

$$\frac{dy}{dx}=\underbrace{4x(2)(1-4x)^{-\frac{3}{2}}}_{(M1)}+\underbrace{4(1-4x)^{-\frac{1}{2}}}_{(M1)}$$

$$=\frac{8x}{\sqrt{(1-4x)^3}}+\frac{4}{\sqrt{1-4x}}$$

$$=\frac{8x+4(1-4x)}{\sqrt{(1-4x)^3}} \quad \text{— (M1)}$$

$$=\frac{4-8x}{\sqrt{(1-4x)^2}} \quad \text{— (A1)}$$

3 The graph $y = a\cos 2x + b$, where a and b are positive integers, has a maximum value of 8 and a minimum value of -2.

(a) Show that $a = 5$ and $b = 3$. [2]

a : amplitude $= \dfrac{\text{max} - \text{min.}}{2}$

$= \dfrac{8-(-2)}{2}$

$= 5$ (B1)

b : $\dfrac{\text{max} + \text{min}}{2}$

$= \dfrac{8+(-2)}{2}$

$= 3$ (B1)

Alternative

$\cos 2x_{max} : 1 \quad \cos 2x_{min} : -1$.

$a(1) + b = 8$

$a + b = 8$ — ①

$a(-1) + b = -2$

$-a + b = -2$ — ②

① + ② : $2b = 6$

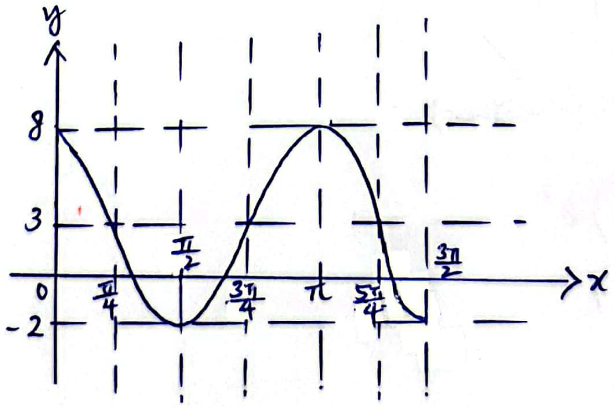
$b = 3$ — ③ — (B1)

Sub ③ into ① : $a + 3 = 8$

$a = 5$ — (B1)

* Awd only if students managed to form sim. eqns correctly.

(b) Hence sketch the graph $y = a\cos 2x + b$ for $0 \le x \le \dfrac{3\pi}{2}$. [3]



B1 : shape, $1\frac{1}{2}$ cycle.

B1 : $(0, 8)$, $(\frac{\pi}{2}, -2)$, $(\pi, 8)$, $(\frac{3\pi}{2}, -2)$

B1 : $(\frac{\pi}{4}, 3)$, $(\frac{3\pi}{4}, 3)$, $(\frac{5\pi}{4}, 3)$.

4 **(a)** By considering the general term of $\left(x+\frac{k}{x}\right)^9$, where k is a positive constant, explain why every term is dependent of x. [2]

$\binom{9}{r} x^{9-r} \left(\frac{k}{x}\right)^r$ — (M1 for considering general term)

$= \binom{9}{r} x^{9-r} k^r x^{-r}$

$= \binom{9}{r} k^r x^{9-2r}$

$9-2r = 0 \Rightarrow r = 4\frac{1}{2}$ (not valid)

Since r is not a whole number | positive integer, then there is no independent term.
∴ Every term is dependent of x. (A1)

(b) Given that the coefficients of x^3 and x^5 are the same, find the value of k. [3]

$9-2r = 3$	$9-2r = 5$
$r = 3$	$r = 2$

(M1 for either)

$\binom{9}{3} k^3 = \binom{9}{2} k^2$ — (M1)

$84k^3 = 36k^2$

$84k^3 - 36k^2 = 0$

$12k^2(7k-3) = 0$

$k = 0$ (rej) or $k = \frac{3}{7}$ (A1)

4 (c) Using the value of k found in (b), find the coefficient of x^7 in the expansion of $\left(1-5x^6\right)\left(x+\frac{k}{x}\right)^9$. [3]

from $\left(x+\frac{k}{x}\right)^9$, find x & x^7 terms.

x^7 term: $9-2r=7$

$2r=2$

$r=1$

$\binom{9}{1}(x)^{9-1}\left(\frac{k}{x}\right)^1$

$= 9(x)^8\left(\frac{3}{7x}\right)$

$= \frac{27x^7}{7}$ — (M1, allow ECF)

x term: $9-2r=1$

$r=4$.

