NATIONAL JUNIOR COLLEGE
SENIOR HIGH 1
Promotional Examinations

NAME	

SUBJECT CLASS	1ma2	REGISTRATION NUMBER	

H2 MATHEMATICS 9758

3 October 2022

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

This paper constitutes 50% of your overall score for SH1 H2 Mathematics.

Write your name, class and registration number in the boxes above.
Please write clearly and use capital letters.

Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.
Up to 2 marks may be deducted for improper presentation.

The number of marks is given in the brackets [ ] at the end of each question or part question.

Question Number	Marks Possible	Marks Obtained
1	3	
2	6	
3	6	
4(a)	4	
4(b)	3	
5	8	
6	9	
7	10	
8	12	
9	12	
10	15	
11	12	
Presentation Deduction		– 1 / –2
TOTAL	100	

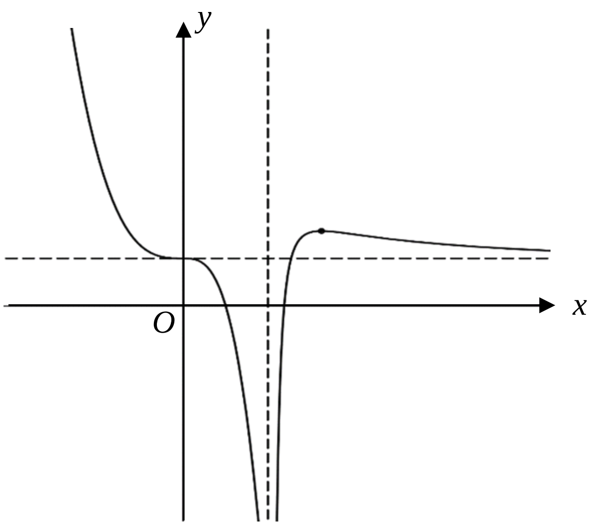
This document consists of **23** printed pages and **5** blank pages.

1 Ben, Caleb and Dylan went to the supermarket in the morning to buy salmon, tuna and swordfish. Ben paid \$246 for 2 kg of salmon, 1 kg of tuna and 3 kg of swordfish. Caleb bought 1 kg of salmon and 1 kg of swordfish. Caleb paid \$18 more than Dylan who bought 1 kg of tuna.

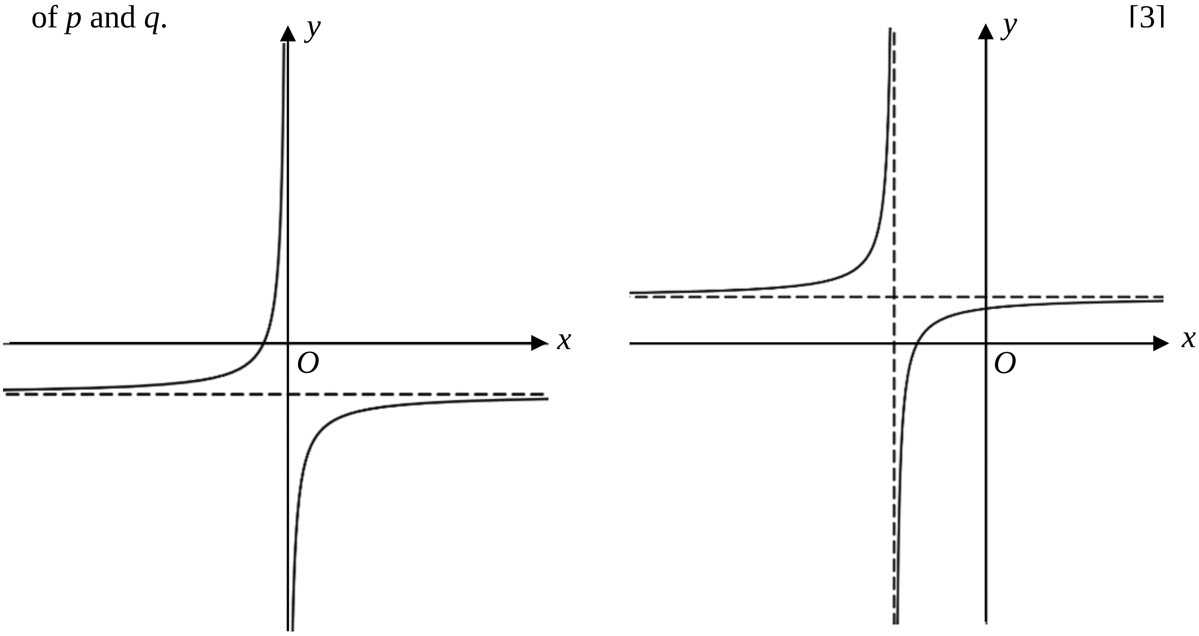
After 12pm, a discount of 15%, 10% and 5% is given for salmon, tuna and swordfish respectively such that the total cost of 1 kg of salmon, 1 kg of tuna and 1 kg of swordfish after discount is \$123.30.

