Candidate Name	Form Class	Index Number

ANG MO KIO SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2024
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

ADDITIONAL MATHEMATICS **4049/01**

Paper 1

26 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is **90**.

For Examiner's Use
90

This document consists of **23** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots\ldots\ \ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The curve $\dfrac{5x}{y} - \dfrac{12y}{x} = \dfrac{-7}{xy}$ and the line $5x - 2y = -7$ intersect at the points A and B. Find the coordinates of A and B. [5]

2 **(a)** Express $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}$ in the form $p+\sqrt{q}$ where p and q are integers. [2]

(b) The diagonals of a rhombus are $\sqrt{30}$ and $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}$ and the perimeter of the rhombus is P. Calculate P^2, giving your answer in the form of $a+b\sqrt{c}$ where a, b, and c are integers. [4]

3 The cost per submarine, \$$y$ in millions, of assembling x submarines can be modelled by $y=\frac{5}{2}x^2-20x+100$, where $x \le 8$.

(i) Explain the meaning of the constant term 100 in this model. [1]

(ii) Express $y=\frac{5}{2}x^2-20x+100$ in the form of $y=a(x-h)^2+k$. [2]

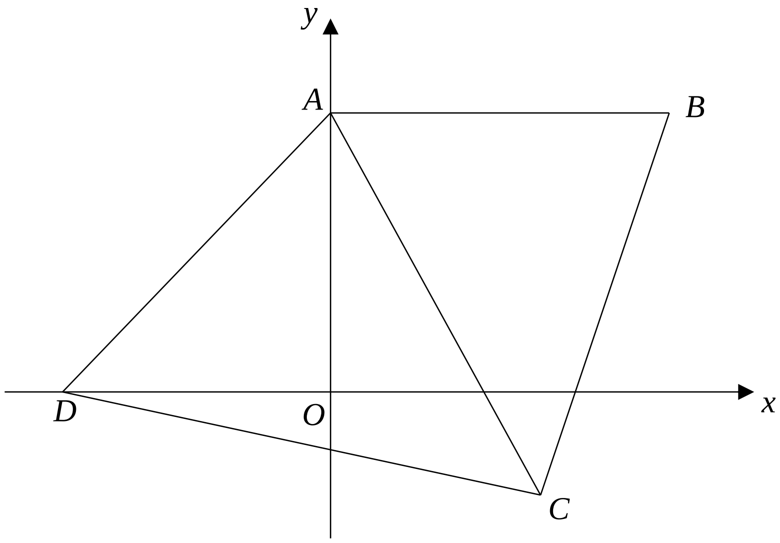
(iii) Explain the significance of $x = h$ in **(ii)**. [1]

4 **(a)** Solve the equation $\log_4 2y - \dfrac{\log_4(y-3)}{2} = 3\log_4 2$. [4]

(b) Given that $\log_{27} z = \log_9 \sqrt{y}$, express z in terms of y. [3]

5 Express $\dfrac{2x^3+6x^2+1}{(x-1)(x+2)^2}$ in partial fractions. [5]

6 The diagram below shows a quadrilateral *ABCD* in which *A* is (0, 6) and *AB* is parallel to the *x*-axis. *D* is a point on the *x*-axis such that the equation of *DC* is $x+5y=-6$. *AC* is perpendicular to the line $2y=x+7$.



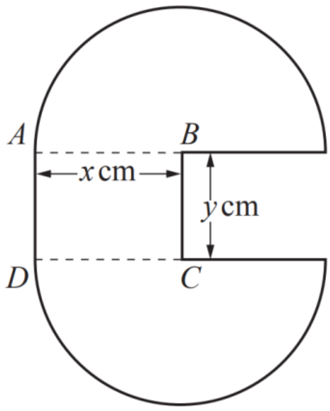
(a) Find the coordinates of point *C*. [4]

(b) Given that the area of triangle ACD is 1.5 times that of triangle ABC, find

(i) the coordinates of point B, [3]

(ii) the perpendicular distance from D to AC. [3]

7 The diagram shows a badge, made of thin sheet metal, consisting of two semi-circular pieces, centres B and C, each of radius x cm. They are attached to each other by a rectangular piece of thin sheet metal, $ABCD$, such that AB and CD are the radii of the semi-circular pieces and that $AD = BC = y$ cm.