$\therefore \binom{9}{4} x^{9-4}\left(\frac{3}{7x}\right)^4$

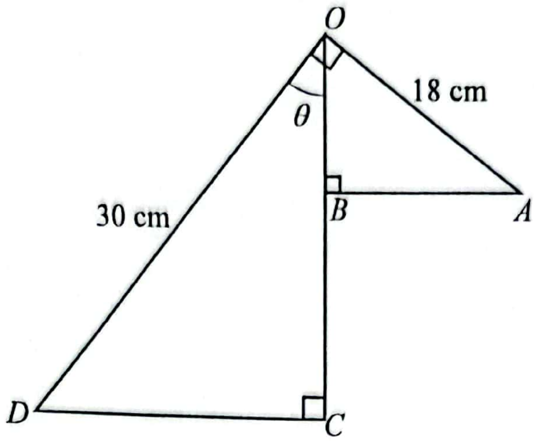
$= \frac{1458x}{343}$ — (M1, allow ECF)

$= (1-5x^6)\left(\ldots + \frac{27x^7}{7} + \frac{1458}{343}x + \ldots\right)$

coeff of x^7: $\frac{27}{7} + (-5)\left(\frac{1458}{343}\right)$

$= -\frac{5967}{343}$ — (A1)

5



The diagram above shows a structure.
The structure has three fixed points O, A and D such that $OA = 18$ cm, $OD = 30$ cm, and angle $AOD = 90°$. The lines AB and DC are perpendicular to OC which makes an angle θ with the line OD. The angle θ can vary in such a way that B lies between the points O and C.

(a) Show that $AB + BC + CD = (12\sin\theta + 48\cos\theta)$ cm. [3]

$\sin\theta = \frac{CD}{30} \Rightarrow CD = 30\sin\theta$

$\cos\theta = \frac{AB}{18} \Rightarrow AB = 18\cos\theta$ } M1 for either.

$\sin\theta = \frac{OB}{18} \Rightarrow OB = 18\sin\theta$

$\cos\theta = \frac{OC}{30} \Rightarrow OC = 30\cos\theta$

$\therefore BC = 30\cos\theta - 18\sin\theta$ — (M1)

$AB + BC + CD = 18\cos\theta + 30\cos\theta - 18\sin\theta + 30\sin\theta$
$= 12\sin\theta + 48\cos\theta$ } (A1)

(b) Express $AB + BC + CD$ in the form $R\sin(\theta + \alpha)$ and $0° \le \alpha \le 90°$. [3]

$12\sin\theta + 48\cos\theta$

$R = \sqrt{12^2 + 48^2}$ — (M1) ; $\theta = \tan^{-1}\left(\frac{48}{12}\right)$ — (M1)

$= \sqrt{2448}$; $= 75.963°$

$12\sin\theta + 48\cos\theta = \sqrt{2448}\sin(\theta + 75.963°)$

$\approx \sqrt{2448}\sin(\theta + 76.0°)$ — (A1)

5 **(c)** Find the maximum value of $AB + BC + CD$ and the corresponding area of the structure. [3]

$(AB + BC + CD)_{max}$: $\sqrt{2448}$ (also accept 49.5) — (m1)

$\therefore \sin(\theta + 75.963°) = 1.$

$\theta = 90° - 75.963°$

$= 14.037°.$ — (m1)

Corresponding area: $\left(\frac{1}{2} \times 30 \times OC \times \sin 14.037°\right) + \frac{1}{2} \times AB \times 18 \times \sin 14.037°$

$= (15 \sin 14.037) \times 30 \cos 14.037° + 9 \sin 14.037° \times 18 \cos 14.037°$

$= 144.00$

$\approx 144\ cm^2$ (3sf) — (A1)