Find the selling price of 1 kg of salmon, tuna and swordfish respectively before discount. [3]

2 **(a)** The diagram shows the graph of $y = \mathrm{g}(x)$ with a stationary point of inflexion at $(0,3)$ and a maximum point at $(4,5)$. The asymptotes of the graph are $x = 2$ and $y = 3$. On a separate diagram, sketch the graph of $y = \mathrm{g}'(x)$, labelling the equations of any asymptotes and the coordinates of any points where the curve crosses the axes. [3]



2 **(b)** The diagram shows the graphs of $y = \mathrm{h}(x)$ and $y = \dfrac{1}{\mathrm{h}(-x-p)} + q$, where p and q are constants. The curve $y = \mathrm{h}(x)$ cuts the axes at $(-1, 0)$ and has asymptotes $x = 0$ and $y = -2$. The curve $y = \dfrac{1}{\mathrm{h}(-x-p)} + q$ has asymptotes $x = -2$ and $y = 1$. Find the values of p and q. [3]



3 **(i)** On the same axes, sketch the graphs of $y = \dfrac{x-b}{x}$ and $y = \dfrac{1}{a}|x-b|$, where $0 < a < 1$ and $b > 1$. State the equations of any asymptotes and the coordinates of the points where the curves cross the axes. [3]

(ii) Hence solve the inequality $\dfrac{x-b}{x} \leq \dfrac{1}{a}|x-b|$. [3]

4 **(a)** Find $\displaystyle\int \frac{x+1}{4+3x^2}\,\mathrm{d}x$. [4]

(b) The region bounded by the curve $(y-1)^2 = x$ and the line $x = 1$ is rotated through 2π radians about the x-axis. Find the volume of the solid obtained, giving your answer correct to 4 decimal places. [3]

5 Referred to the origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. The points O, A, B and C do not lie on the same plane. In triangle ABC, points Q and R are the midpoints of BC and AC respectively.

(i) Find a vector equation of the line BR in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. [2]

(ii) Given that line AQ has equation $\mathbf{r} = \mathbf{a} + \mu(-2\mathbf{a} + \mathbf{b} + \mathbf{c}), \mu \in \mathbb{R}$. Hence find, in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, the position vector of the point G where the lines BR and AQ meet. [2]

(iii) It is given further that the point S has position vector $2\mathbf{b} - \mathbf{a}$ and $\angle AOB = \frac{2\pi}{3}$. Find, in terms of $|\mathbf{a}|$ and $|\mathbf{b}|$, the length of projection of $\overrightarrow{OS}$ onto $\overrightarrow{OA}$. [4]

6 **(i)** Use the substitution $x = m\tan t$ to find $\int \frac{1}{\sqrt{m^2 + x^2}}\,\mathrm{d}x$, where m is a positive constant and $0 \le t < \frac{\pi}{2}$. [4]

(ii) Find $\int \frac{x}{\sqrt{m^2 + x^2}}\,\mathrm{d}x$. [2]

(iii) Hence, find $\int \frac{x}{\sqrt{m^2 + x^2}}\tan^{-1}\left(\frac{x}{m}\right)\mathrm{d}x$. [3]

7 A curve C has equation $3x^2 + 2xy - y^2 + k = 0$ where k is a non-zero constant.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x + y}{y - x}$. [2]

(ii) Find the range of values of k such that C has no stationary points. [3]

(iii) It is given that $k < -1$. The lines $x = \alpha$ and $x = \beta$ are tangent to C, where $\alpha < \beta$. Find α and β in terms of k. Hence show that $x = \beta$ does not meet C again. [5]

8 The function f is defined by

$$f(x)=\begin{cases} x-4 & \text{for } \quad x \le 1, \\ \cos x & \text{for } \quad 1 < x \le \pi. \end{cases}$$

(i) Show that f has an inverse. [3]

(ii) Find f^{-1}. [3]

(iii) Evaluate $f^2\left(\frac{\pi}{2}\right)$ and find f^2. [4]

(iv) Find $f^{2022}(0)$. [2]

9 A curve D has parametric equations

$$x = k\left(2\cos\theta - \cos 2\theta\right)$$
$$y = -k\sin 2\theta$$

for $0 < \theta \le \pi$, where k is a positive constant.