(i) Given that the area of the badge is 40 cm^2, show that the perimeter, P cm, of the badge is given by $P = 2x + \dfrac{80}{x}$. [4]

(ii) Given that x can vary, find the stationary value of P, and determine whether this value is a maximum or a minimum. [6]

8 The roots of a cubic equation $F(x)=0$ are -1, 2 and 5. When $F(x)$ is divided by $x-3$, the remainder is 30.

(i) Find the remainder when $F(x)$ is divided by $x+3$. [4]

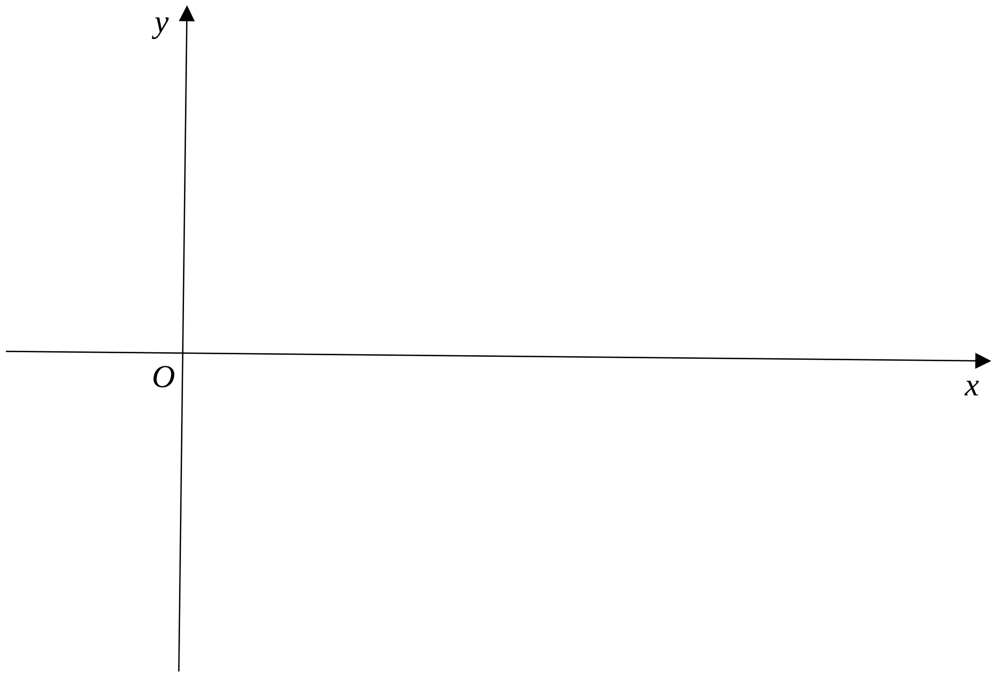
(ii) Solve the equation $F\left(\sqrt{m}\right) = 0$. [2]

9 It is given that $\mathrm{f}(x) = 3\cos 2x + 1$ and $g(x) = \sin\left(\dfrac{x}{2}\right)$.

(i) State the amplitude and period of $\mathrm{f}(x)$. [2]

(ii) Sketch, on the same axes, the graphs of $y = \mathrm{f}(x)$ and $y = g(x)$

for $0^\circ \leq x \leq 360^\circ$. [4]