6 (a) The straight line $y - x = -5$ intersects the curve $2x^2 - y^2 = 2xy + 11$ at the points A and B. Find the coordinates of A and B. [3]

$y = x - 5$ — ①

$2x^2 - y^2 = 2xy + 11$ — ②

Sub ① into ②: $2x^2 - (x-5)^2 = 2x(x-5) + 11.$ — (M1)

$2x^2 - (x^2 - 10x + 25) = 2x^2 - 10x + 11$

$x^2 + 10x - 25 = 2x^2 - 10x + 11$

$x^2 - 20x + 36 = 0$

$(x-18)(x-2) = 0$

$[\therefore x = 2 \text{ or } 18]$. — (A1)

at $x = 2$, $y = -3$

$x = 18$, $y = 13$ } (A1)

$\therefore A(2, -3)$, $B(18, 13)$

- 1m if students do not write coordinates of A and B.

(b) Find the exact values of the constant k for which the line $y = k(x+1)$ is a tangent to the curve $y = x^2 + 6x + 3k$. [3]

$x^2 + 6x + 3k = k(x+1)$

$x^2 + 6x - kx + 3k - k = 0$

$x^2 + (6-k)x + 2k = 0$

$(6-k)^2 - 4(1)(2k) = 0$ — (M1 for applying $b^2 - 4ac$ correctly
M1 for equating $b^2 - 4ac$ to 0)

$36 - 12k + k^2 - 8k = 0$

$k^2 - 20k + 36 = 0$

$(k-18)(k-2) = 0$

$k = 18$ or 2 — (A1)

6 (c) Find the range of values of a for which $y = (a-6)x^2 - 8x + a$ lies completely above the x axis. [4]

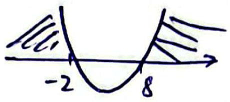
$(-8)^2 - 4(a-6)(a) < 0$ — (M1 for $b^2-4ac < 0$, penalise under "presentation" if students use "8" instead of "-8").

$64 - 4a^2 + 24a < 0$

$-4(a^2 - 6a - 16) < 0$

$a^2 - 6a - 16 > 0$

$(a-8)(a+2) > 0$ — (M1 for correct factorization)

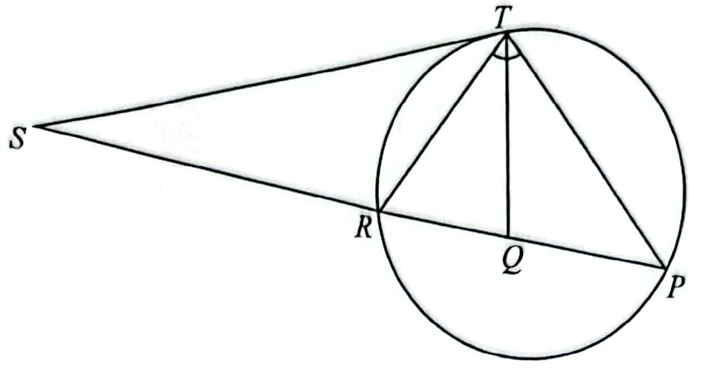


$a < -2$ and $a > 8$ — (M1)

Since graph lies above x-axis, $a > 6$.

$\therefore$ $a > 8$ —(A1)

7



In the diagram, PS is a straight line intersecting the circle at P and R. T lies on the circle and TS is a tangent to the circle. Q lies on PS such that the line TQ bisects angle RTP.

(a) Prove that triangle STR and triangle SPT are similar. [2]

$\angle STR = \angle SPT$ (alt. segment theorem). — (M1)

$\angle TSR = \angle PST$ (common $\angle$).

By AA-similarity test, ΔSTR and ΔSPT are similar. — (A1)

(b) Prove that triangle QST is isosceles. [3]

$\angle STR = \angle SPT = \alpha$

$\angle RTQ = \angle PTQ = \beta$.

(with ref. to ΔTQP)

$\angle RQT = \alpha + \beta$ (ext. $\angle$ of Δ)

$= \angle SQT$. } (M1)

$\angle STQ = \angle STR + \angle RTQ$

$= \alpha + \beta$ } (M1)

Since $\angle STQ = \angle SQT = \alpha + \beta$,

ΔQST is an isos. Δ. } (A1)

7 (c) Prove that $SQ^2 - SR^2 = SR \times RP$. [3]

fr(a) $\frac{ST}{SP} = \frac{SR}{ST}$ — (M1)

$ST^2 = SR \times SP$.