(i) Find, in terms of k, the exact coordinates of the stationary points of D. [3]

(ii) Sketch the graph of D, labelling, in terms of k, the points where D meets the x-axis. [2]

(iii) Show that the area enclosed by the x-axis and the part of D above the x-axis, is given by

$$k^2\int_{\theta_1}^{\theta_2}\left(2\sin^2 2\theta - 2\sin\theta\sin 2\theta\right)\,d\theta,$$

where θ_1 and θ_2 should be stated. [3]

(iv) Hence find, in terms of k, the exact area enclosed by the x-axis and the part of D above the x-axis. [4]

10 A player controls an avatar in an exploration game to find treasures. In one of the stages, the avatar scales a cliff. The sloping surfaces of the cliff can be modelled by plane p_1 with equation $x-5z=-2$ and plane p_2 with equation $4x+y-nz=30$, where n is a constant. The top surface of the cliff can be modelled by plane p_3 with equation $x-12z=-450$ which contains the point $T(30,\ 30,\ 40)$, where a treasure is located. The figure below shows a side view of the cliff.

Locations of points $(x,\ y,\ z)$ are defined relative to a point $(0,\ 0,\ 0)$ at the foot of the cliff and it is assumed that the avatar travels from a point to another using the shortest path. The avatar expends 1 unit of stamina for every 2 units travelled.

(i) The ground in the game can be modelled by the plane with equation $z=0$. Given that p_2 is steeper than p_1 relative to the ground, find the range of values of n^2. [4]

It is given further that $T(30,\ 30,\ 40)$ also lies in p_2.

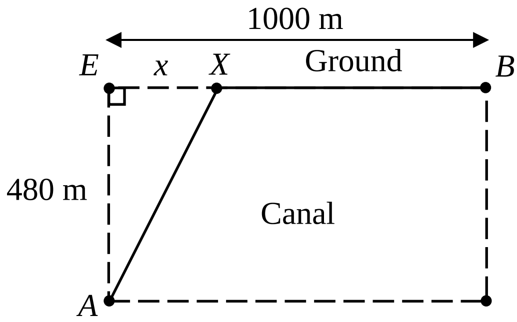
(ii) Show that $n=3$. [1]

(iii) The boundary where p_1 and p_2 meet can be modelled by the line l. Verify that a vector equation of boundary l is $\mathbf{r}=\begin{pmatrix}8\\4\\2\end{pmatrix}+\lambda\begin{pmatrix}5\\-17\\1\end{pmatrix},\lambda\in\mathbb{R}$. [2]

(iv) The player resumes the game with the avatar on p_1. The avatar moves towards B, a point on boundary l such that BT is minimum. Find the exact position vector of B. Hence, find the total stamina expended by the avatar to travel from B to T. [5]

(v) From the point T, the avatar travels along a path parallel to $\begin{pmatrix}10\\3a\\9a\end{pmatrix}$, where a is a constant. Another treasure is located at $R(90,\ 0,\ 45)$ on p_3. Determine whether the avatar will eventually reach this treasure. [3]

11 Engineers need to lay pipes to connect two factories A and B that are separated by a canal of uniform width 480 m as shown in the diagram. They plan to lay the pipes under the canal in a line from A to X and then under the ground in a line from X to B. The cost of laying the pipes under the canal is five times the cost of laying the pipes under the ground.



The point E is such that AE is perpendicular to the line XB, where EB has a length of 1000 m. X is between E and B such that EX has a length of x m. The cost per metre of laying the pipes under the ground is 180 dollars.

(i) Show that the total cost C, in dollars, of laying the pipes from A to B is given by $C = 900\sqrt{230400 + x^2} + 180(1000 - x)$. [2]

(ii) In the context of the question, find the range of values of x such that $\frac{\mathrm{d}C}{\mathrm{d}x} > 0$. [3]

Let the angle at which the pipes are joined, $\angle AXB$, be θ. For engineering reasons, $\frac{2\pi}{3} \le \theta \le \frac{3\pi}{4}$.

(iii) State an equation relating θ and x. Hence find the range of values of x when $\frac{2\pi}{3} \le \theta \le \frac{3\pi}{4}$. [3]

(iv) By considering part **(ii)** and part **(iii)**, show that $\frac{\mathrm{d}C}{\mathrm{d}\theta} > 0$ for $\frac{2\pi}{3} \le \theta \le \frac{3\pi}{4}$. [3]

(v) Hence, deduce the value of θ which gives the minimum total cost. [1]