(iii) State the value of k for which the equation

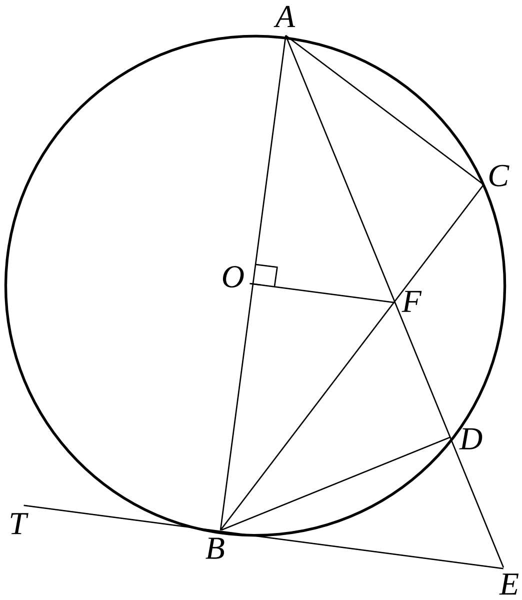
$3\cos 2x + 1 = \sin\left(\dfrac{x}{2}\right) + k$ has 7 solutions for $-360^\circ \leq x \leq 360^\circ$. [1]

10 In an industrial process, water flows at a constant rate of $p\,\text{cm}^3/\text{s}$ into a funnel. The water flows out through a small hole in the funnel at a constant rate of $q\,\text{cm}^3/\text{s}$, where $q < p$. The volume of the water in the funnel at time t seconds is $V\,\text{cm}^3$. Initially, the funnel is empty.

(i) Express $\frac{\mathrm{d}V}{\mathrm{d}t}$ in terms of p and q. [1]

(ii) It is given further that $V = \frac{1}{3}\pi h^3$, where h is the vertical height of the funnel in contact with water. In the case where $p = 90$ and $q = 10$, find the rate of change of the vertical height of the funnel in contact with the water, where $h = 5$. [3]

11 In the figure below, AB is a diameter of the circle with centre O. Chords AD and BC intersect at F. AD produced meets the tangent to the circle, TBE at E. AE is an angle bisector of angle BAC.



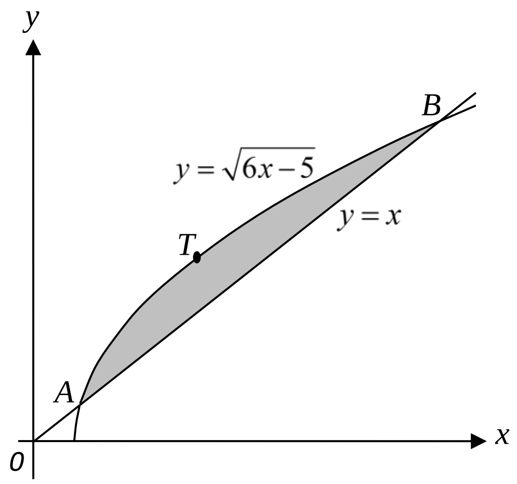
(a) Prove that $\angle CBD = \angle DBE$. [3]

Given that $\angle AOF = 90°$, prove that

(b) **(i)** triangle *AOF* is similar to triangle *ADB,* [2]

(ii) $2(AO)^2 = AF \times (AF + FD)$. [2]

12



The diagram above shows part of the curve $y=\sqrt{6x-5}$ intersecting the line $y=x$ at points A and B. A point T lies on the curve and the tangent at point T is parallel to the line $y=x$.

(i) Find the coordinates of T. [5]

(ii) Find the area of the shaded region enclosed by the line and curve. [6]

13 **(i)** Prove the identity $\dfrac{\sin\theta}{1-\cos\theta}-\dfrac{1}{\tan\theta}=\operatorname{cosec}\theta$. [4]

(ii) Hence, solve the equation $\dfrac{\sin 2A}{1-\cos 2A}-\dfrac{1}{\tan 2A}=9\sin 2A$ for $0 < A < \pi$. [4]

END OF PAPER

$\dfrac{\sin\theta}{1-\cos\theta}-\dfrac{1}{\tan\theta}=9\sin\theta$

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