$ST^2 = SR \times (SR + RP)$ — (M1)

$= SR^2 + SR \times RP$.

Since $ST = SQ$,

$SQ^2 = SR^2 + SR \times RP$

$SQ^2 - SR^2 = SR \times RP$ (A1)

8

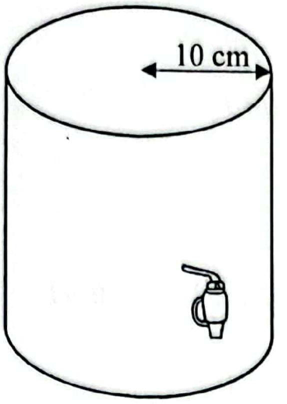


Diagram I

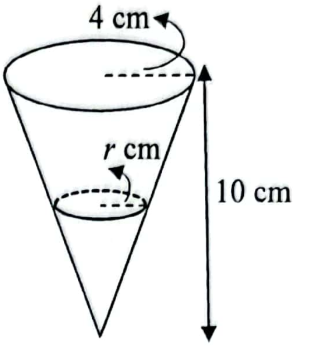


Diagram II

Diagram I shows a water dispenser, in the shape of a cylinder of radius 10 cm, which dispenses water at a constant rate into an empty conical cup, as shown in diagram II, of radius 4 cm and height 10 cm. The depth of the water in the dispenser decreases at a rate of 0.0015 cm/s.
After t seconds, radius of the horizontal surface of the water in the conical cup is r cm.

(a) Show that the volume of water in the conical cup increases at a rate of $\frac{3\pi}{20}$ cm³/s. [2]

Rate of water out from dispenser = Rate of increase of water in cup.

$= \pi(10\text{cm})(10\text{cm})(0.0015 \text{ cm/s})$ — (M1)

$= \frac{3\pi}{20} \text{ cm}^3/\text{s}$. — (A1)

8 (b) Express the volume of water in the conical cup in terms of r. [2]

$V = \frac{1}{3}\pi r^2 h$

$\frac{r}{4} = \frac{h}{10} \Rightarrow h = \frac{5r}{2}$ — (M1)

$V = \frac{1}{3}\pi r^2\left(\frac{5r}{2}\right)$

$= \frac{5\pi r^3}{6}$ — (A1)

(c) At the instant where the volume of the water in the conical cup is $\frac{20\pi}{3}$ cm^3, find the rate of change in the radius of the horizontal surface of the water in the conical cup. [4]

$\frac{dv}{dr} = \frac{5\pi}{6}(3r^2)$

$= \frac{5\pi r^2}{2}$ — (M1)

When $V = \frac{20\pi}{3}$,

$\frac{5\pi r^3}{6} = \frac{20\pi}{3}$ — (M1)

$r^3 = 8$

$r = 2$

$\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$

$\frac{3\pi}{20} = \frac{5\pi r^2}{2} \times \frac{dr}{dt}$ — (M1)

$\frac{dr}{dt} = \frac{3\pi}{20} \div \frac{5\pi r^2}{2}$

at $r = 2$, $\frac{dr}{dt} = \frac{3\pi}{20} \div \frac{5\pi(2)^2}{2}$

$= \frac{3}{200}$ — (A1)

9 The mass, m mg, of a radioactive substance decreases with time, t hours. Measured values of m and t are recorded in the following table.

t (hours)	2	4	6	8	10
m (mg)	48.2	41.5	35.5	33.7	26.5

m and t are related by the equation $m = m_o e^{-kt}$, where m_o and k are constants.

It is known that one of the readings in the table is recorded wrongly.

(a) On the grid found on page 17, plot ln m against t and draw a straight line graph. Use your graph to identify the reading which has been recorded wrongly. [3]

Wrong recording : $(8, 3.52)$
also accept: $m = 33.7$, $t = 8$ — B1

(b) Use your graph to estimate

(i) a value of m to replace the incorrect recording of m found in part **(a)**, [1]

$\ln m = 3.43$ (also accept : 3.42, 3.425, 3.435, 3.44)

$m = 30.9$ (3sf) (also accept : 30.6, 30.7, 31.0, 31.2) — (B1)

(ii) the value of k and m_o. [4]

$\ln m = \ln m_o e^{-kt}$

$\ln m = -kt + \ln m_o$ — (M1)

gradient: $-k$, y-int: $\ln m_o$.

$\ln m_o = 4.03$ (also accept 4.02, 4.025, 4.035, 4.04)

$m_o = 56.3$ (3sf) (also accept: 55.7, 56.0, 56.5, 56.8)

(A1)

$-k = \dfrac{3.85-3.45}{2.4-7.8}$ — (M1)

$k = \dfrac{2}{27}$ (accept from 0.0655 to 0.084)

(A1)

(c) Explain what m_o represents. [1]

m_o represents the amount of radioactive substance present at the start. (B1)

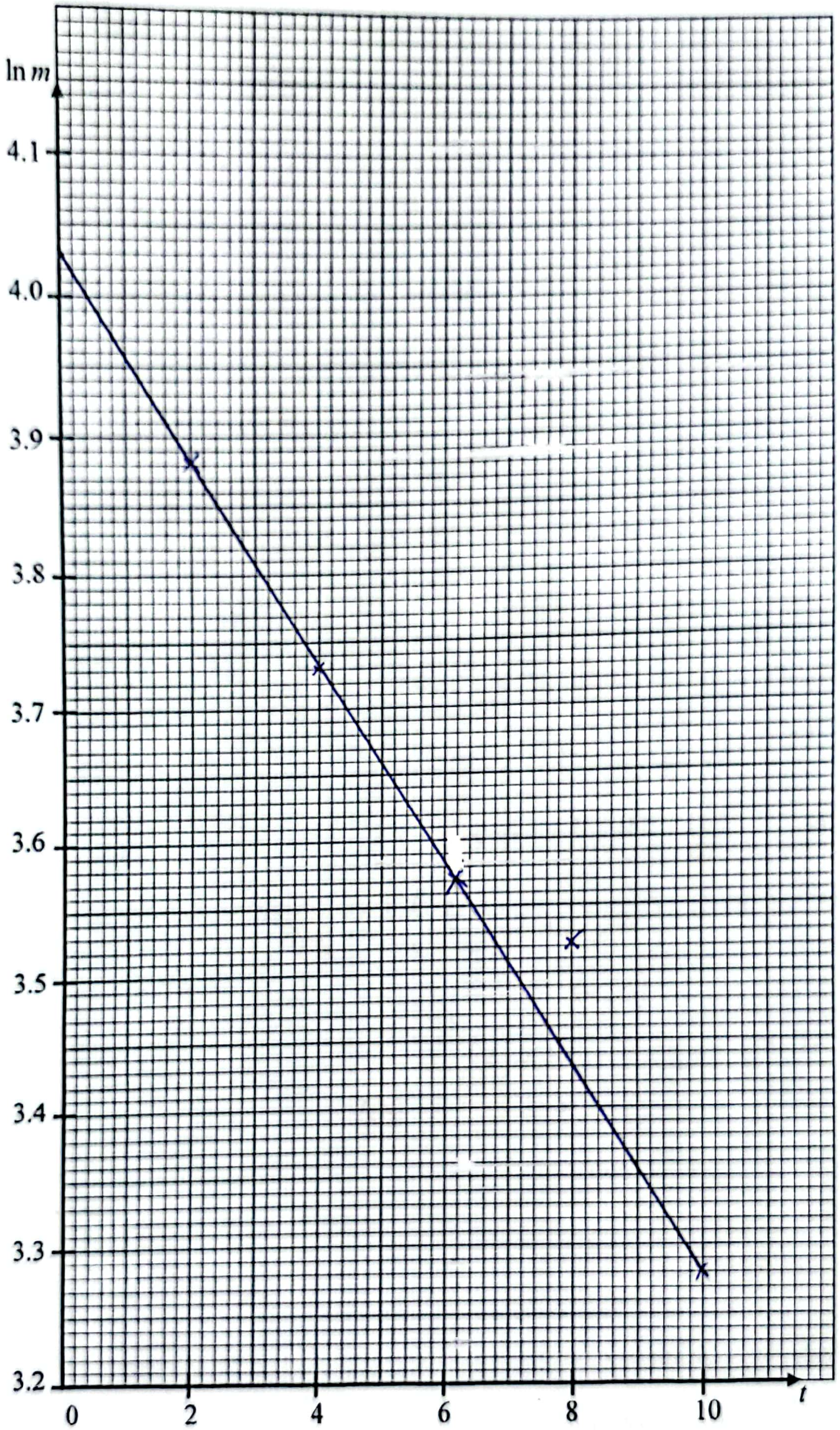
(d) Ben claims that the mass of the radioactive substance reaches 0 mg after 10 to 12 hours. Do you agree with Ben? Explain your answer. [1]

Disagree. $e^{-kt} > 0$ for all real val. of t.

$\therefore m = m_o e^{-kt} > 0$.

Since $m > 0$, there is no t such that $m = 0$. — B1.

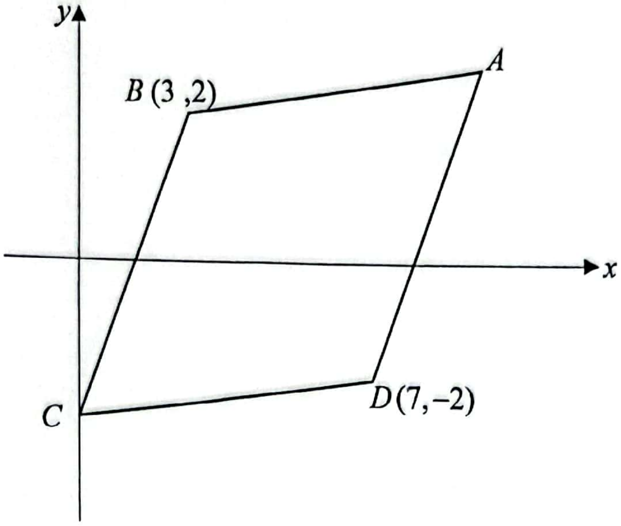
Alternative: extrapolating the graph such that it intersects x-axis does not result in the x-int. val. as $\ln m \neq 0$.



B1 - correct points plotted.

B1 - st. line joining all pts except 4th pt

10



The diagram shows a quadrilateral with vertices A, $B(3,2)$, C and $D(7,-2)$.
C lies on the y-axis.
AC is a perpendicular bisector of BD.

(a) Show that the equation of AC is $y = x - 5$. [3]

$m_{BD} = \dfrac{2-(-2)}{3-7} = -1$

m_{AC} : $(-1) \div (-1) = 1$. — (M1)

midpt of BD : $\left(\dfrac{3+7}{2}, \dfrac{2+(-2)}{2} \right)$ — (M1)

$= (5, 0)$

Eqn of AC : $y - 0 = 1(x-5)$

$y = x - 5$ — (A1)

(b) State the coordinates of C. [1]

$(0, -5)$ —(B1)

10 (c) Given that triangle ABD has an area of 16 units^2, find the coordinates of A. [4]

let coordinates of A be (x, y)

Area of $\triangle ABD$: $\frac{1}{2}\begin{vmatrix} x & 3 & 7 & x \\ y & 2 & -2 & y \end{vmatrix}$

$= \frac{1}{2}(2x - 6 + 7y - 3y - 14 + 2x)$ — (M1)

$= \frac{1}{2}(4x + 4y - 20)$

$= 2x + 2y - 10$.

$2x + 2y - 10 = 16$ — (M1)

$x + y = 13$ — ①

$y = x - 5$ — ②

Sub ② into ①: $x + x - 5 = 13$ — (M1)

$2x = 18$

$x = 9$

$\therefore y = 4$ $\therefore A(9, 4)$ — (A1)

(d) Explain whether $ABCD$ is a rhombus. [2]

$m_{AB} = \frac{4-2}{9-3} = \frac{1}{3}$

$m_{CD} = \frac{-2-(-5)}{7-0} = \frac{3}{7}$ (M1)

Since $m_{AB} \neq m_{CD}$, AB is not parallel to CD.
$\therefore$ ABCD is not a rhombus. (A1)

Alt.: $AB = \sqrt{(9-3)^2 + (4-2)^2}$

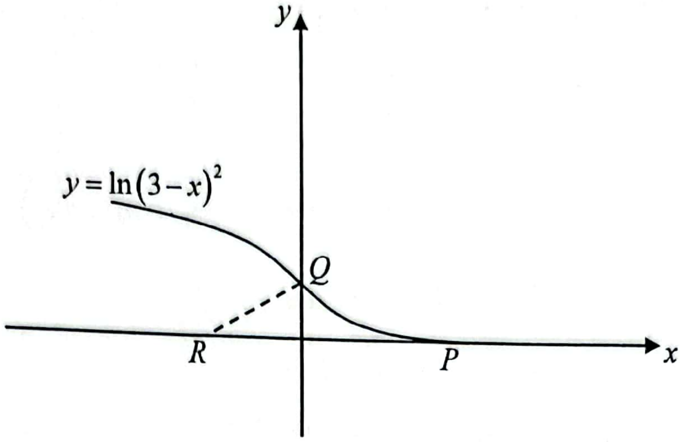
$= \sqrt{40}$

$CD = \sqrt{(7-0)^2 + (-2+5)^2}$ (M1)

$= \sqrt{58}$

Since $AB \neq CD$, not all sides are of equal length.
$\therefore$ ABCD is not a rhombus. (A1)

11 The diagram shows part of the curve $y = \ln(3-x)^2$ which crosses the axes at P and Q.



The normal to the curve at Q meets x-axis at R. Find the area of the region bounded by the curve, RQ and the x-axis.

[12]

$\ln(3-x)^2 = 0$
$(3-x)^2 = 1$
$3-x = 1 \text{ or } -1$
$x = 2 \text{ or } 4$
(M1) (rej)

Also accept

$2\ln(3-x) = 0$
$3-x = 1$
$x = 2$

$\therefore P(2, 0)$

at $x=0$, $y = \ln 9$ — (M1)

$\therefore Q(0, \ln 9)$

$\frac{dy}{dx} = 2\left(\frac{1}{3-x}\right)(-1)$

$= -\frac{2}{3-x}$ — (M1)

at $x=0$, $\frac{dy}{dx} = -\frac{2}{3}$.

$\therefore$ M normal : $\frac{3}{2}$ — (M1)

Equation of normal: $y = \frac{3}{2}x + \ln 9$ — (M1)

at $y=0$, $0 = \frac{3}{2}x + \ln 9$

$\frac{3}{2}x = -\ln 9$

$x = -\frac{2}{3}\ln 9$ — (A1)

$\therefore R\left(-\frac{2}{3}\ln 9, 0\right)$

Continuation of working space for question 11.

Area of ΔORQ: $\frac{1}{2} \times \ln 9 \times \frac{2}{3} \ln 9$ — (M1)

$= \frac{1}{3}(\ln 9)^2$

$= 1.6092$ } (A1, accept either)

$y = \ln(3-x)^2$

$\frac{y}{2} = \ln(3-x)$

$e^{\frac{y}{2}} = 3 - x$

$x = 3 - e^{\frac{y}{2}}$ — (M1)

∴ Area bounded by curve, x & y axes

$\Rightarrow \int_0^{\ln 9} 3 - e^{\frac{y}{2}} \, dy$

$= \left[3y - \frac{e^{\frac{y}{2}}}{\left(\frac{1}{2}\right)} \right]_0^{\ln 9}$ — (M1)

$= \left[3y - 2e^{\frac{y}{2}} \right]_0^{\ln 9}$

$= 3\ln 9 - 2e^{\frac{1}{2}\ln 9} - \left(3(0) - 2e^0\right)$

$= 3\ln 9 - 6 + 2$

$= 3\ln 9 - 4$ (also accept 2.5916) — (A1)

∴ total area: $(3\ln 9 - 4) + \frac{1}{3}(\ln 9)^2$

$= 4.20$ (3 sf) — (A1)

--- **End of Paper** ---